Tutorial 1

1. Express each of the following events in terms of the events A, B and C as well as the operations of complementation, union and intersection:
(a) at least one of the events A, B, C occurs;
(b) at most one of the events A, B, C occurs;
(c) none of the events A, B, C occurs;
(d) all three events A, B, C occur;
(e) exactly one of the events A, B, C occurs;
(f) events A and B occur, but not C;
(g) either event A occurs or, if not, then B also does not occur.

2. Alice and Bob each choose at random a number between zero and one according to the uniform probability law. Consider the following events:
A = {The magnitude of the difference of the two numbers is greater than 1/3.}
B = {At least one of the numbers is greater than 1/4.}
C = {The sum of the two numbers is 1.}
Find the following probabilities: P(A), P(B), P(C).

3. David, an IC Freshman, makes one to five new friends every week, with equal probability. The number of friends he makes during each week is independent from all other weeks. We are concerned with two consecutive weeks.
Let event A be “David made a total of 10 friends during the two weeks”. Let event B be “David made more than five friends during the two weeks”.
(a). Are event A and B independent?
(b). Let C be the event “David made exactly 5 friends during the first week”. Are A and B independent, conditioned on C?

4. X and Y are independent random variables and both of them are uniform distributed within [-a, a]. What is the probability density function of Z=X+Y?

5. Let X be a point chosen randomly within a disc of radius 5. Let Y be the distance from X to the center of the disc. Assuming X is uniformly distributed in the disc, find pdf, CDF, expected value and variance of Y.