Error Performance Bounds for Routing Algorithms in Wireless Cooperative Networks

Zhengguo Sheng*, Zhiguo Ding†, Kin K Leung*
*Department of Electrical and Electronic Engineering, Imperial College, UK
†School of Electrical, Electronic and Computer Engineering, Newcastle University, UK
Email: {zhengguo.sheng06,kin.leung}@imperial.ac.uk, z.ding@newcastle.ac.uk

Abstract—**Cooperative diversity has emerged as a promising approach to improving reception reliability by realizing spatial diversity gains for nodes with single antenna. We consider here cooperative ad-hoc wireless networks where communications between two nodes can be assisted by a single relay using two time slots. This paper continues our investigation of PHY techniques and cross-layer routing algorithms in such networks. Specifically, we investigate here the optimal relay location for cooperative link in networks with infinite node density. By using this result, we analyze the error performance bound for routing algorithms in infinitely dense networks. Furthermore, we study the performance bounds for regularly dense networks with linear topology. Theoretical analysis shows that the proposed routing algorithm performs close to the optimal error performance.**

I. INTRODUCTION

Cooperative transmission (CT) has gained much attention as an effective technique to combat multi-path fading and enhance receiver reliability in wireless communication systems [1], [2]. The key feature of cooperative transmission is to encourage single-antenna devices to share their antennas cooperatively such that a virtual antenna array can be constructed and, hence, reception reliability can be boosted significantly.

Two common approaches to cooperative diversity are amplify-and-forward (AF) and decode-and-forward (DF). The first scheme can be viewed as repetition coding from two separate transmitters, except that the relay transmitter amplifies its own receiver noise. While for the second scheme, the relay fully decodes and retransmits the received signal to the destination (and possibly transmits decoding errors). The destination can employ a variety of combining techniques to achieve diversity gain from cooperation. Due to the enhanced performance, we explore the decode-and-forward scheme [2] in this work. Although various cooperative transmission schemes [3]–[6] have been developed to further improve the bandwidth efficiency of cooperative diversity, it is still not clear how such performance gain at the physical layer can benefit the upper layers, which is the major issue to be explored in this paper.

The performance analysis of cooperative networks has revealed many interesting results including diversity gain [2], [7], power reduction [8]–[10] and outage probability for CT over Rayleigh-fading channels [2], [11]. For sufficiently high signal-to-noise ratios, this paper derives the outage probability for a single decode-and-forward cooperative link as well as a route composed of two or more such links. The problem of optimal relay location is then investigated and further evaluations on different routing algorithms are shown at the end.

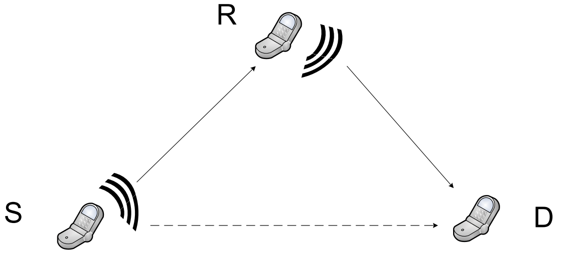

Fig. 1. Illustration of a cooperative link

The outline of this paper is organized as follows: in Section II, the system and channel model used for cooperative transmission is defined and the optimal relay location problem for a single link is analyzed for the networks with infinite node density. Section III describes the proposed cooperative routing algorithm and compares it with optimal solution. Section IV is routing performance evaluation, which we compare the outage performance of two algorithms from two different network scenarios. Section V provides simulation results and the paper is concluded in Section VI.

II. SYSTEM MODEL AND OUTAGE PROBABILITY

We consider a cooperative network in Figure 1, where a source node communicates with a destination node with the help of one relay. Each node is equipped with a omnidirectional antenna. Here, relay transmission is a main feature of cooperative communication.

Such transmissions employ an identical transmission power for both source and relay and are carried out using two time slots as follows. In the first time slot, the source broadcasts its data to the relay and the destination. In the second time slot, the relay transmits the signal it received in the previous time slot, if the SNR exceeds a threshold; otherwise, the source retransmits the signal. Thus an ACK from relay to source is assumed. Two time slots are used to transmit and relay a given data signal to avoid RF capture effects when simultaneously transmitting and receiving in the same frequency band. As a

---

This research was sponsored by US Army Research laboratory and the UK Ministry of Defence and was accomplished under Agreement Number W911NF-06-3-0001. The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the US Army Research Laboratory, the U.S. Government, the UK Ministry of Defense, or the UK Government. The US and UK Governments are authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation hereon.

result, the destination receives two independent copies of the same packets transmitted through different wireless channels, from which diversity gain can be achieved by combining the data copies using MRC (Maximum Ratio combining). The process repeats when the source starts to send new data in the third time slot to the relay and destination. A detailed time schedule is provided in Figure 3. In essence, data communication from the source through the relay to the destination is accomplished by this cooperative link (CL), which consists of the direct link (dashed line) between the source and the destination, and the links (solid lines) going through the relay.

It is shown in [2] that the full second-order of diversity can be achieved from such CL strategy. Such cooperative communications also provide significant improvement to reception reliability, which becomes an important criterion to measure the performance of various cooperative transmission schemes as will be examined in the rest of this paper.

Our channel model incorporates path loss and Rayleigh fading as follows

$$y_j = \mathrm{a}_{ij} x_i + n_j \tag{1}$$

where x_i is the signal transmitted by the transmitter and n_j is additive white Gaussian noise, with variance σ_n^2, at the receiver. The channel gain a_{ij} between the nodes i and j is modelled as $\mathrm{a}_{ij} = h_{ij}/d_{ij}^{k/2}$, where d_{ij} is the distance between the nodes i and j, k is the path-loss exponent and h_{ij} captures the channel fading characteristics. In addition, the channel fading parameter h_{ij} is assumed to be independent and identically distributed (i.i.d), complex Gaussian with zero mean and unit variance, across pairs of time slots.

A. Direct Transmission

To establish baseline performance, direct transmission is considered in our system model. The channel capacity between a source S and a destination D is

$$I_D = \log(1 + \rho |\mathrm{a}_{s,d}|^2) \tag{2}$$

where $\rho = E_b/N_0$ is defined as the normalized transmission power. Since for Rayleigh fading, $|\mathrm{a}_{s,d}|^2$ is exponentially distributed with parameter $d_{s,d}^k$. Thus, the outage probability satisfies

$$\begin{aligned} p_D^{out} = \Pr[I_D < R] &= 1 - \exp\left(-\frac{(2^R - 1) d_{s,d}^k}{\rho}\right) \\ &\approx d_{s,d}^k \left(\frac{2^R - 1}{\rho}\right) \end{aligned} \tag{3}$$

for large ρ. Where R is the desired data rate in bit/s/Hz, which is defined by the quality of service (QoS) requirement.

B. DF Cooperative Transmission

Let $d_{s,d}$, $d_{s,r}$ and $d_{r,d}$ to be the respective distances among the source, relay and destination. During the first time slot, the destination receives $y_{d,1} = \frac{h_{s,d}}{d_{s,d}^{k/2}} x_s + n_d$ from the source node, where x_s is the information transmitted by the source and n_d is white noise. During the second time slot, the destination node receives

$$y_{d,2} = \begin{cases} \frac{h_{s,d}}{d_{s,d}^{k/2}} x_s + n_d, & \text{if } |\frac{h_{s,r}}{d_{s,r}^{k/2}}|^2 < f(\rho) \\ \frac{h_{r,d}}{d_{r,d}^{k/2}} x_r + n_d, & \text{if } |\frac{h_{s,r}}{d_{s,r}^{k/2}}|^2 \geq f(\rho) \end{cases} \tag{4}$$

where $f(\rho) = (2^{2R} - 1)/\rho$ can be derived from direct transmission and is analogous to (3). In this protocol, the relay transmits only if the SNR exceeds a threshold; otherwise, the source retransmits in the second time slot. We thus implicitly assume a mini-slot at the beginning of the second slot during which ACKs are sent error-free from relay to source.

Consider that a relay node is selected and can perform perfect decoding when the received SNR exceeds a threshold, the channel capacity of this cooperative link can be shown as

$$I_C = \begin{cases} \frac{1}{2}\log(1 + 2\rho |\mathrm{a}_{s,d}|^2), & |\mathrm{a}_{s,r}|^2 < f(\rho) \\ \frac{1}{2}\log(1 + \rho |\mathrm{a}_{s,d}|^2 + \rho |\mathrm{a}_{r,d}|^2), & |\mathrm{a}_{s,r}|^2 \geq f(\rho) \end{cases} \tag{5}$$

It is worth noting that the same noise variance is assumed at both relay and destination. Therefore, the outage event is given by $I_C < R$ and the outage probability becomes

$$\begin{aligned} & p_C^{out} = \Pr[I_C < R] \\ & = \Pr[|\mathrm{a}_{s,r}|^2 < f(\rho)] \Pr[2|\mathrm{a}_{s,d}|^2 < f(\rho)] \\ & + \Pr[|\mathrm{a}_{s,r}|^2 \geq f(\rho)] \Pr[|\mathrm{a}_{s,d}|^2 + |\mathrm{a}_{r,d}|^2 < f(\rho)] \end{aligned} \tag{6}$$

By computing the limit, we obtain a closed form expression of (6)

$$\begin{aligned} \frac{1}{f^2} p_C^{out} &= \underbrace{\frac{1}{f}\Pr[|\mathrm{a}_{s,r}|^2 < f]}_{\mathbf{1}} \underbrace{\frac{1}{f}\Pr[2|\mathrm{a}_{s,d}|^2 < f]}_{\mathbf{2}} \\ & + \underbrace{\Pr[|\mathrm{a}_{s,r}|^2 \geq f]}_{\mathbf{3}} \underbrace{\frac{1}{f^2}\Pr[|\mathrm{a}_{s,d}|^2 + |\mathrm{a}_{r,d}|^2 < f]}_{\mathbf{4}} \end{aligned} \tag{7}$$

where $f = f(\rho)$, $\mathbf{1} \longrightarrow d_{s,r}^k$, $\mathbf{2} \longrightarrow d_{s,d}^k/2$, $\mathbf{3} \longrightarrow 1$, $\mathbf{4} \longrightarrow d_{s,d}^k d_{r,d}^k/2$. Because $f(\rho) = (2^{2R} - 1)/\rho$, we have the outage probability between the source and the destination

$$p_C^{out} = \frac{1}{2} d_{s,d}^k (d_{s,r}^k + d_{r,d}^k) \frac{(2^{2R} - 1)^2}{\rho^2} \tag{8}$$

It is worth noting that outage probability is actually a lower bound of bit error rate and will be used throughout the paper.

C. Optimal Relay Location for CL

Through the above definition, it is clear that relay selection is crucial for the performance of cooperative transmission. This is so because a good quality relay yields strong multi-user diversity gain, thus potentially enhancing the system performances (i.e., outage probability, transmission power and data rate). To derive the error performance bounds, we assume here that the network under consideration has infinite node density. We first develop a theoretical analysis to provide new insights into the optimal relay location that minimizes the outage probability. We then provide simulation results to

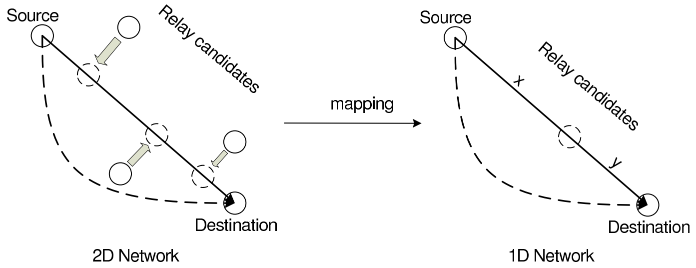

Fig. 2. Network mapping

illustrate the effects on system performance by using different relays.

For the assumed infinitely dense two-dimensional network, which allows the selection of relay node at any location, the following optimization problem can help us determine the optimal relay to minimize the outage probability for a given pair of source and destination nodes.

First of all, it is clear from (8) that better outage performance is always achieved by selecting a relay node on the straight line connecting the source and the destination. Therefore, with the infinite node density (i.e., nodes exist everywhere), the optimal relay could be only located on the line between source and destination and the problem in a two-dimensional network is equivalent to the problem in a single-dimensional network, which is shown in Figure 2. The optimization problem is to find the optimal relay that minimizes the end-to-end outage probability p_{out} of cooperative link with the constraint on the distances among the associated nodes, as formulated as follows

$$\begin{cases} \min & p_{out} = \frac{(2^{2R}-1)^2}{2\rho^2} D^k (x^k + y^k) \\ \text{s.t.} & x + y = D \end{cases} \tag{9}$$

where x, y and D are distance among the source-relay, relay-distance and source-destination, respectively, which is shown in Figure 2. For further simplification, the problem in (9) leads to

$$\begin{cases} \min & x^k + y^k \\ \text{s.t.} & x + y = D \end{cases} \tag{10}$$

By applying the Lagrangian multipliers to this problem, we obtain

$$L = x^k + y^k + \lambda(x + y - D)\,, \tag{11}$$

Taking the first order condition, we have

$$\frac{\partial L}{\partial x} = kx^{k-1} + \lambda = 0, \quad \frac{\partial L}{\partial y} = ky^{k-1} + \lambda = 0\,. \tag{12}$$

Hence $x = y = \sqrt[k-1]{-\lambda/k}$. Substituting the results into (9), we then have $x = y = D/2$. Clearly, in order to achieve the best outage performance in this linear network, the relay node is always chosen at the middle between each pair of source and destination nodes.

III. COOPERATIVE ROUTING ALGORITHMS

Based on the characteristics of cooperative transmission analyzed at Section II, we propose here a distributed routing algorithm to establish a cooperative route in an arbitrary network that ensures each link p_{out} below a certain target level (constraint). Table I describes the routing algorithm in detail.

TABLE I
PROPOSED COOPERATIVE ROUTING ALGORITHM

Initialize: Select the best possible relay node and establish one cooperative link from the source to the destination to minimize the link outage p_{out}. Calculate the p_{out} according to (8) and compare it with the target outage probability (constraint).
Repeat: If any link p_{out} along the constructed route is larger than the target error rate, new relay selections are triggered among the source-relay and relay-destination of that link to improve its p_{out} performance. Re-compute each link p_{out} of the new constructed cooperative links.
Stop: If all the link p_{out} are equal or smaller than the target error rate, then the cooperative route is finalized. Otherwise, continue with the *repeat* step.

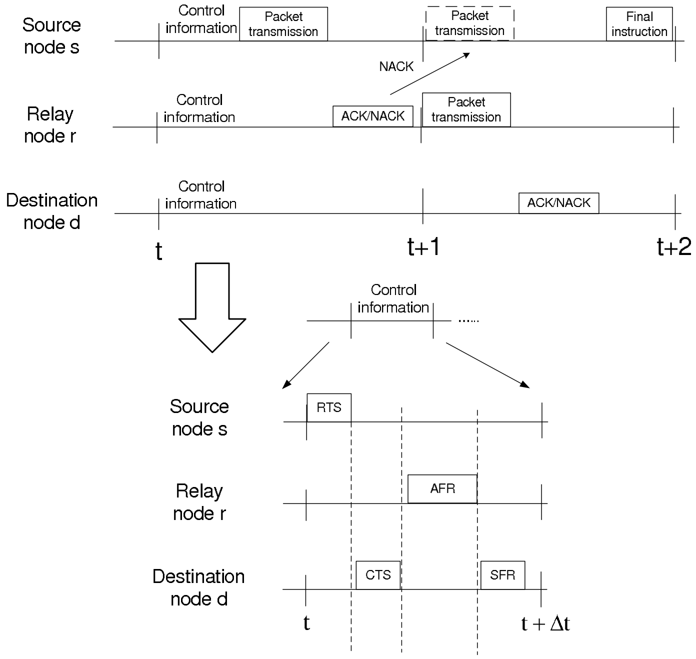

Fig. 3. A timing diagram of cooperative transmission

A. Description of Proposed Cooperative Routing Algorithm

The timing schedule of the proposed cooperative algorithm is shown in Figure 3. As a distributed routing algorithm, each relay node as a monitor periodically broadcasts a HELLO packet to its source-destination pair to measure the link performance. When an improvement is necessary, the relay sends a NOTIFICATION to its source and destination and triggers new relay selections between the source-relay and the relay-destination links. Such "control information" needs to be synchronized among the source, relay and destination before packet transmission.

To fit the non-infrastructure nature of ad-hoc networks, it is desirable to devise a distributed mechanism to choose the relay node with the best incoming and outgoing channel condition among candidate nodes without using a central controller.

In the proposed algorithm, relays use similar carrier sensing scheme [12] and go through a backoff period before sending received data to the destination. According to (8), since the outage performance can be estimated by $d_{s,d}$, $d_{s,r}$, $d_{r,d}$, we propose a new relay selection scheme as following:

- RTS (ready-to-send): source broadcasts a RTS packet to the rests of nodes, each candidate relay node can estimate its $d_{s,r}$ and incoming channel condition through the received SNR strength. Meanwhile, the destination node can estimate $d_{s,d}$.
- CTS (clear-to-send): destination node responses a CTS packet with information $d_{s,d}$, each candidate relay node can estimate $d_{r,d}$ and outgoing channel condition and extract $d_{s,d}$ from this packet.
- AFR (apply-for-relay): relays which hear both RTS and CTS can qualify for relay selection and set a back-off time which is proportional to p_{out}. The back-off period for each relay is chosen such that the smaller the p_{out}, the shorter the back-off time is. After the first back-off time expired, the corresponding relay node will broadcast an acknowledgment (AFR) to other nodes which will quit the competition and refrain for next competition. As a result, the selection of back-off periods at various relay nodes ensures that the best quality relay will be the one responsible for forwarding the data to the destination node.
- SFR (select-for-relay): destination finally sends a confirmation (SFR) to avoid hidden relays problem.

B. Comparison with Centralized Optimal Routing Algorithm

Typically, delay is strongly related to the number of hops in a route. In the context of cooperative networks, one hop can be a direct link or a cooperative link, as defined in above. A meaningful routing problem is to find a route with no more than N hops that minimizes the outage performance in the cooperative networks. Based on the analysis of the optimal relay location problem in Section II, the p_{out} for the cooperative link can be minimized by locating the relay at the middle of node pair associated with the link. For a route with multiple cooperative links, it is obvious that "straight line" routes can achieve better outage performance than any other curve-shaped routes. Furthermore, one can observe that the route that minimizes the p_{out} must have the maximum allowable number of hops N. By assuming that the error performances among links are independent in a given cooperative network, the end-to-end (ETE) outage probability is given by

$$p_{\text{ETE}} = 1 - \prod_{i \in N} (1 - p_{out}^i) \tag{13}$$

where p_{out}^i denotes outage probability for the cooperative link i. For small outage probabilities $p_{out}^i \ll 1$, we make the following approximation

$$p_{\text{ETE}} \approx \sum_{i \in N} p_{out}^i \tag{14}$$

Based on these observations, the routing optimization problem becomes

$$\begin{cases} \min & p_{\text{ETE}} = \sum_{n=1}^{N} {d_{ij}^n}^{2k} \frac{(2^{2R}-1)^2}{2^k \rho^2} \\ \text{s.t.} & \sum_{n=1}^{N} d_{ij}^n = D \end{cases} \tag{15}$$

where d_{ij}^n is the distance between node i and j associated with the nth link in the route, D is the total distance along the route from the source to the destination. We can then simplify the problem and obtain the Lagrangian for this problem as

$$L = \sum_{n=1}^{N} {d_{ij}^n}^{2k} + \lambda (D - \sum_{n=1}^{N} d_{ij}^n)\,, \tag{16}$$

The conditions for optimality are

$$\frac{\partial L}{\partial d_{ij}^n} = 2k {d_{ij}^n}^{2k-1} - \lambda = 0\,, \tag{17}$$

Hence $d_{ij}^n = \sqrt[2k-1]{\lambda / 2k}$. Substituting the results into (15) yields $d_{ij}^n = D/N$. Clearly, in order to achieve the best outage performance, the cooperative links of the optimal routing are uniformly distributed along the line between the source and the destination node.

IV. ROUTING PERFORMANCE EVALUATION

In this section, we analyze our proposed routing algorithm and compare with the optimal routing solution in two special scenarios: networks with infinite node density and those with finite node density in linear topology. Furthermore, we derive bounds performance for the end-to-end p_{out}.

A. Infinitely Dense Network

Theorem 1: For infinitely dense network where node exists at any location, the outage probability for the proposed routing with N hops is

$$p_{\text{ETE}} \sim \Theta\left(\frac{1}{A^{2k-1}}\right)$$

where A, being perfect power of 2, is largest integer that smaller than the total number of hops N and k is the pass loss exponent.

Proof: Suppose the total number of hops is N and the distance between the source and destination is D.

If $log_2(N) =$ integer, then

$$p_{\text{ETE}} = N \frac{(2^{2R}-1)^2}{2\rho^2} (\frac{D}{N})^k (2 \frac{D^k}{2^k N^k}) = \frac{(2^{2R}-1)^2 D^{2k}}{2^k \rho^2 N^{2k-1}}\,. \tag{18}$$

Otherwise, determine two nearest integers A and B which next to N and satisfy $A < N < B$. Both A and B are perfect power of 2. Therefore, we have

$$p_{\text{ETE}} = \frac{(2^{2R}-1)^2 D^{2k} (N-A)}{2^k \rho^2 B^{2k-1} A} + \frac{(2^{2R}-1)^2 D^{2k} (2A-N)}{2^k \rho^2 A^{2k-1} A}\,, \tag{19}$$

Using the relationship $B = 2A$, we have

$$p_{\text{ETE}} = \frac{(2^{2R}-1)^2 D^{2k}}{2^k \rho^2 A^{2k}} (\frac{N-A}{2^{2k-1}} + 2A - N)\,. \tag{20}$$

It is not difficult to observe that $p_{\text{ETE}} \sim \Theta\left(\frac{1}{A^{2k-1}}\right)$. □

Motivated by such conclusions above, we can find the performance of our proposed routing algorithm and optimal solution in 2D infinitely dense networks, which are shown in Figure 7. We observe that the proposed algorithm exhibits performance close to optimal, especially when the hop number N satisfies $log_2(N) =$ integer.

It is worth pointing out that we include an outage constraint in our proposed routing protocol for the following reasons: First, our proposed algorithm starts with routes with a small number of hops. Implicitly, it does not explore routes with an excessive number of hops. Instead, our algorithm achieves a good trade-off and balance between the hop count (which relates to delay) and end-to-end p_{out} for routing, in order to achieve acceptable system performance. Second, for a given end-to-end outage constraint, we can reduce the total number of nodes involved. Hence other benefits such as energy saving, communication traffic reducing could be realized.

B. Regularly Dense Network

Following the ideas above, we compare the minimum end-to-end p_{out} achieved by our proposed algorithm with that of the optimal routing solution for a regularly dense linear network scenario. We consider a linear topology where nodes are located at equal distance from each other on a straight line. We assume that this distance between two adjunct nodes is D and the total number of nodes is n.

Before proceeding further, let us define a gap ratio g, as the normalized difference between the outage probability for the best route established by our proposed algorithm and that of the optimal route

$$g = \frac{p_{\text{proposed}} - p_{\text{optimal}}}{p_{\text{optimal}}} \tag{21}$$

The following theorem compares the performance of the routing algorithm to the optimal route.

Theorem 2: For a regular linear network with n nodes (k=2),

$$g = \begin{cases} 0, & \text{if } \log_2(n-1) or \log_2(n) = \text{integer} \\ \frac{11}{4}, & \text{if } \log_2(\frac{n-1}{3}) = \text{integer} \\ \frac{33}{2(n-1)}, & \text{otherwise for an odd number nodes} \end{cases}$$

Proof: In order to achieve the minimal end-to-end outage probability in such a regular linear topology, the optimal solution for cooperative routing is shown below.

For even n: There is an odd number of links. Hence, the outage probability of optimal routing according to Section III-B that can achieve the minimal end-to-end outage probability from the source to the destination is

$$p_{\text{optimal}} = \frac{(2^{2R}-1)^2}{2\rho^2} D^{2k}(2^k n - 2k + 2 + 6^k + 3^k)\,. \tag{22}$$

For odd n: There is an even number of links. Hence, all the cooperative links can be equally distributed. Then, the corresponding minimal end-to-end outage probability from the source to the destination is

$$p_{\text{optimal}} = \frac{(2^{2R}-1)^2 2^{k-1} D^{2k}(n-1)}{\rho^2}\,. \tag{23}$$

The proposed cooperative route is slightly different from the optimal solution and is more complicated to analyze. Using our proposed routing algorithm, we obtain the minimal end-to-end p_{out} as follows

(1). If $log_2(n) =$ integer or $log_2(n-1) =$integer, there is no difference between the route generated by the proposed algorithm and the optimal solution.

(2). If $log_2(\frac{n-1}{3}) =$ integer, the gap ratio is $g = \frac{11}{4}$.

For any value n which satisfies the above condition, we then can obtain the end-to-end $p_{\text{proposed}} = \frac{(2^{2R}-1)^2}{2\rho^2} D^{2k}(n2^k - 2^k 13 + 3^k 4 + 2^{k+2} 3^k)$ by using the proposed algorithm. Placing this into with (21) yields $g = 11/4$. However, compared with the optimal solution, we can reduce $2^{log_2 \frac{n-1}{3} - 1}$ hops and $(n-1)/3$ nodes involved.

Otherwise: the gap ratio for odd number nodes is $\frac{33}{2(n-1)}$.

This proof is similar as above. Using the same argument, the end-to-end outage probability is $p_{\text{proposed}} = \frac{(2^{2R}-1)^2}{2\rho^2} D^{2k}(n2^{k-1} - 2^{k-1}7 + 3^k + 6^k)$ and the gap ratio is $\frac{33}{2(n-1)}$. However, compared with the optimal solution, we can reduce 1 hop and 2 nodes involved. ☐

In general, Theorem 2 tells us that the proposed routing algorithm can have a p_{out} close optimal. For example, for the first case where $n-1$ or n is perfect power of 2, the proposed algorithm yields the exactly same p_{out} as the optimal route. The gap ratio can be close to zero for the third case where the number of nodes is large enough. In addition to error performance, the proposed routing algorithm also provides advantage of delay reducing. For example, for the second case, compare with optimal solution, we can reduce $2^{log_2 \frac{n-1}{3} - 1}$ hops and $(n-1)/3$ nodes involved when compared with optimal solution. For the third case, we can reduce 1 hop and 2 nodes involved.

V. SIMULATION RESULTS

In this section, we present some simulation results to further demonstrate the performance of cooperative transmission.

A. Outage Performance on Relay Locations

The impacts of relay positions, path loss, data rate and signal-to-noise ratio, on the outage probability is depicted in Figure 4. We choose the distance between the source and the destination to be 100m. When the relay is located on the line between source-destination pair, the plotted curves show that the optimal relay is placed halfway between the source and the destination for the minimal p_{out}. Under the same parameters (i.e., data rate, path loss and signal-to-noise ratio), cooperative transmission is shown to achieve much better error performance than direct transmission (i.e., the solid horizontal line).

However, such optimal location could be affected by other factors, e.g., interference. Let us consider how interference affects the optimal relay location. Since we assume that each node uses the same transmission power, the SINR is given by

$$\rho' = \text{SINR} = \frac{P_T}{P_0 + P_I} = \frac{P_T}{P_0 + P_T |\mathrm{a}_{i,d}|^2}\,, \tag{24}$$

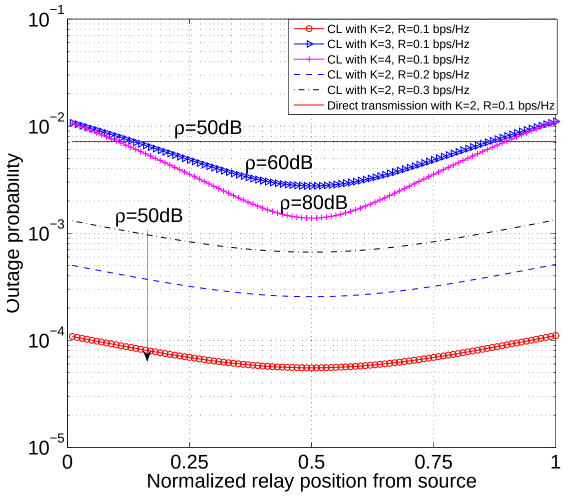

Fig. 4. Outage probability versus normalized relay position

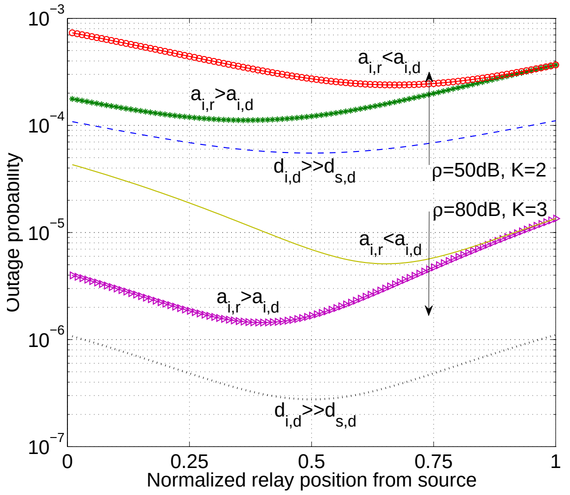

Fig. 5. Interference impacts on the outage probability

where i and d denote the interference and destination nodes, respectively. P_I is the interfering power at the destination. Since for Rayleigh fading, $|\mathrm{a}_{i,d}|^2$ is exponentially distributed with parameter $d_{i,d}^k$. By taking the average of $|\mathrm{a}_{i,d}|^2$ and assuming the white noise power $P_0 \ll P_I$, then (24) becomes

$$\rho' = \mathrm{SINR} \approx \frac{P_T}{P_I} = \frac{\rho}{\rho/d_{i,d}^k} = d_{i,d}^k\,, \tag{25}$$

Therefore, with consideration of interference, Appendix A shows that the outage probablity of CL is

$$p_{out} \approx (\frac{d_{s,r}^k d_{s,d}^k d_{i,d}^k}{2d_{i,r}^k} + (1 - d_{s,r}^k \gamma / d_{i,r}^k)\frac{d_{s,d}^k d_{r,d}^k}{2})(\gamma / d_{i,d}^k)^2\,. \tag{26}$$

where $\gamma = 2^{2R} - 1$. Different to the scenario without interference, the outage probability is no longer a function of transmission power.

Figure 5 depicts the outage performance as a function of relay position. We assume that the interfering node is located 300m to the right of the destination. Result confirms that interference does affect the optimal relay location. Our analysis further shows that in a general network, if the channel condition between the interfering node i and the relay is much better than the channel between the node i and the destination, (i.e., interference on the relay will be more significant than on the destination), the best relay should be away from the node i or close to the source, otherwise it should be close to destination. This is because when the interferer-relay channel is better, in order to guarantee that the relay can successfully decode information from the source, the relay should be away from the interfering node and close to the source. In contrast, when interferer-destination channel is better, the noise impacts on the destination will be significant. In order to combat the noise and to minimize the p_{out} at the receiver, the relay should be located close to the destination. When the interfering node is far away from the cooperative link ($d_{i,d} \gg d_{s,d}$), it is clear that the interference effect is negligible and the optimal relay is located halfway between the source and the destination.

B. Proposed Cooperative Routing Algorithm

Figure 6 shows a routing example which is established by our proposed algorithm. The 100 nodes are uniformly distributed in $1000m \times 1000m$ topology with the source and destination nodes located at the top left corner (node 1) and the bottom right corner (node 100), respectively. Due to the long distances and the given outage constraint 3×10^{-2}, we set transmission power to noise ratio to 50dB. The green dash line (located toward the upper right direction) is the Distance-Vector (DV) routing, whereas the combined red lines represent the proposed cooperative routing. For example, the cooperative link between node 1 and 19 uses node 26 as its relay. As shown in this figure, our proposed algorithm establishes a totally different route path compared with the DV routing algorithm. Furthermore, when compared with 9 hops and 10% end-to-end p_{out} for the DV algorithm, the route generated by our proposed algorithm yields much better performance in terms of delay and outage probability: 5 hops and 3% end-to-end p_{out}.

Moreover, under the same network assumption with finite number of nodes, Figure 7 illustrates the end-to-end outage performance in terms of number of hops. It is shown that for cooperative routing, the end-to-end p_{out} improves as the number of hops in the selected route increases. It also shows that cooperative transmission can achieve better p_{out} performance than DV algorithm. Such implies that our proposed algorithm can generate routes with a smaller number of hops and satisfactory end-to-end p_{out} when compared with the optimal solution from the DV algorithm. Such performance of the infinite node case can be treated as a low bound performance of the proposed algorithm.

VI. CONCLUSIONS

This paper continues our investigation of PHY techniques and cross-layer routing algorithms in cooperative networks where communications between two nodes can be assisted by

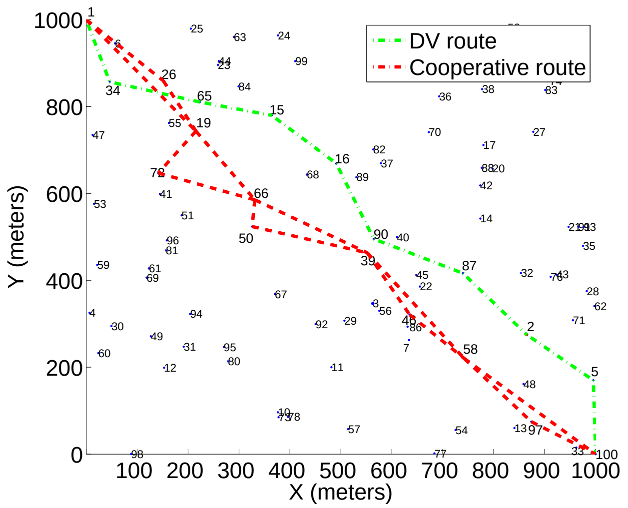

Fig. 6. Routing Comparison between Proposed Algorithm and Destination-Sequenced Distance-Vector (DSDV) algorithm

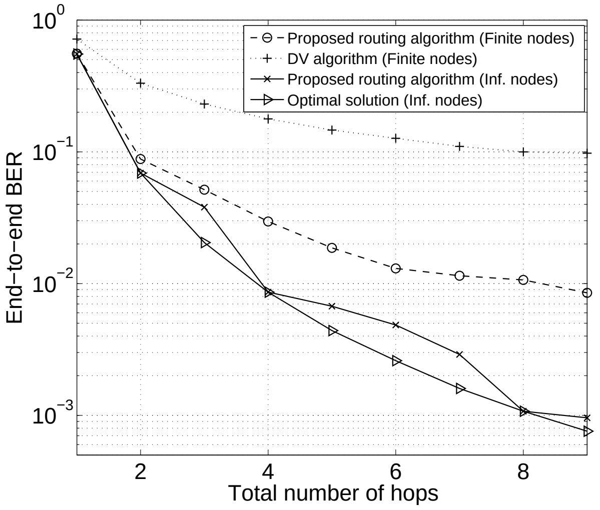

Fig. 7. End-to-end p_{out} versus the number of hops in the route for R=0.1 bps/Hz

a single relay using two time slots. Specifically, we investigate here the optimal relay location for cooperative link in networks with infinite node density. By using this result, we analyze the upper-bound error performance for routing algorithms in the infinitely dense networks. Furthermore, we study the performance bounds for regularly dense networks with linear topology. Theoretical analysis shows that the proposed routing algorithm performs close to the optimal error performance.

APPENDIX A

Consider that a relay node is selected randomly. Hence the mutual information of this cooperative link with interference can be shown as

$$I_C = \begin{cases} \frac{1}{2}\log(1+2\rho'_d|\mathrm{a}_{s,d}|^2), & |\mathrm{a}_{s,r}|^2 < G(\rho'_r) \\ \frac{1}{2}\log(1+\rho'_d|\mathrm{a}_{s,d}|^2+\rho'_d|\mathrm{a}_{r,d}|^2), & |\mathrm{a}_{s,r}|^2 > G(\rho'_r) \end{cases} \tag{27}$$

where ρ'_r and ρ'_d are transmission power to interference ratios at relay and destination and $G(\rho'_r) = (2^{2R}-1)/\rho'_r$. The first case in above corresponding to the relay that is connected by poor links and not being able to decode and the source is repeating its transmission. The second case corresponds to the relay's ability to decode and repeat the source transmission. Here, the maximum average mutual information is that repetition coding from the source and relay to the destination node. Therefore, the outage probability becomes a sum

$$\begin{aligned} P_C^{out} &= Pr[I_C < R] \\ &= Pr[|\mathrm{a}_{s,r}|^2 < G(\rho'_r)]Pr[2|\mathrm{a}_{s,d}|^2 < G(\rho'_d)] \\ &+ Pr[|\mathrm{a}_{s,r}|^2 \geq G(\rho'_r)]Pr[|\mathrm{a}_{s,d}|^2 + |\mathrm{a}_{r,d}|^2 < G(\rho'_d)]\,, \end{aligned} \tag{28}$$

Here, we compute a closed form expression for (28). Then, the large SINR behaviour of (28) by computing the limit: (Let $G(\rho'_r) = G_r$ and $G(\rho'_d) = G_d$)

$$\begin{aligned} \frac{1}{G_d^2}P_C^{out} &= \underbrace{\frac{1}{G_d}Pr[|\mathrm{a}_{s,r}|^2 < G_r]}_{1}\underbrace{\frac{1}{G_d}Pr[2|\mathrm{a}_{s,d}|^2 < G_d]}_{2} \\ &+ \underbrace{Pr[|\mathrm{a}_{s,r}|^2 \geq G_r]}_{3}\underbrace{\frac{1}{G_d^2}Pr[|\mathrm{a}_{s,d}|^2 + |\mathrm{a}_{r,d}|^2 < G_d]}_{4}\,, \end{aligned} \tag{29}$$

where $1 \longrightarrow d_{s,r}^k|\mathrm{a}_{i,r}|^2/|\mathrm{a}_{i,d}|^2$, $2 \longrightarrow d_{s,d}^k/2$, $3 \longrightarrow 1 - d_{s,r}^k(2^{2R}-1)|\mathrm{a}_{i,r}|^2$, $4 \longrightarrow d_{s,d}^k d_{r,d}^k/2$. Taking the average of $|\mathrm{a}_{i,j}|^2$, we have the result.

REFERENCES

[1] A. Scaglione, D. Goeckel, and J. N. Laneman, "Cooperative communications in mobile ad-hoc networks: Rethinking the link abstraction," *IEEE Signal Processing Magazine*, vol. 23, pp. 18–29.

[2] J. N. Laneman, D. N. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. On Information Theory*, vol. 50(12), Dec. 2004.

[3] A. Ribeiro, X. Cai, and G. B. Giannakis, "Symbol error probabilities for general cooperative links," *IEEE Trans. on Wireless Communications*, vol. 4(3), May 2005.

[4] E. Zimmermann, P. Herhold, and G. Fettweis, "A novel protocol for cooperative diversity in wireless networks," in *5th European wireless conference*, 2004.

[5] A. K. Sadek, Z. Han, and K. J. R. Liu, "A distributed relay-assignment algorithm for cooperative communication in wireless networks," in *IEEE International Conference on Communications (ICC) 2006*, 2006.

[6] G. Jakllari, S. V. Krishnamurthy, M. Faloutsos, P. V. Krishnamurthy, and O. Ercetin, "A cross-layer framework for exploiting virtual miso links in mobile ad hoc networks," *IEEE Trans. on MOBILE COMPUTING*, vol. 6(6), 2007.

[7] Z. Ding and K. K. Leung, "Cross-layer routing optimization for wireless networks with cooperative diversity," *IEEE PIMRC*, 2008.

[8] A. E. Khandani, "Cooperative routing in wireless networks," Ph.D. dissertation, MIT, 2004.

[9] K. W. F. Li and A. Lippman, "Energy-efficient cooperative routing in multi-hop wireless ad hoc networks," in *IEEE IPCCC*, 2006.

[10] A. S. Ibrahim, Z. Han, and K. J. R. Liu, "Distributed energy- efficient cooperative routing in wireless networks," in *IEEE Global Telecommunication Conference, Globecom*, 2007.

[11] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Information Theory*, vol. 49, pp. 2415–2425, Oct. 2003.

[12] S. Zhu and K. K. Leung, "Jdistributed cooperative routing for uwb ad-hoc networks," in *IEEE International Conference on Communication, ICC*, 2007.