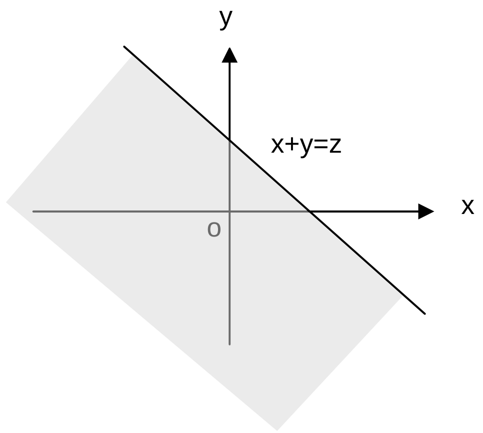
1. Probability desity function (pdf) of Z=X+Y:

Solution: Suppose the joint pdf is $f(x,y)$, then PDF of Z=X+Y is:

$$F_z(z) = P\{X+Y \le z\} = \int\int_{x+y\le z} f(x,y)dxdy$$

As can be seen from figure below, the shadow half is integral region:



According to the figure above, PDF of Z could be:

$$F_z(z) = \int_{-\infty}^{+\infty}[\int_{-\infty}^{z-y} f(x,y)dx]dy$$

Let's fix z and y, let x=u-y, then

$$F_z(z) = \int_{-\infty}^{+\infty}[\int_{-\infty}^{z} f(u-y,y)du]dy$$

$$= \int_{-\infty}^{z}[\int_{-\infty}^{+\infty} f(u-y,y)dy]du$$

Therefore, pdf of Z is: (x and y are symmetric)

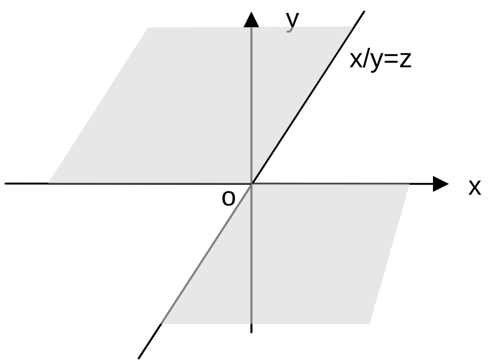
$$f_z(z) = F_z'(z) = \int_{-\infty}^{+\infty} f(z-y,y)dy \text{ or } = \int_{-\infty}^{+\infty} f(x,z-x)dx$$

2. Probability desity function (pdf) of Z=X/Y:

Solution: Suppose the joint pdf is $f(x,y)$, then PDF of Z=X/Y is:

$$F_z(z) = P\{X/Y \le z\} = \int\int_{x/y\le z} f(x,y)dxdy$$

The shadow integral region is shown below:



According to the figure above, PDF of Z could be:

$$F_z(z) = \int_{-\infty}^{0}[\int_{yz}^{+\infty} f(x,y)]dy + \int_{0}^{+\infty}[\int_{-\infty}^{yz} f(x,y)dx]dy$$

Let's fix z and y, let u=x/y, then x=uy, $dx = y \cdot du$

$$F_z(z) = \int_{-\infty}^{0} [\int_{z}^{+\infty} y \cdot f(uy, y) du] dy + \int_{0}^{+\infty} [\int_{-\infty}^{z} y \cdot f(uy, y) du] dy$$

$$= \int_{-\infty}^{z} [\int_{0}^{+\infty} y \cdot f(uy, y) dy - \int_{-\infty}^{0} y \cdot f(uy, y) dy] du$$

Finally, we have:

$$f_z(z) = \int_{0}^{+\infty} y \cdot f(zy, y) dy - \int_{-\infty}^{0} y \cdot f(zy, y) dy$$

$$= \int_{-\infty}^{+\infty} |y| \cdot f(zy, y) dy$$