Joint Beamforming and Power Management for Nonregenerative MIMO Two-Way Relaying Channels

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***Abstract*—We consider multiple-input–multiple-output (MIMO) two-way relaying channels where a pair of multiantenna users wishes to exchange information with the help of a nonregenerative multiantenna relay. A low-complexity joint beamforming and power management scheme is proposed. The proposed beamformers first align the channel matrices of the user pair and then decompose the aligned channel into parallel subchannels. Two power management issues, i.e., power allocation and power control, are addressed in this paper. First, sum-rate optimizing power allocation is proposed to allocate power between all subchannels and nodes. Second, quality of service (QoS) satisfying power control is proposed to minimize the total transmission power in the network. Simulation results justify that the proposed joint beamforming and power management scheme delivers better sum-rate performance or consumes lower transmission power when compared with existing schemes.**

***Index Terms*—Beamforming, convex optimization, power allocation, power control, two-way relaying.**

I. Introduction

TWO-WAY relaying is a spectrally efficient technique to enable information exchange between two users. Two-way relaying protocols, such as those based on decode-and-forward (DF) [2], amplify-and-forward (AF) [3], and estimate-and-forward (EF) relaying [4], are able to fulfill two-way information exchange in only two phases. Specifically, both users concurrently transmit in the same channel during the first phase, whereas the relay broadcasts the processed mixture to both users in the second phase. Each user utilizes the knowledge of the previously transmitted message, known as self-interference, to decode the received mixture. In comparison, conventional one-way relaying consumes four orthogonal channel uses to complete the information exchange.

For practical consideration, nonregenerative relaying, i.e., AF relaying, is desirable when compared with other relaying methods. This is due to the fact that nonregenerative relaying has lower complexity and lower processing delay and incurs lower signal processing power when compared with regenerative relaying, i.e., DF and EF relaying. Attracted by the benefits of multiantenna in enhancing system capacity and reliability, subsequent works on nonregenerative two-way relaying extend to a multiantenna scenario. References [5] and [6] considered sum-rate optimizing AF-based beamforming and power allocation at the relay, for the case where only the relay is equipped with multiple antennas. Reference [7] generalized the scenario to include multiple pairs of single-antenna users and demonstrated that AF-based beamforming at the relay is able to address cochannel interference and improve throughput and reliability.

Meanwhile, [8] and [9] looked into the case with a single pair of multiantenna users and a nonregenerative multiantenna relay. Reference [8] proposed a sum-rate maximizing beamforming design at the relay subject to a fixed power constraint. However, the number of antennas required at the relay is twice the number of antennas needed at each user due to the zero-forcing criterion. Reference [9] studied the joint design of the beamformer at the relay and the decoder at the users to minimize the sum of the mean-square error (MSE) subject to an individual power constraint at each node. The possibility of beamforming at the users was not explored in [9].

The joint design of the transmit and receive beamformers in the multiple-input–multiple-output (MIMO) two-way relaying channels was studied in [10] and [11]. It was recognized in [10] and [11] that the sum-rate expression is not jointly concave with respect to the transmit and receive beamforming matrices, which complicates the optimization of the beamforming matrices. Reference [10] proposed an iterative searching algorithm based on the gradient-descent method to find the locally optimal solution for the beamformers at the users and the relay satisfying individual power constraints with equality. The algorithm has to be extensively repeated with different starting points to increase the probability of finding the best locally optimal solution that corresponds to the globally optimal solution. Meanwhile, [11] proposed an alternate optimization (A-Opt) technique that combines searching algorithms and convex optimization techniques to find locally optimal beamformers at the

Manuscript received May 11, 2011; revised August 23, 2011; accepted October 2, 2011. Date of publication October 20, 2011; date of current version December 9, 2011. This work was supported by the U.S. Army Research Laboratory and the U.K. Ministry of Defence under Agreement W911NF-06-3-0001. The work of C. Y. Leow was supported by Universiti Teknologi Malaysia and the Ministry of Higher Education Malaysia. The work of Z. Ding was supported by the U.K. Engineering and Physical Sciences Research Council under Grant EP/I037423/1. This paper was presented in part at the IEEE International Conference on Communications, Kyoto, Japan, June 5–9, 2011. The review of this paper was coordinated by Prof. E. Bonek.

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Digital Object Identifier 10.1109/TVT.2011.2172009

users when the beamformer at the relay is fixed, and *vice versa*, until convergence is reached. Similar to [10], the algorithms that were proposed in [11] to find locally optimal solutions have to be repeated multiple times to increase the probability of reaching the globally optimal solution. One major drawback of the algorithms that were proposed in [10] and [11] is the expensive computation cost involved in determining the beamforming matrices. The problem dimension of the algorithms in [10] and [11], which quadratically grows with the number of antennas, significantly increases the computational complexity when the number of antennas is large. Another shortcoming is the high computation overhead involved following the fact that both [10] and [11] required extensive repetition of a local optimization procedure with different initial points to achieve the globally optimal solution. In addition to that, [10] and [11] only considered the case where each node is subject to a fixed individual power constraint. The possible performance gain of implementing joint power allocation (JPA) at all nodes subject to a total network power constraint remains unexplored.

The JPA problem has been investigated in a one-way relaying scenario, e.g., in [12] and [13]. However, the solutions cannot be directly applied to a two-way relaying scenario due to the fundamental difference in the transmission protocol. In two-way relaying, the relay receiver needs to account for the superposition of the transmissions from both users, whereas the transmit beamformer at the relay needs to forward information to both users simultaneously. It was remarked in [10] that channel decomposition for substream power allocation, i.e., water filling, is not possible in MIMO two-way relaying channels. Furthermore, the joint transmit and receive beamforming design in MIMO two-way relaying channels involves all three nodes in the network. In one-way relaying, only the source and the relay are involved in transmit beamforming, whereas the relay and the destination participate in receive beamforming.

On the other hand, the capability of the MIMO system to support multiple parallel substreams enables spatial multiplexing of several traffic types with various predefined quality of service (QoS) [14]. The QoS constraints can be target signal-to-noise ratios (SNRs), target data rates, target error rates, etc. The QoS requirement depends on the type of traffic. For instance, in multimedia applications, real-time video traffic requires a higher target data rate than real-time audio traffic. In certain applications, i.e., real-time control, real-time surveillance, etc., successful information delivery defined by QoS constraints is more important than that by power constraints. All these lead to the problem of fulfilling the QoS constraints with the lowest amount of power [14]. Reference [15] studied the power control (power minimization) problem subject to SNR constraints for the MIMO one-way relaying channels, whereas [16] investigated the joint beamforming and power control problem subject to per-user signal-to-interference-and-noise ratio constraints for the multiuser MIMO one-way relaying channels. Nonetheless, the power control problem with QoS constraints in the MIMO two-way relaying channels remains unexplored.

In this paper, we consider a nonregenerative two-way relaying scenario consists of a pair of multiantenna users and a multiantenna relay, all equipped with M antennas. We propose a low-complexity transmit and receive beamforming design that allows the use of single-input–single-output (SISO) decoders and enables JPA and joint power control (JPC) in the network. The proposed beamforming design based on subchannel alignment enables us to investigate the following two power management issues: 1) JPA problem, which maximizes the sum rate subject to a predefined total power constraint in the network; and 2) JPC problem, which minimizes the total transmit power consumption in the network subject to preset QoS constraints, i.e., target data rates. Such network power allocation and power minimization are critical in limiting the total interference incurred in a coverage area that is usually regulated by the authority. To enable the power allocation problem to be efficiently solved using convex optimization techniques [17], a concave upper bound is derived. On the other hand, the power control problem in the form of a geometric program is transformed into convex form solvable using convex optimization techniques. The results show that the ergodic sum rate of the proposed scheme significantly outperforms baseline schemes. The results also reveal the performance gain of the proposed scheme over a comparable scheme [11] when SNRs of the nodes are asymmetrical. In addition, the results fully support the claim that the proposed scheme is more energy efficient in satisfying the QoS constraints when compared with the baseline scheme.

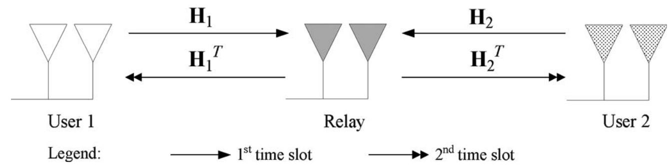

Fig. 1. Example of a two-way relaying scenario where each node is equipped with $M = 2$ antennas. The symbols above the arrows represent the channel matrices, whereas the directions of the arrows indicate the directions of data flows.

II. System Model and Protocol Description

Consider a scenario where two multiantenna users wish to exchange information with the help of a nonregenerative multiantenna relay. We are interested in the full spatial multiplexing case where all nodes are equipped with M antennas. This configuration commonly occurs in ad hoc and sensor networks where nodes have the same number of antennas and the relay is selected from idle users in the network. Fig. 1 shows an example of MIMO two-way relaying channels with $M = 2$. All channels undergo independent and identically distributed (i.i.d.) quasi-static Rayleigh fading and assume channel reciprocity. The receiver is corrupted by circularly symmetric additive white Gaussian noise. A half-duplex constraint is assumed throughout this paper, and it is realized through time-division duplexing. It is assumed that the relay has full knowledge of the channel state information (CSI) of both the relay-to-user channels, whereas each user knows his and his partner's user-to-relay channels.

The transmission protocol can be described in two time slots. Fig. 1 summarizes the transmission flow of the proposed protocol. In the first time slot, both users transmit linear precoded information vectors to the relay, i.e., user i transmits $\mathbf{F}_i\mathbf{x}_i$, where $\mathbf{F}_i \in \mathbb{C}^{M\times M}$ is the transmit beamforming matrix of user i, and $\mathbf{x}_i \in \mathbb{C}^{M\times 1}$ is the information-bearing vector of user i

with normalized covariance, i.e., $E\{\mathbf{x}_i\mathbf{x}_i^H\} = \mathbf{I}_M$, where $E\{.\}$ denotes the expected value, and $[\cdot]^H$ represents the Hermitian transpose. The design of $\mathbf{F}_i$ will be discussed in the following section. The signal observed by the relay can be expressed as

$$\mathbf{r} = \mathbf{H}_1\mathbf{F}_1\mathbf{x}_1 + \mathbf{H}_2\mathbf{F}_2\mathbf{x}_2 + \mathbf{n}_r \tag{1}$$

where $\mathbf{H}_i \in \mathbb{C}^{M\times M}$ is the channel from user i to the relay, and $\mathbf{n}_r \in \mathbb{C}^{M\times 1}$ is the noise vector observed by the relay. In the second time slot, the relay broadcasts the linear precoded observation to both users, i.e., $\mathbf{Wr}$, where $\mathbf{W} \in \mathbb{C}^{M\times M}$ is the joint receive and transmit beamforming matrix at the relay, and $\mathbf{r} \in \mathbb{C}^{M\times 1}$ is the observation in the first time slot, as expressed in (1). Relay beamforming matrix $\mathbf{W}$ will be discussed in the following section. The signal received by user i is

$$\mathbf{y}_i = \mathbf{H}_i^T\mathbf{W}(\mathbf{H}_1\mathbf{F}_1\mathbf{x}_1 + \mathbf{H}_2\mathbf{F}_2\mathbf{x}_2 + \mathbf{n}_r) + \mathbf{n}_i \tag{2}$$

where $\mathbf{n}_i \in \mathbb{C}^{M\times 1}$ is the noise vector observed by user i. Each user performs linear postprocessing on the received mixture broadcast by the relay, i.e., user i calculates $\mathbf{G}_i\mathbf{y}_i$, where $\mathbf{G}_i \in \mathbb{C}^{M\times M}$ is the receive beamforming matrix for user i. Self-interference of user i contains the information transmitted by user i in the previous time slot, i.e., $\mathbf{G}_1\mathbf{H}_1^T\mathbf{W}\mathbf{H}_1\mathbf{F}_1\mathbf{x}_1$ is the self-interference of user 1. Using the principle of analog network coding [3], the self-interference is subtracted from the mixture, and the desired information vector can be decoded.

III. Beamforming Design

Here, the proposed low-complexity design of the transmit beamformer at users $\mathbf{F}_i\ \forall i = \{1, 2\}$, the joint receive and transmit beamformer at relay $\mathbf{W}$, and the receive beamformer at users $\mathbf{G}_i\ \forall i = \{1, 2\}$, are described. The objective of the beamforming design is to decompose the channels into M parallel subchannels, which not only facilitates substream power allocation and power control but also enables the use of simple SISO decoders at the users.

A. Design of $\mathbf{F}_i$

Recall that, in a conventional point-to-point MIMO system, the optimal transmit and receive beamformers are designed by means of singular value decomposition (SVD), such that the channel matrix is decomposed into parallel subchannels (or eigenmodes) to enable optimal power sharing among subchannels [18]. However, this cannot be directly implemented in the two-way relaying scenario considered here, as mentioned in [10]. This is due to the fact that the relay (acting as a MIMO receiver) is not able to simultaneously separate the subchannels from user 1 and user 2, which have different channel directions.

To address the aforementioned issue, we propose the idea of subchannel alignment in the design of $\mathbf{F}_i$ to ensure that the kth subchannel of user 1 and the kth subchannel of user 2 occupy the same signal subspace. Specifically, we propose the following structure for the transmit beamformer of user i:

$$\mathbf{F}_i = \tilde{\mathbf{F}}_i\mathbf{V}_i\mathbf{\Sigma}_i \tag{3}$$

where alignment matrix $\tilde{\mathbf{F}}_i \in \mathbb{C}^{M\times M}$ is obtained from subchannel alignment[1]

$$\mathbf{H}_1\tilde{\mathbf{F}}_1 = \mathbf{H}_2\tilde{\mathbf{F}}_2. \tag{4}$$

The subchannel alignment problem can be solved as follows:

$$\begin{bmatrix} \tilde{\mathbf{F}}_1 \\ -\tilde{\mathbf{F}}_2 \end{bmatrix} = \text{Null Space}\,[\,\mathbf{H}_1 \quad \mathbf{H}_2\,] \tag{5}$$

where the computation of the null space vectors can be found in [19]. Matrix $\mathbf{V}_i \in \mathbb{C}^{M\times M}$ in (3) is the right singular matrix obtained from the SVD of $\mathbf{H}_i\tilde{\mathbf{F}}_i$, i.e., $\mathbf{H}_i\tilde{\mathbf{F}}_i = \mathbf{U}_i\mathbf{\Lambda}_i\mathbf{V}_i^H$, where $\mathbf{U}_i \in \mathbb{C}^{M\times M}$ is the left singular matrix, and $\mathbf{\Lambda}_i \in \mathbb{R}^{M\times M}$ is the diagonal matrix of singular values. Diagonal matrix $\mathbf{\Sigma}_i \in \mathbb{R}^{M\times M}$ in (3) is the transmit power allocation matrix of user i. The transmit power consumption of user i is $\|\mathbf{F}_i\|_F^2$, where $\|.\|_F$ denotes the Frobenius norm. Due to subchannel alignment[2] in (4), $\mathbf{U}_1\mathbf{\Lambda}_1\mathbf{V}_1^H = \mathbf{U}_2\mathbf{\Lambda}_2\mathbf{V}_2^H$. We omit the subscripts of $\mathbf{U}_i\mathbf{\Lambda}_i\mathbf{V}_i^H$ for simplicity of notation. The received signal at the relay in (1) reduces to

$$\mathbf{r} = \mathbf{U\Lambda}(\mathbf{\Sigma}_1\mathbf{x}_1 + \mathbf{\Sigma}_2\mathbf{x}_2) + \mathbf{n}_r. \tag{6}$$

B. Design of $\mathbf{W}$

The design of joint receive and transmit beamformer $\mathbf{W}$ ensures that the received signal in (6) can be decomposed into parallel substreams. To achieve the objective of subchannel decomposition, we propose the following structure:

$$\mathbf{W} = \mathbf{U}^*\mathbf{\Sigma}_r\mathbf{U}^H \tag{7}$$

where $\mathbf{U}$ is the left singular matrix of $\mathbf{H}_i\tilde{\mathbf{F}}_i$, and diagonal matrix $\mathbf{\Sigma}_r \in \mathbb{R}^{M\times M}$ is the transmit power allocation matrix at the relay. Notice that the use of subchannel alignment discussed in the previous section enables the relay to decompose the channels of user 1 and user 2 simultaneously. The total transmit power consumption at the relay is $\|\mathbf{W}\mathbf{H}_1\mathbf{F}_1\|_F^2 + \|\mathbf{W}\mathbf{H}_2\mathbf{F}_2\|_F^2 + \sigma_r^2\|\mathbf{W}\|_F^2$, where σ_r^2 is the receiver noise power at the relay.

C. Design of $\mathbf{G}_i$

The design of receive beamformer $\mathbf{G}_i$ is to ensure that the received signal in (2) can be decomposed into parallel substreams. Specifically, we propose the following structure:

$$\mathbf{G}_i = \mathbf{V}^T\tilde{\mathbf{F}}_i^T \tag{8}$$

which is the transposition of transmit beamforming matrix $\mathbf{F}_i$ but without the power allocation matrix. The signal obtained by

[1]Note that $\tilde{\mathbf{F}}_i$ is not unique. However, the multiplication of $\tilde{\mathbf{F}}_i$ with a unitary matrix (rotation matrix) does not change the singular values of the effective channels, i.e., $\mathbf{\Lambda}_i$ remains the same.

[2]Although subchannel alignment reduces the channel eigenvalues, the loss is negligible when the SNR is large. This is analogous to the zero-forcing transmit beamforming operation in the MIMO broadcast channels: the effective channel eigenvalues reduce after the zero-forcing operation. When the SNR is asymptotically large, the loss is negligible, and the zero-forcing beamformer approaches the optimal multiplexing gain [20], [21].

user i in (2) after receive beamforming can be expressed as

$$\mathbf{G}_i\mathbf{y}_i = \mathbf{\Lambda}^2\mathbf{\Sigma}_r(\mathbf{\Sigma}_1\mathbf{x}_1 + \mathbf{\Sigma}_2\mathbf{x}_2) + \tilde{\mathbf{n}}_i \tag{9}$$

where $\tilde{\mathbf{n}}_i = \mathbf{\Lambda}\mathbf{\Sigma}_r\mathbf{U}^H\mathbf{n}_r + \mathbf{V}^T\tilde{\mathbf{F}}_i^T\mathbf{n}_i$ is the effective noise observed by user i. As described in Section II, each user is able to decode the desired information by subtracting the self-interference from the mixture. For instance, user 1 is able to decode $\mathbf{x}_2$ by subtracting self-interference $\mathbf{\Lambda}^2\mathbf{\Sigma}_r\mathbf{\Sigma}_1\mathbf{x}_1$ from the received mixture.

IV. JPA

Here, we investigate the JPA problem using the proposed beamforming scheme. First, the SNR of each subchannel is derived. Second, the JPA problem is formulated using the sum-rate criterion, and the convexity of the optimization problem is investigated. Since the objective function is nonconcave, we derive an upper bound to approximate the original objective function. The final sections discuss the proposed power allocation strategies and the comparable scheme.

A. Subchannel SNR Derivation

From (9), it can be observed that the channel matrices are decomposed into M parallel subchannels. Here, the SNR of each subchannel is derived. Denote the transmit power allocation matrix of user 1 by $\mathbf{\Sigma}_1 = \text{diagonal}(\sqrt{a_1}, \ldots, \sqrt{a_M})$, the transmit power allocation matrix of user 2 by $\mathbf{\Sigma}_2 = \text{diagonal}(\sqrt{b_1}, \ldots, \sqrt{b_M})$, the transmit power allocation matrix of the relay by $\mathbf{\Sigma}_r = \text{diagonal}(\sqrt{c_1}, \ldots, \sqrt{c_M})$, the diagonal matrix of singular values by $\mathbf{\Lambda} = \text{diagonal}(\sqrt{\lambda_1}, \ldots, \sqrt{\lambda_M})$, and $\hat{\mathbf{F}}_i = \tilde{\mathbf{F}}_i\mathbf{V}_i$. Variables a_k, b_k, and c_k represent the kth substream power allocation factors for user 1, user 2, and the relay, respectively. Assuming the SISO decoder is used to decode each parallel substream, the SNR of the kth subchannel of user 1 can be expressed as follows:

$$\gamma_{1,k} = \frac{\lambda_k^2 b_k c_k}{\sigma_r^2\lambda_k c_k + \sigma_1^2\sum_{j=1}^M \left|\hat{\mathbf{F}}_1(j,k)\right|^2} \tag{10}$$

where σ_1^2 is the noise power at user 1 receiver. Similarly, the SNR of the kth subchannel of user 2 can be written as

$$\gamma_{2,k} = \frac{\lambda_k^2 a_k c_k}{\sigma_r^2\lambda_k c_k + \sigma_2^2\sum_{j=1}^M \left|\hat{\mathbf{F}}_2(j,k)\right|^2} \tag{11}$$

where σ_2^2 is the noise power at user 2 receiver. Assuming Gaussian coding, the instantaneous data rate (or mutual information) of user i can be expressed as

$$R_i = \frac{1}{2}\sum_{k=1}^M \log_2(1+\gamma_{i,k}). \tag{12}$$

B. Sum-Rate Optimization

Sum-rate criterion, i.e., $R_1 + R_2$, is the optimization criterion used in this paper. All transmissions in the network are subject to total network power constraint P, which is expressed as follows:

$$\sum_{i=1}^2 \left(\|\mathbf{F}_i\|_F^2 + \|\mathbf{W}\mathbf{H}_i\mathbf{F}_i\|_F^2\right) + \sigma_r^2\|\mathbf{W}\|_F^2 \le P. \tag{13}$$

The joint power constraint is the summation of the transmit power consumption at user 1, user 2, and the relay. The joint power constraint expression can be simplified. Specifically, the transmit power consumption at the relay can be simplified as

$$\|\mathbf{W}\mathbf{H}_1\mathbf{F}_1\|_F^2 + \|\mathbf{W}\mathbf{H}_2\mathbf{F}_2\|_F^2 + \sigma_r^2\|\mathbf{W}\|_F^2 = \sum_{k=1}^M\left(\lambda_k a_k c_k + \lambda_k b_k c_k + \sigma_r^2 c_k\sum_{j=1}^M |\mathbf{U}(j,k)|^2\right).$$

Note that $\sum_{j=1}^M |\mathbf{U}(j,k)|^2 = 1$. Similarly, the transmit power consumption at user 1 and user 2 can be simplified as $\|\mathbf{F}_1\|_F^2 = \sum_{k=1}^M\sum_{j=1}^M a_k|\hat{\mathbf{F}}_1(j,k)|^2$ and $\|\mathbf{F}_2\|_F^2 = \sum_{k=1}^M\sum_{j=1}^M b_k|\hat{\mathbf{F}}_2(j,k)|^2$, respectively. To further simplify the expression, we represent $\tilde{a}_k = a_k\sum_{j=1}^M|\hat{\mathbf{F}}_1(j,k)|^2$, $\tilde{b}_k = b_k\sum_{j=1}^M|\hat{\mathbf{F}}_2(j,k)|^2$, and $\tilde{c}_k = c_k(\lambda_k a_k + \lambda_k b_k + \sigma_r^2)$ as the effective kth substream power allocation factors for user 1, user 2, and the relay, respectively.

The JPA problem using the sum-rate criterion can be formulated as follows:

$$\underset{\tilde{a}_k,\tilde{b}_k,\tilde{c}_k \quad \forall k=\{1,\ldots,M\}}{\text{maximize}} \quad \frac{1}{2}\sum_{k=1}^M\left(\log_2\left(1+\frac{t_{1,k}\tilde{b}_k\tilde{c}_k}{t_{2,k}\tilde{a}_k + t_{3,k}\tilde{b}_k + t_{4,k}\tilde{c}_k + t_{5,k}}\right) + \log_2\left(1+\frac{u_{1,k}\tilde{a}_k\tilde{c}_k}{u_{2,k}\tilde{a}_k + u_{3,k}\tilde{b}_k + u_{4,k}\tilde{c}_k + u_{5,k}}\right)\right) \tag{14}$$

$$\text{subject to} \quad \sum_{k=1}^M(\tilde{a}_k + \tilde{b}_k + \tilde{c}_k) \le P, \quad \tilde{a}_k \ge 0,\ \tilde{b}_k \ge 0,\ \tilde{c}_k \ge 0 \quad \forall k = \{1,\ldots,M\} \tag{15}$$

where constants $t_{1,k} = \lambda_k^2$, $t_{2,k} = \sigma_1^2\beta_k\lambda_k$, $t_{3,k} = \sigma_1^2\alpha_k\lambda_k$, $t_{4,k} = \sigma_r^2\beta_k\lambda_k$, $t_{5,k} = \sigma_1^2\sigma_r^2\alpha_k\beta_k$, $u_{1,k} = t_{1,k} = \lambda_k^2$, $u_{2,k} = \sigma_2^2\beta_k\lambda_k$, $u_{3,k} = \sigma_2^2\alpha_k\lambda_k$, $u_{4,k} = \sigma_r^2\alpha_k\lambda_k$, $u_{5,k} = \sigma_2^2\sigma_r^2\alpha_k\beta_k$, $\alpha_k = \sum_{j=1}^M|\hat{\mathbf{F}}_1(j,k)|^2$, and $\beta_k = \sum_{j=1}^M|\hat{\mathbf{F}}_2(j,k)|^2$. The optimization problem can be solved using convex optimization techniques [17] if the constraints are convex and the objective is concave. The inequality of the power constraint in (15) is affine, hence convex (and concave) with respect to (w.r.t.) all input parameters $\tilde{a}_1,\ldots,\tilde{a}_M$, $\tilde{b}_1,\ldots,\tilde{b}_M$, and $\tilde{c}_1,\ldots,\tilde{c}_M$. However, it can be shown that the objective function in (14) is nonconcave w.r.t. all input parameters.

To ease the difficulty in solving the power allocation problem, we derive a concave upper bound of the original objective function, which can be efficiently solved using convex optimization techniques. The following theorem summarizes the concavity of the derived upper bound.

Theorem 1: The following upper bound of the objective function is jointly concave w.r.t. input parameters $\tilde{a}_1, \ldots, \tilde{a}_M$, $\tilde{b}_1, \ldots, \tilde{b}_M$, and $\tilde{c}_1, \ldots, \tilde{c}_M$:

$$f_{\text{upper}} = \frac{1}{2}\sum_{k=1}^{M} \log_2\left(1 + \frac{(\tilde{a}_k + \tilde{b}_k)\tilde{c}_k}{t_{a,k}(\tilde{a}_k + \tilde{b}_k) + t_{b,k}\tilde{c}_k}\right) + \frac{1}{2}\sum_{k=1}^{M} \log_2\left(1 + \frac{(\tilde{a}_k + \tilde{b}_k)\tilde{c}_k}{u_{a,k}(\tilde{a}_k + \tilde{b}_k) + u_{b,k}\tilde{c}_k}\right) \quad (16)$$

where constants $t_{a,k} = \min(t_{2,k}, t_{3,k})/t_{1,k}$, $t_{b,k} = t_{4,k}/t_{1,k}$, $u_{a,k} = \min(u_{2,k}, u_{3,k})/u_{1,k}$, and $u_{b,k} = u_{4,k}/u_{1,k}$.

Proof: Refer to the Appendix. ■

Remark 1: Power allocation factors $\tilde{a}_1, \ldots, \tilde{a}_M, \tilde{b}_1, \ldots, \tilde{b}_M$, and $\tilde{c}_1, \ldots, \tilde{c}_M$, which were obtained by solving the concave upper bound in (16), are suboptimal solutions to the original problem in (14). The approximation in (16) enables the transmission power being dynamically allocated between the users and the relay, whereas the user pair shares identical power allocation factors. Since the positive sum of the power allocation factors of both users, i.e., $\tilde{a}_k + \tilde{b}_k$, can be represented as a single power allocation factor, the result is a combination of dynamic power sharing between the users and the relay, which is coupled with equal power sharing between users.

C. Proposed Power Allocation Strategies

Here, two JPA strategies are proposed.

1) Proposed JPA I: This proposed JPA I computes the power allocation factors by solving the sum-rate optimization problem in (14) and (15). As discussed in the previous section, the objective function in (14) is nonconcave. In this case, the locally optimal solution does not necessarily correspond to the globally optimal solution. The globally optimal solution can be found with a certain probability by means of randomization-based global optimization [12]. For each channel realization, multiple random starting vectors are generated, and the locally optimal solution for each starting vector is computed using the convex optimization techniques, i.e., the interior-point method [17]. The globally optimal solution for each channel realization is the maximum of all locally optimal solutions. Since this method requires the use of multiple random starting vectors, a centralized node, i.e., the relay, will compute the power allocation factors and distribute them to other nodes.

2) Proposed JPA II: The proposed JPA II computes the power allocation factors by solving the concave upper bound in (16). The power allocation factors can be efficiently calculated using the convex optimization techniques, i.e., the interior-point method [17]. Since the upper-bound objective function in (16) is concave, the locally optimal solution obtained using convex optimization corresponds to the globally optimal solution. The computed power allocation factors that correspond to the globally optimal solution of (16) are then substituted back into the original objective function in (14) to obtain the achievable sum rate. With the CSI knowledge of the channels, each node is able to compute the power allocation factors locally, without any cooperation between nodes.

D. Comparable Scheme: A-Opt [11]

The best comparable scheme for the nonregenerative MIMO two-way relaying channels is the A-Opt scheme that was proposed in [11]. Due to the fact that the sum-rate expression is nonconcave, [11] proposed the A-Opt scheme, which alternately computes locally optimal source beamformers for fixed relay beamformers and locally optimal relay beamformers for fixed source beamformers until convergence is reached. Several searching algorithms were proposed in [11] to determine locally optimal beamforming matrices subject to individual power constraints and assuming the use of perfect MIMO decoders at the users. Although the A-Opt scheme is able to achieve the best sum-rate under individual power constraints and symmetric SNR, it is computationally expensive to determine the beamforming matrices. Generally, the problem dimension of the searching algorithms in [11] quadratically grows with the number of antennas. This significantly increases the computational complexity when a higher number of antennas are used. In comparison, the problem dimension of our proposed JPA I and II schemes is linear with the number of antennas. Furthermore, due to the fact that the sum rate is nonconcave for any fixed source beamformers, the searching algorithms in [11] have to be repeated multiple times with different starting points to increase the probability of finding the globally optimal relay beamformer. This further increases the computational overhead.

To obtain the simulation results for the A-Opt scheme, we employ the weighted minimum MSE algorithm that was proposed in [11] to compute the relay beamforming matrix and use a semidefinite program solver in the CVX toolbox [22], [23] to compute the user beamforming matrices. Each node is subject to individual power constraint $P/3$, which sums up to a joint power constraint P to enable fair comparison with our proposed methods.

We do not include [10] in the comparison because it is suboptimal when compared with [11] for the case assuming individual power constraints. In [10], the suboptimal minimum MSE receiver was used, whereas in [11], the optimal maximum-likelihood receiver was assumed.

V. JPC

Here, we consider the power control problem in guaranteeing the predetermined QoS constraints of the two-way information transmission. First, the JPC problem is presented. Second, several power control strategies are proposed.

A. JPC Optimization Problem

The objective of the power control policy is to minimize the total power consumption in the network, subject to the kth substream rate constraint of user 1, i.e., $R_{1,k}$, and the kth substream rate constraint of user 2, i.e., $R_{2,k}\ \forall k = \{1, \ldots, M\}$. Specifically, the kth substream rate constraint of user 1 and user 2 can be expressed as $(1/2)\log_2(1+\gamma_{1,k}) \geq R_{1,k}$ and $(1/2)\log_2(1+\gamma_{2,k}) \geq R_{2,k}$, respectively, where the kth substream SNRs $\gamma_{1,k}$ and $\gamma_{2,k}$ can be found in (10) and (11), respectively. The kth substream rate constraints serve as criteria to guarantee the QoS specified on the kth data stream. Notice that the kth substream rate constraints can be also expressed as kth substream SNR constraints, i.e., $\gamma_{1,k} = 2^{2R_{1,k}} - 1$ and $\gamma_{2,k} = 2^{2R_{2,k}} - 1, \forall k = \{1,\ldots,M\}$.

Recall that the power consumption at user 1 is a function of the power allocation factors at user 1, i.e., $P_1(\tilde{a}_1,\ldots,\tilde{a}_M) = \sum_{k=1}^{M}\tilde{a}_k$, where $\tilde{a}_k = a_k\sum_{j=1}^{M}|\hat{\mathbf{F}}_1(j,k)|^2$. Similarly, the power consumption at user 2 is $P_2(\tilde{b}_1,\ldots,\tilde{b}_M) = \sum_{k=1}^{M}\tilde{b}_k$, where $\tilde{b}_k = b_k\sum_{j=1}^{M}|\hat{\mathbf{F}}_2(j,k)|^2$, whereas the power consumption at the relay is $P_r(\tilde{c}_1,\ldots,\tilde{c}_M) = \sum_{k=1}^{M}\tilde{c}_k$, where $\tilde{c}_k = c_k(\lambda_k a_k + \lambda_k b_k + \sigma_r^2)$. After some algebraic manipulations, the power control approach that minimizes the total power consumption in the network subject to the substream rate constraints can be formulated as follows:

$$\underset{\tilde{a}_k,\tilde{b}_k,\tilde{c}_k \quad \forall k=\{1,\ldots,M\}}{\text{minimize}}$$

$$P_1(\tilde{a}_1,\ldots,\tilde{a}_M) + P_2(\tilde{b}_1,\ldots,\tilde{b}_M) + P_r(\tilde{c}_1,\ldots,\tilde{c}_M) \quad (17)$$

subject to

$$\gamma_{1,k}\left(r_{1,k}\tilde{a}_k\tilde{b}_k^{-1}\tilde{c}_k^{-1} + r_{2,k}\tilde{c}_k^{-1} + r_{3,k}\tilde{b}_k^{-1} + r_{4,k}\tilde{b}_k^{-1}\tilde{c}_k^{-1}\right) \le 1 \quad \forall k = \{1,\ldots,M\} \quad (18)$$

$$\gamma_{2,k}\left(s_{1,k}\tilde{c}_k^{-1} + s_{2,k}\tilde{a}_k^{-1}\tilde{b}_k\tilde{c}_k^{-1} + s_{3,k}\tilde{a}_k^{-1} + s_{4,k}\tilde{a}_k^{-1}\tilde{c}_k^{-1}\right) \le 1 \quad \forall k = \{1,\ldots,M\} \quad (19)$$

where constants $r_{1,k} = \sigma_1^2\beta_k\lambda_k^{-1}$, $r_{2,k} = \sigma_1^2\alpha_k\lambda_k^{-1}$, $r_{3,k} = \sigma_r^2\beta_k\lambda_k^{-1}$, $r_{4,k} = \lambda_k^{-2}\sigma_1^2\sigma_r^2\alpha_k\beta_k$, $s_{1,k} = \sigma_2^2\beta_k\lambda_k^{-1}$, $s_{2,k} = \sigma_2^2\alpha_k\lambda_k^{-1}$, $s_{3,k} = \sigma_r^2\alpha_k\lambda_k^{-1}$, $s_{4,k} = \lambda_k^{-2}\sigma_2^2\sigma_r^2\alpha_k\beta_k$, $\alpha_k = \sum_{j=1}^{M}|\hat{\mathbf{F}}_1(j,k)|^2$, and $\beta_k = \sum_{j=1}^{M}|\hat{\mathbf{F}}_2(j,k)|^2$.

The optimization problem is in the form of a geometric program, since the objective is an affine function (which can be generalized as a posynomial function [17]) and the constraints are posynomial functions. Although the original problem is not convex, the geometric program can be transformed into equivalent convex form by means of change of variables and logarithmic transformation of the constraint functions [17].

B. Proposed Power Control Strategies

Here, two power control strategies using the proposed beamforming scheme are presented, namely, JPC and equal power control (EPC).

1) JPC: The proposed JPC corresponds to solving the power minimization problem in (17) subject to rate constraints in (18) and (19). As discussed in the previous section, the equivalent power control problem in convex form can be efficiently solved using convex optimization techniques, i.e., the interior-point method [17]. The locally optimal solution obtained by convex optimization corresponds to the globally optimal solution. The computation of the power allocation factors is locally performed at each node.

2) EPC: In the proposed EPC, each substream is allocated with a common power allocation factor, subject to the rate constraints in (18) and (19). Under this suboptimal strategy, the power allocation factors have the following relationship: $\tilde{a}_k = \tilde{b}_k = \tilde{c}_k = \theta^* \, \forall k = \{1,\ldots,M\}$, where θ^* is the common power allocation factor. Common power allocation factor θ^* can be analytically obtained by solving the following quadratic equations:

$$\theta^2 + \gamma_{1,k}(r_{1,k} + r_{2,k} + r_{3,k})\theta + \gamma_{1,k}r_{4,k} = 0 \quad (20)$$

$$-\theta^2 + \gamma_{2,k}(s_{1,k} + s_{2,k} + s_{3,k})\theta + \gamma_{2,k}s_{4,k} = 0 \quad (21)$$

$\forall k = \{1,\ldots,M\}$. From (20), the discriminant of the quadratic equation can be expressed as $\triangle = \gamma_{1,k}^2(r_{1,k} + r_{2,k} + r_{3,k})^2 + 4\gamma_{1,k}r_{4,k} > 0$ since all constants ($\gamma_{1,k}$ and $r_{i,k}$ $\forall i = 1,2,3,4$) are positive. When $\triangle > 0$, the real roots are $\theta_1 = \gamma_{1,k}(r_{1,k} + r_{2,k} + r_{3,k})/2 - \sqrt{\triangle}/2 < 0$ and $\theta_2 = \gamma_{1,k}(r_{1,k} + r_{2,k} + r_{3,k})/2 + \sqrt{\triangle}/2 > 0$. We know that the power allocation factor is always positive, i.e., $\theta > 0$; therefore, we choose $\theta = \theta_2$. Following similar steps, the real root for (21) is $\theta = \gamma_{2,k}(s_{1,k} + s_{2,k} + s_{3,k})/2 + \sqrt{\triangle}/2$, where $\triangle = \gamma_{2,k}^2(s_{1,k} + s_{2,k} + s_{3,k})^2 + 4\gamma_{2,k}s_{4,k}$. Common power allocation factor θ^* is the maximum of all real roots of $2M$ quadratic equations previously stated to ensure that all rate constraints are satisfied. The total power consumption in the network is $3M\theta^*$.

VI. Numerical Results

Here, we present the numerical results of the proposed joint beamforming and power management scheme in comparison with existing schemes. Optimization problems discussed in the previous sections are solved using the nonlinear optimization toolbox in MATLAB, i.e., using function $fmincon$ and the interior-point method. The numerical results are organized into two sections. In the first section, the ergodic sum rates of various schemes with a fixed total power constraint are simulated using the Monte Carlo method. Refer to Section IV for details of the JPA formulation. In the second section, various power control schemes with fixed rate constraints are simulated using the Monte Carlo method. See Section V for details of the JPC formulation.

A. Power Allocation With a Fixed Total Power Constraint

Here, the simulation results of the proposed JPA, baseline, and comparable schemes are generated to study the relationship between the ergodic sum rate and parameters such as SNR, number of antennas, and path loss. The baseline relaying schemes used for comparison are the pure AF scheme and the MIMO one-way relaying scheme. In the pure AF two-way relaying scheme, the relay simply forwards the power normalized observation to the users, without beamforming and power allocation. In the MIMO one-way relaying scheme, a DF relay is used, and the information exchange consumes four time slots. All baseline schemes assume equal power allocation subject to total transmission power P. The comparable scheme is the A-Opt scheme. Refer to Section IV-D.

Fig. 2 shows the ergodic sum rate versus reference SNR $(1/\sigma_1^2 = 1/\sigma_2^2 = 1/\sigma_r^2)$ of the proposed JPA schemes in comparison with existing schemes. The reference SNR is defined as the inverse of the noise power. In the subsequent discussion, we use the term SNR to imply the reference SNR. In this

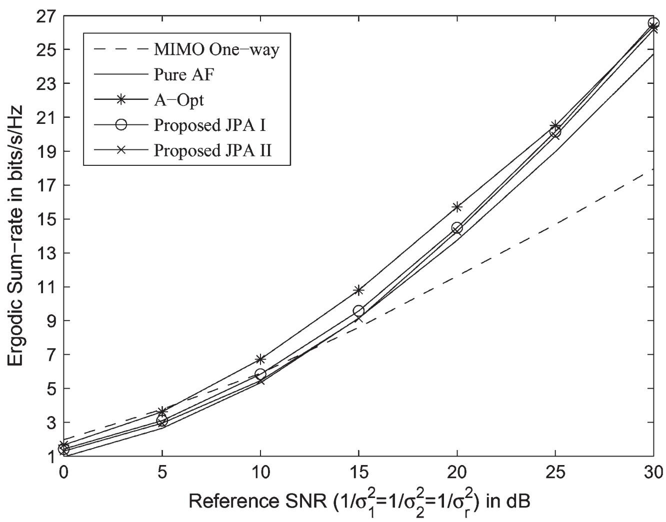

Fig. 2. Ergodic sum rate versus SNR for fixed $M = 4$ and $P = 3$.

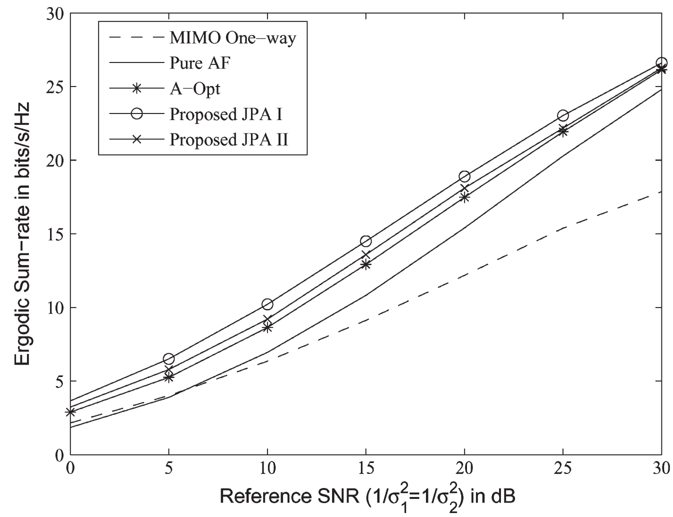

Fig. 3. Ergodic sum rate versus SNR for fixed $M = 4$, $P = 3$, and $1/\sigma_r^2 = 30$ dB.

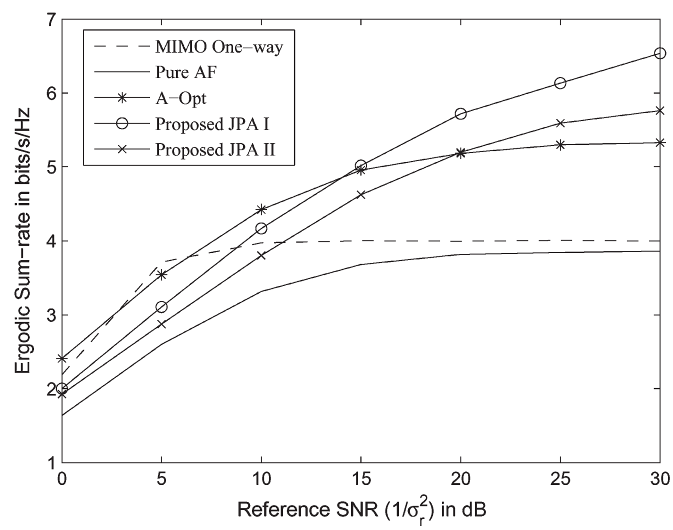

Fig. 4. Ergodic sum rate versus SNR for fixed $M = 4$, $P = 3$, and $1/\sigma_1^2 = 1/\sigma_2^2 = 5$ dB.

simulation, the SNRs at all nodes are assumed to be symmetrical (equal noise power), i.e., $1/\sigma_1^2 = 1/\sigma_2^2 = 1/\sigma_r^2$. The fixed parameters are the number of antennas, i.e., $M = 4$, and the total power constraints, i.e., $P = 3$ W. From the figure, it can be observed that the proposed JPA I and II schemes perform close to the A-Opt scheme at high SNR. In the range of low to medium SNR, the performance gain contributed by the proposed JPA schemes over the baseline pure AF scheme is limited when compared with the A-Opt scheme. This is due to the fact that the choice of beamforming directions in the proposed JPA schemes is suboptimal when compared with the A-Opt scheme. However, at high SNR, the suboptimal beamforming directions do not prevent the proposed JPA schemes from delivering a significant performance gain against the pure AF scheme through dynamic allocation of power among substreams and nodes. It can be also observed that the proposed JPA II performs close to the proposed JPA I. This indicates that the upper bound in Theorem 1 is a good approximation of the original problem in (14). The performance gaps between the two-way relaying schemes (pure AF, A-Opt, and proposed JPA I and II) and the one-way relaying scheme enlarge with the increase in SNR. Evident from the slope of the sum-rate curves, the two-way relaying schemes are able to achieve a higher multiplexing gain due to a more efficient use of bandwidth.

The following simulations study the ergodic sum rates of the various schemes when the SNRs at the users and the relay are asymmetrical (unequal noise power). Fig. 3 shows the ergodic sum rate versus SNR at the users $(1/\sigma_1^2 = 1/\sigma_2^2)$ when the SNR at the relay is fixed at $1/\sigma_r^2 = 30$ dB. Other fixed parameters are $M = 4$ and $P = 3$ W. From the figure, it can be observed that the proposed JPA I achieves the best ergodic sum rate, closely followed by the proposed JPA II. The A-Opt scheme does not perform better than the proposed JPA schemes. This is due to the fact that, in the A-Opt scheme, each node is allocated with a fixed amount of power that does not correlate with the asymmetric SNRs. In comparison, the proposed JPA schemes respond to the asymmetric SNRs by allocating power dynamically among nodes and substreams. The JPA between nodes provides another dimension of improvement. Fig. 4 shows the ergodic sum rate versus SNR at the relay $(1/\sigma_r^2)$ when the SNRs at the users are fixed at $1/\sigma_1^2 = 1/\sigma_2^2 = 5$ dB. Other fixed parameters are $M = 4$ and $P = 3$ W. From the figure, it is clear that the proposed JPA schemes deliver significant performance gain over the pure AF scheme. The proposed JPA I and II schemes deliver a higher ergodic sum rate than the A-Opt scheme when the SNR is greater than 15 and 20 dB, respectively. This supports that dynamic power allocation between the users and the relay is able to utilize the asymmetric SNRs between nodes to obtain better sum-rate performance. At low SNR, the proposed schemes do not perform as good as the A-Opt scheme due to the use of suboptimal beamforming directions. It is interesting to observe that, at low SNR, the baseline MIMO one-way relaying scheme performs as good as the A-Opt scheme. At low SNR, the performance of the two-way relaying schemes (pure AF, A-Opt, and proposed JPA I and II) is limited by not only the noise at the users but also the propagated noise from the relay. As a result, the

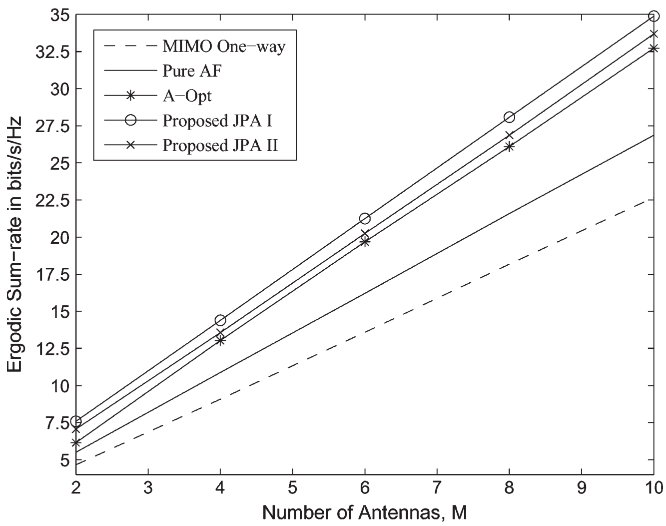

Fig. 5. Ergodic sum rate versus number of antennas M for fixed $1/\sigma_1^2 = 1/\sigma_2^2 = 15$ dB, $1/\sigma_r^2 = 30$ dB, and $P = 3$.

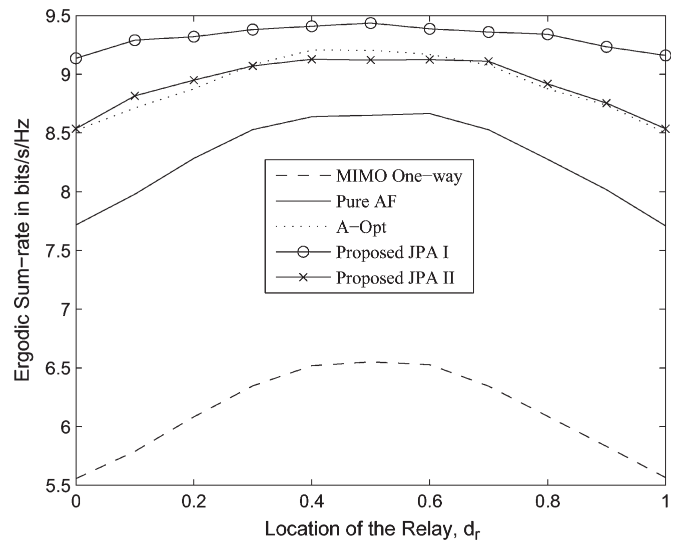

Fig. 6. Ergodic sum rate versus relay location d_r for fixed $M = 2$, $1/\sigma_1^2 = 1/\sigma_2^2 = 1/\sigma_r^2 = 30$ dB, $P = 3$, and $\alpha = 4$.

two-way relaying schemes do not perform better than the one-way relaying scheme at low SNR.

Fig. 5 shows the ergodic sum rate versus number of antennas M of various schemes. The fixed parameters are $1/\sigma_1^2 = 1/\sigma_2^2 = 15$ dB, $1/\sigma_r^2 = 30$ dB, and $P = 3$ W. Generally, the ergodic sum rates of all schemes linearly increase with the number of antennas M. As the number of antennas at all nodes is simultaneously increased, the number of independent data streams supportable in the network increases. In other words, the multiplexing gain linearly grows with M. Among all power allocation schemes, the proposed JPA I achieves the best sum-rate performance, closely followed by the proposed JPA II. The proposed JPA schemes outperform the A-Opt scheme owing to the JPA between nodes. From the figure, it can be observed that the gaps between the proposed schemes and the pure AF scheme enlarge for increasing M. This shows that JPA is vital in delivering a better data rate in a system with a high multiplexing gain.

In the next simulation, the effect of large-scale path loss to the ergodic sum rate is investigated. The path loss is integrated in the channel model as $(1/\sqrt{d_i^\alpha})\mathbf{H}_i$, where d_i is the distance between user i and the relay, α is the path-loss exponent, and $\mathbf{H}_i$ is the channel matrix between user i and the relay (as shown in Section II), where its entries are i.i.d. Rayleigh distributed. A simple line network is considered where the relay is placed in between the users. Fig. 6 shows the ergodic sum rate versus relay location d_r of various schemes. The distance between user 1 and the relay is $d_1 = 1 + d_r$, whereas the distance between user 2 and the relay is $d_2 = 2 - d_r$. The constant offset of 1 in d_i is introduced to ensure that the receive power is upper bounded by the transmit power. The fixed parameters are $M = 2$, $\alpha = 4$, $1/\sigma_1^2 = 1/\sigma_2^2 = 1/\sigma_r^2 = 30$ dB, and $P = 3$ W. In general, all schemes achieve their best sum rate when the relay is located in the middle of the users. The proposed JPA I delivers the best sum-rate performance and has the least sensitivity toward the variation of the location of the relay. The proposed JPA II performs close to JPA I, but it is more sensitive to unequal path loss due to the fact that both users are allocated with an equal amount of power. Refer to the remark of Theorem 1. The A-Opt scheme has similar performance as the proposed JPA II. The baseline MIMO one-way relaying scheme displays the worst sum-rate performance and the highest sensitivity toward unequal path loss.

B. Power Control With Fixed Substream Rate Constraints

Here, the simulation results of various schemes are presented to illustrate the relationship between the average total transmit power consumption and parameters such as SNR and target data rate. The baseline scheme used for comparison is the MIMO one-way relaying scheme. The MIMO one-way relaying scheme uses a DF relay and consumes four time slots to complete the information exchange. To enable fair comparison, the users and the relay in the MIMO one-way relaying scheme jointly minimize the total power consumption subject to substream rate constraints of user 1 and user 2.

Fig. 7 shows the average total transmit power consumption (in watts) in the network versus reference SNR $(1/\sigma_1^2 = 1/\sigma_2^2 = 1/\sigma_r^2)$ of various schemes. The substream target data rate is assumed to be symmetrical, i.e., $R_{1,k} = R_{2,k} = R\,\forall k = \{1, \ldots, M\}$. The fixed parameters are the target data rate, i.e., $R = 2$ bits/s/Hz, and the number of antennas, i.e., $M = 2$. From the figure, it is obvious that the proposed JPC is the most energy-efficient scheme, whereas the baseline MIMO one-way relaying scheme is the most energy-consuming scheme. When the SNR increases, the average total power of all schemes exponentially decreases. Note that the y-axis is in logarithmic scale. At higher SNR, the noise power at each substream is lower; therefore, the target data rate can be easily fulfilled with a lower amount of power. The proposed EPC performs better than the baseline scheme but does not perform as good as the proposed JPC. EPC is inferior when compared with JPC because the common power allocation factor used in EPC is the largest power allocation factor among all substream power allocation factors. The common power allocation factor is used

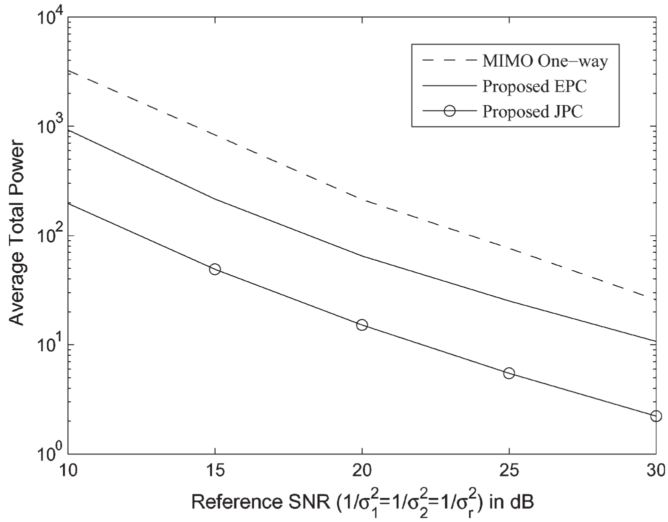

Fig. 7. Average total power consumption versus SNR when $R = 2$ bits/s/Hz and $M = 2$.

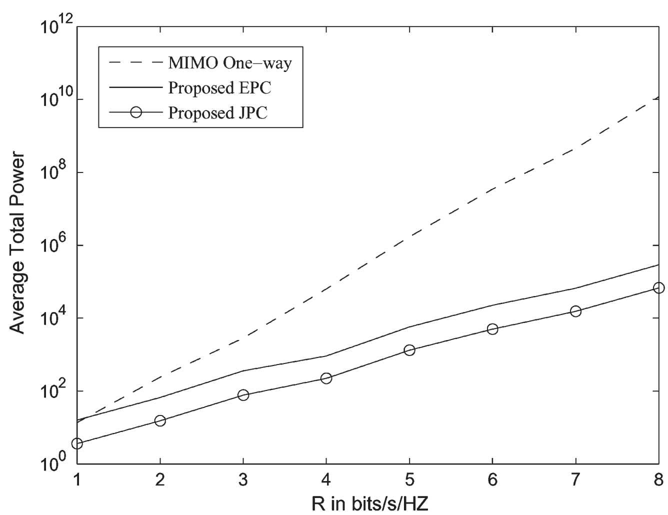

Fig. 8. Average total power versus target rate R when reference SNR $1/\sigma_1^2 = 1/\sigma_2^2 = 1/\sigma_r^2 = 20$ dB and $M = 2$.

to ensure that all subchannels (including the worst subchannel) satisfy the predefined rate constraints. Refer to Section V-B2.

Fig. 8 shows the average total transmit power consumption (in watts) in the network versus target data rate R. Similar to the previous figure, a symmetrical substream target data rate is assumed, i.e., $R_{1,k} = R_{2,k} = R \, \forall k = \{1, \ldots, M\}$. The fixed parameters are $1/\sigma_1^2 = 1/\sigma_2^2 = 1/\sigma_r^2 = 20$ dB and $M = 2$. From the figure, it can be seen that the proposed JPC scheme consumes the lowest amount of power, closely followed by the proposed EPC scheme. In general, the average total power consumption of all schemes increases with data rate R. Recall that the data rate is a logarithmic function of signal power, for any fixed noise power. The baseline MIMO one-way relaying scheme displays the most drastic increase in total power as a function of data rate. In comparison, the proposed JPC and EPC demonstrate subtle increase in power as a function of data rate. These observations verify that the proposed schemes are able to deliver significant power saving, particularly in a system demanding high data rates.

VII. Conclusion

Joint beamforming and power management in the nonregenerative MIMO two-way relaying channels has been studied in this paper. Based on the idea of subchannel alignment, transmit and receive beamformers were designed such that the channel pair can be decomposed into parallel subchannels to enable JPA and JPC. JPA dynamically allocated power to all substreams and nodes to maximize the sum rate, subject to a total power constraint. On the other hand, JPC minimized the total transmission power while satisfying the predefined target data rates. The convexity of the power allocation and power control formulations were determined. The nonconcave power allocation utility function was approximated by a concave upper bound, whereas the power control formulation was transformed from a geometric program into equivalent convex form to facilitate the use of efficient convex optimization techniques in solving the optimization problems. Simulation results demonstrate that the proposed joint beamforming and power management scheme is able to deliver significant sum-rate improvement or achieve substantial transmission power saving when compared with existing schemes.

Appendix
Proof of Theorem 1

The upper bound of the objective function can be derived from the inequality of the kth substream SNR of user 1, i.e., $\gamma_{1,k} = t_{1,k}\tilde{b}_k\tilde{c}_k/(t_{2,k}\tilde{a}_k + t_{3,k}\tilde{b}_k + t_{4,k}\tilde{c}_k + t_{5,k})$, and the kth substream SNR of user 2, i.e., $\gamma_{2,k} = u_{1,k}\tilde{a}_k\tilde{c}_k/(u_{2,k}\tilde{a}_k + u_{3,k}\tilde{b}_k + u_{4,k}\tilde{c}_k + u_{5,k})$. Represent $x = \tilde{a}_k$, $y = \tilde{b}_k$, and $z = \tilde{c}_k$ and omit subscript k of constants $t_{j,k}$ for simplicity, the kth substream SNR of user 1 can be expressed as $\gamma_{1,k} = t_1yz/(t_2x + t_3y + t_4z + t_5)$. Using the fact that $x \geq 0$, the kth substream SNR of user 1 can be upper bounded as follows:

$$\frac{t_1yz}{t_2x + t_3y + t_4z + t_5} \leq \frac{t_1(x+y)z}{t_2x + t_3y + t_4z + t_5} \quad (22)$$

$$\leq \frac{t_1(x+y)z}{t_n(x+y) + t_4z + t_5} \quad (23)$$

$$\leq \frac{(x+y)z}{t_a(x+y) + t_bz} \quad (24)$$

where the second inequality is obtained by choosing $t_n = \min(t_2, t_3)$, and the third inequality is obtained by omitting t_5 and normalizing $t_a = t_n/t_1$ and $t_b = t_4/t_1$. Since $y \geq 0$, similar steps can be applied to the kth substream SNR of user 2 to get the following upper bound:

$$\frac{u_1xz}{u_2x + u_3y + u_4z + u_5} \leq \frac{(x+y)z}{u_a(x+y) + u_bz} \quad (25)$$

where $u_a = \min(u_2, u_3)/u_1$, and $u_b = u_4/u_1$.

Denote f_{upper} as the upper-bound objective function obtained using the derived kth substream SNR inequality. The proof of the concavity of f_{upper} is described in the following. Since the positive weighted sum of any convex (or concave) functions preserves convexity (or concavity) [17], we only need to check the concavity of the kth substream rate function upper bound of user 1, i.e., $\log_2(1 + (x + y)z/(t_a(x + y) + t_b z))$, and the kth substream rate function upper bound of user 2, i.e., $\log_2(1 + (x + y)z/(u_a(x + y) + u_b z))$. We focus on developing the proof of the concavity of the rate function of user 1. Define function $h(w, z) = \log_2(1 + wz/(t_a w + t_b z))$ and $w(x, y) = x + y$. From the Hessian matrix of function h

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 h}{\partial w^2} & \frac{\partial^2 h}{\partial w \partial z} \\ \frac{\partial^2 h}{\partial z \partial w} & \frac{\partial^2 h}{\partial z^2} \end{bmatrix}$$

we have the following inequalities:

$$\frac{\partial^2 h}{\partial w^2} = -\frac{t_b z^2 \left(2t_a^2 w + 2t_a wz + 2t_a t_b z + t_b z^2\right)}{\ln 2 (t_a w + t_b z)^2 (t_a w + t_b z + wz)^2} \leq 0 \quad (26)$$

$$\frac{\partial^2 h}{\partial z^2} = -\frac{t_a w^2 \left(t_a w^2 + 2t_a t_b w + 2t_b wz + 2t_b^2 z\right)}{\ln 2 (t_a w + t_b z)^2 (t_a w + t_b z + wz)^2} \leq 0 \quad (27)$$

whereas the determinant of the Hessian matrix can be represented by the following inequality:

$$\det(\mathbf{H}) = \frac{2t_a t_b w^2 z^2 (\ln 2)^{-2}}{(t_a w + t_b z)^2 (t_a w + t_b z + wz)^3} \geq 0. \quad (28)$$

since constants $t_a \geq 0$ and $t_b \geq 0$ and variables $w \geq 0$ and $z \geq 0$. Recall that the determinant of the Hessian matrix corresponds to the product of two eigenvalues. For a concave function, each eigenvalue is nonpositive. By observing the inequalities from (26)–(28), matrix $\mathbf{H}$ is proved to be a negative semidefinite matrix. This indicates that function $h(w, z)$ is concave w.r.t. (w, z). Recall that a composition with an affine function preserves concavity (or convexity) [17]. Since function $w(x, y)$ is affine, we can conclude that function $h(w(x, y), z) = \log_2(1 + (x + y)z/(t_a(x + y) + t_b z))$ is concave w.r.t. (x, y, z). Similar steps can be used to prove the concavity of the rate function of user 2, i.e., $\log_2(1 + (x + y)z/(u_a(x + y) + u_b z))$. Since concavity is closed under positive summation, it can be concluded that upper-bound objective function f_{upper} is jointly concave w.r.t. all input parameters $\tilde{a}_1, \ldots, \tilde{a}_M$, $\tilde{b}_1, \ldots, \tilde{b}_M$, and $\tilde{c}_1, \ldots, \tilde{c}_M$, and the theorem is proved. ■

ACKNOWLEDGMENT

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