Tutorial 4

1. $X(t)$ and $A(t)$ are independent random variables, one of them with 0 mean. Assume $Y(t) = X(t) + A(t)$. Find the autocorrelation relationship of $Y(t)$, $X(t)$ and $A(t)$.

2. Consider a stationary scalar output process $\{y_k\}$ and vector state process $\{x_k\}$ governed by the equations:

$$\begin{cases} x_{k+1} = A \cdot x_k + b \cdot e_k \\ y_k = c^T \cdot x_k \end{cases}$$

Here, A is a given $n \times n$ matrix and b and c are given $n-$vector. $\{e_k\}$ is a sequence of zero mean, uncorrelated random variables, each with unit variance. Develop formulae for the covariance matrix of x_k and the variance of y_k :

$$R_x(0) = E[x_k x_k^T] \text{ and } R_y(0) = E[y_k^2]$$

3. Speech Predictor: Let $X_1, X_2, X_3, \ldots$ be a sequence of samples of a speech voltage waveform. Suppose that $\{x_k\}$ represents a stationary, second order, stochastic process where $E[x_k] = 0$ and $E[x_k^2] = \sigma^2 > 0$ for all $k = 1, 2, 3\ldots$ The sample X_{k-2} and X_{k-1} are used to predict X_k with the least mean square error. That is, for $k = 3, 4, 5\ldots$

$$\hat{X}_k = aX_{k-1} + bX_{k-2}$$

is the least squares predictor of X_k where a and b are constants.
Find a and b in terms of the variance σ^2 and covariances of $\{x_k\}$

4. Given the stationary property of the process, the covariance of $\{X_k\}$ depend on the "distance" between the time indices, but not on the specific index values. Following above question, we define for all i and j :

$$\text{cov}(X_i, X_j) = \rho_{|i-j|} \cdot \sigma^2$$

where $\rho_k > 0$ and $\rho_0 = 1$. (For example, $\text{cov}(X_i, X_{i+2}) = \rho_2 \cdot \sigma_X^2$) Find a and b in terms of ρ_1 and ρ_2.