Solution 2

1. Solution: $A-B=A\bar{B}=A(\Omega-B)=A\Omega-AB=A-AB$

$$(A\bigcup B)-B=(A\bigcup B)\bigcap\bar{B}=A\bar{B}\bigcup B\bar{B}$$
$$=(A-B)\bigcup\phi=A-B$$

2. Yes. Since $A\subset B$, P(A$\bigcap$B)=P(A). This is equal to $P(A)\cdot P(B)$ when P(B)=1 or P(A)=0

3. $P(Y=1)=\frac{1}{3}$, $P(Y=2)=\frac{2}{3}$, then $E[Y]=1\cdot\frac{1}{3}+2\cdot\frac{2}{3}=\frac{5}{3}$.

4. Let $x_i=\begin{cases}1, & \text{if } i^{th} \text{ man selects his own hat}\\ 0, & \text{otherwise}\end{cases}$.

$$X=\sum_{i=1}^{N}x_i \quad\Rightarrow\quad E[X]=\sum_{i=1}^{N}E[x_i],$$

For each i, $E[x_i]=\frac{1}{N}\cdot 1+\frac{N-1}{N}\cdot 0=\frac{1}{N}$ $\quad\Rightarrow E[x]=\sum_{i=1}^{N}E[x_i]=N\cdot\frac{1}{N}=1$

5. $P(R=1/S=1)=0.9$, $P(R=1/S=0)=0.05$, $P(S=0)=0.6,\quad P(S=1)=0.4$

According to Bayes' rule,

$$P(S=1/R=1)=\frac{P(R=1/S=1)\cdot P(S=1)}{P(R=1)}$$
$$=\frac{P(R=1/S=1)\cdot P(S=1)}{P(R=1/S=1)\cdot P(S=1)+P(R=1/S=0)\cdot P(S=0)}$$
$$=\frac{0.9\cdot 0.4}{0.9\cdot 0.4+0.05\cdot 0.6}=0.923$$