Tutorial 2

1. Show: $A - B = A - AB$, $(A \cup B) - B = A - B$

2. Let A and B be events such that $A \subset B$. Can A and B be independent?

3. Let X be uniformly distributed in the unit interval [0,1]. Consider the random variable Y=g(x), where

$$g(x) = \begin{cases} 1, & \text{if } x \leq 1/3 \\ 2, & \text{if } x > 1/3 \end{cases}$$

   Find the expected value of Y.

4. At a party, N men throw their hats in the center of a room. The hats are mixted up, and each man takes turn to randomly select one and then returns it to the center. Assume that all hats are different. Find the expected number of men that select their own hats.

5. A communication link transmits digits of 0 and 1. With probability 0.9, the receiver can correctly detect a digit 1 when it is in fact sent. However, with probability of 0.05, the receiver incorrectly detects a digit 1 when a digit 0 is in fact sent. If digits 0 and 1 are actually sent with probability 0.6 and 0.4, respectively, what is the probability the sent digit is 1, given that the receiver does detect a digit 1? $= (A - B) \cup \phi = A - B$ (Hint: Use Bayes' rule)