Signals and Systems

Lecture 1

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Teacher's coordinates

Teacher:

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Logistics of the course

- Lectures - 15 hours over 8-9 weeks

- Problem Classes – 7-8 hours over 8-9 weeks

- Assessment – 100% by examination in June

- Handouts in the form of pdf slides are available at http://www.commsp.ee.ic.ac.uk/~tania/

- Text Books:

  - B.P. Lathi, "Linear Systems and Signals", 2nd Ed., Oxford University Press

  - A.V. Oppenheim & A.S. Willsky "Signals and Systems", Prentice Hall

Aims and Objectives of Signals and Systems

- The concepts of signals and systems arise in a variety of fields such as, communications, aeronautics, bio-engineering, energy, circuit design and others.

- Although the physical attributes of the signals and systems involved in the above disciplines are different, all signal and systems have basic features in common.

- **The aim of this course is to provide the fundamental and universal tools for the analysis of signals.**

- Furthermore, the course aims at teaching the analysis and design of basic systems independently of the domain of application.

Aims and Objectives cont.

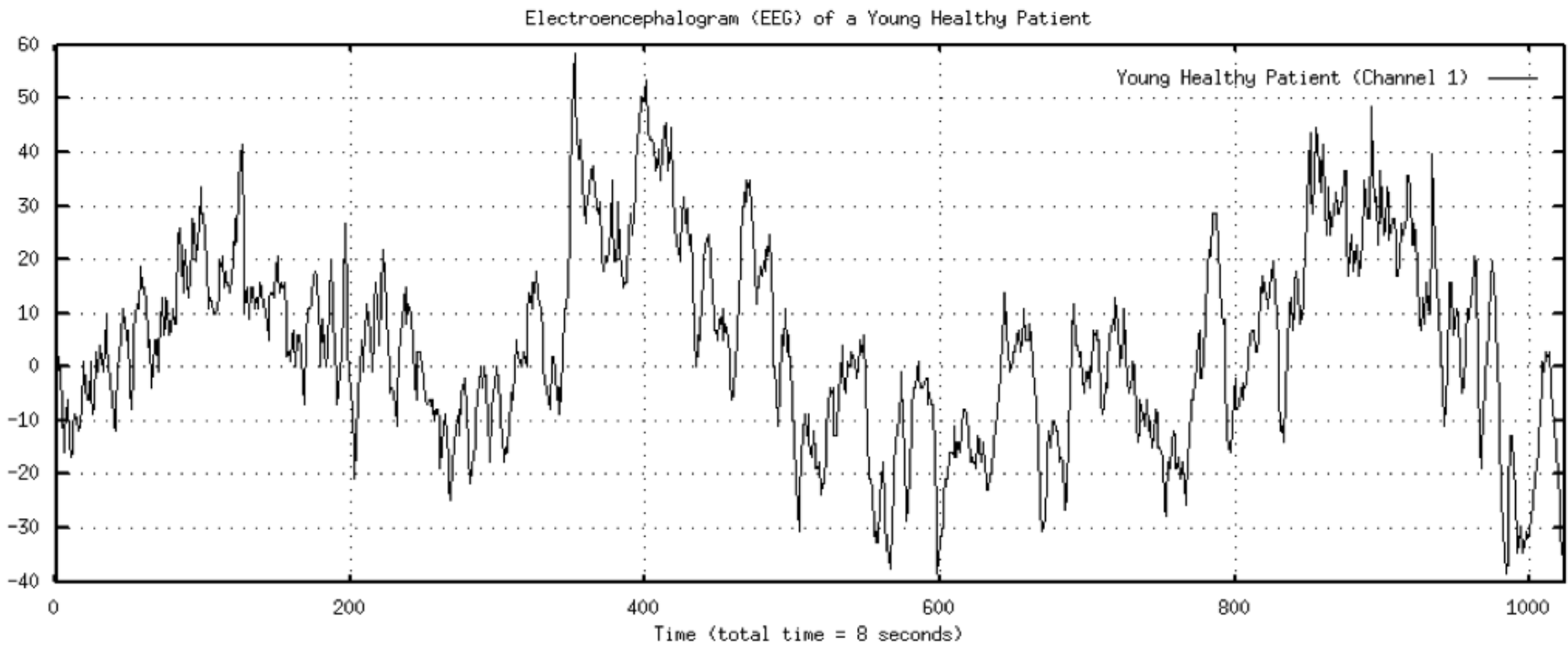By the end of the course, you will have understood:

- Basic signal analysis

- Basic system analysis

- Time-domain system analysis including convolution

- Laplace and Fourier Transform

- System analysis in Laplace and Fourier domains

- Filter design

- Sampling theory

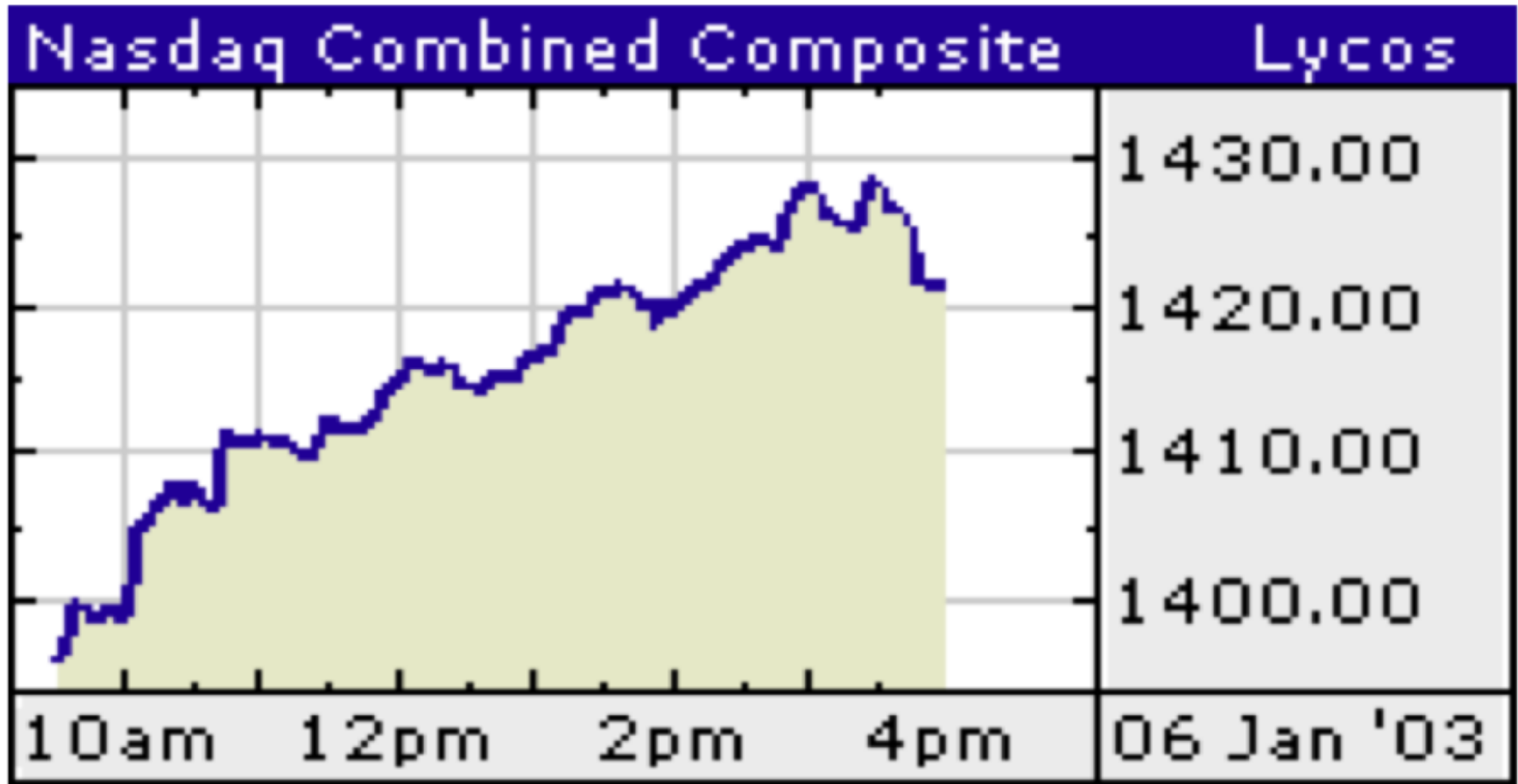- Basics on z-transform

Today's Lecture

In this lecture we will talk about:

- Examples of signals
- Some useful signal operations
- Classification of signals
- Some specific widely used signals:
    - Unit step function
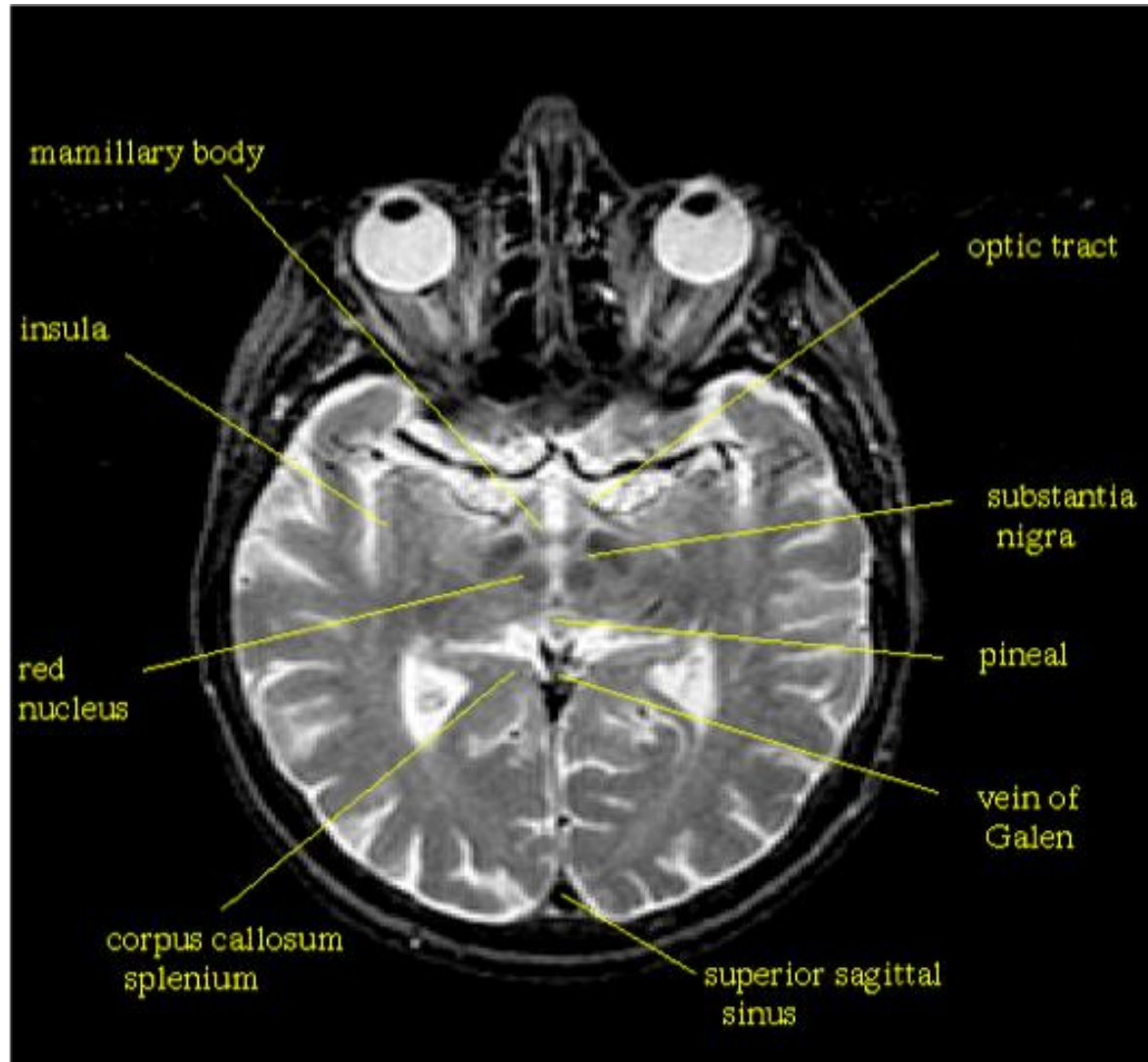   - Unit impulse function
    - The exponential function

Example of an Electro-Enchephalo-Gram (EEG) signal

Electroencephalogram (EEG) of a Young Healthy Patient

Example of a stock market signal

Nasdaq Combined Composite — Lycos

1430.00
1420.00
1410.00
1400.00

10am   12pm   2pm   4pm   06 Jan '03

# Magnetic Resonance Image (MRI).
This is a signal in 2 dimensions.

Useful signal operations: time shifting

- Consider a signal $x(t)$ in continuous – time shown in the figure right.

- The signal $x(t)$ may be delayed by T units of time; in that case the signal $\phi(t) = x(t - T)$ is obtained.

- The signal $x(t)$ may be advanced by T units of time; in that case the signal $\phi(t) = x(t + T)$ is obtained.
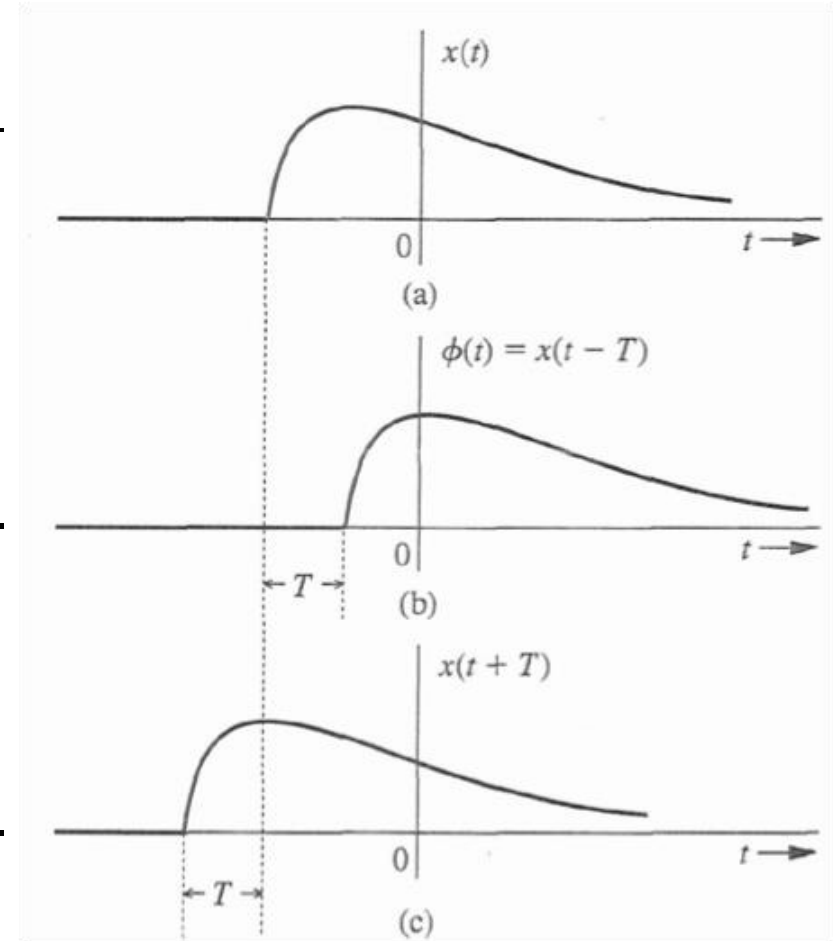
Useful signal operations cont.: time scaling

- Consider a signal $x(t)$ in continuous – time shown in the figure right.

- The signal $x(t)$ may be compressed by a factor of 2; in that case the signal $\phi(t) = x(2t)$ is obtained.

- The signal $x(t)$ may be stretched by a factor of 2; in that case the signal $\phi(t) = x\left(\frac{t}{2}\right)$ is obtained.
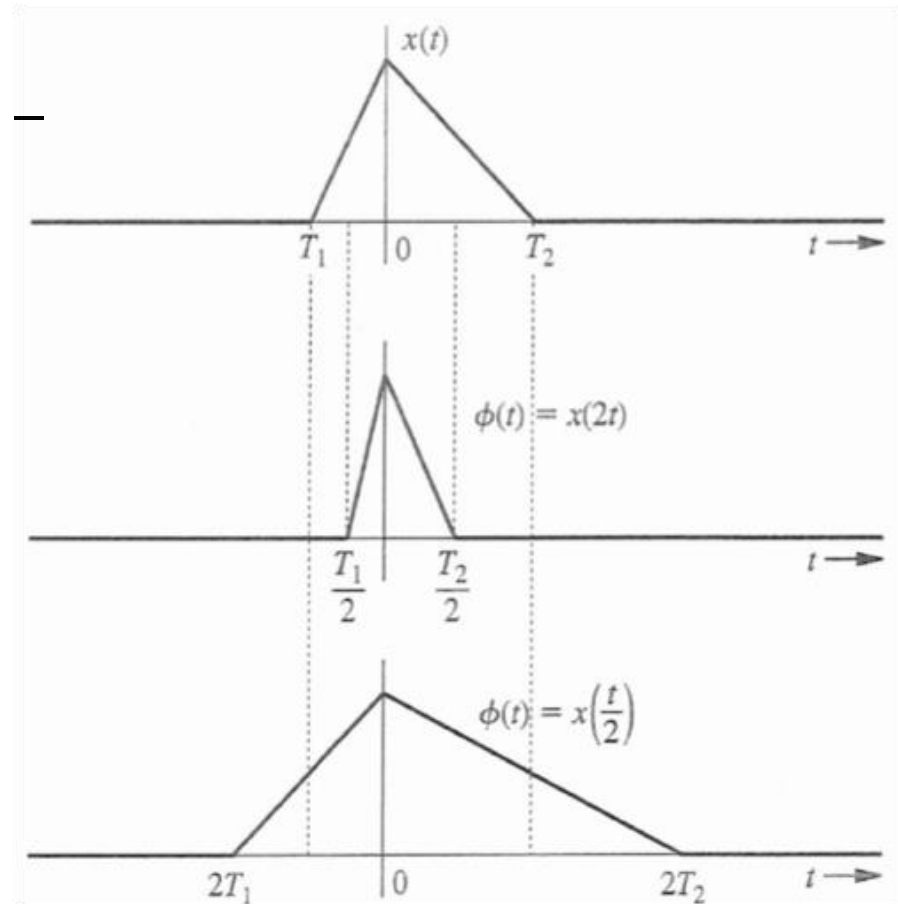
Classification of signals

- Signals may be classified into:

    - Energy and power signals
    - Discrete – time and continuous – time signals
    - Analogue and digital signals
    - Deterministic and probabilistic signals
    - Periodic and aperiodic signals
    - Even and odd signals

Energy of a signal

- How do we measure the "size" of a signal $x(t)$?

  - By the signal energy E_x

  $$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt$$

  - For a complex-valued signal the above relationship becomes:

  $$E_x = \int_{-\infty}^{\infty} |x^2(t)|\,dt$$

  - The energy of a signal must be finite. This implies that:
  $$\lim_{t \to \pm\infty} |x(t)| = 0$$

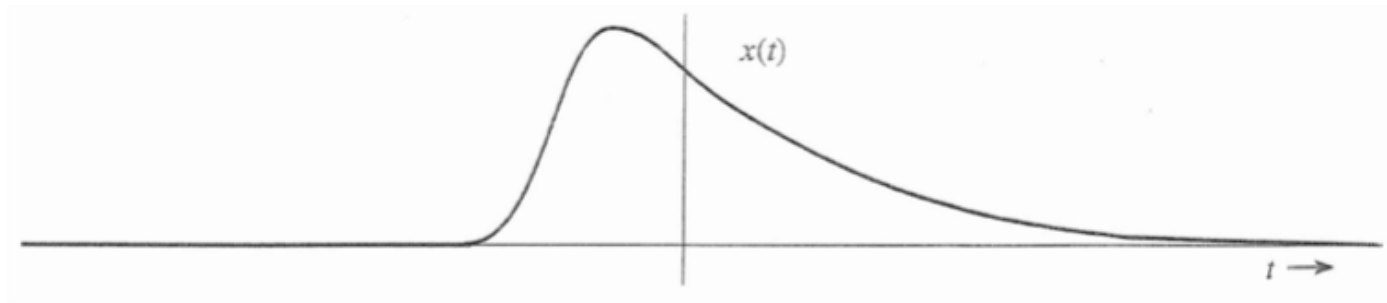- If $\lim_{t \to \pm\infty} |x(t)| \neq 0$ we must use the signal power instead

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt$$

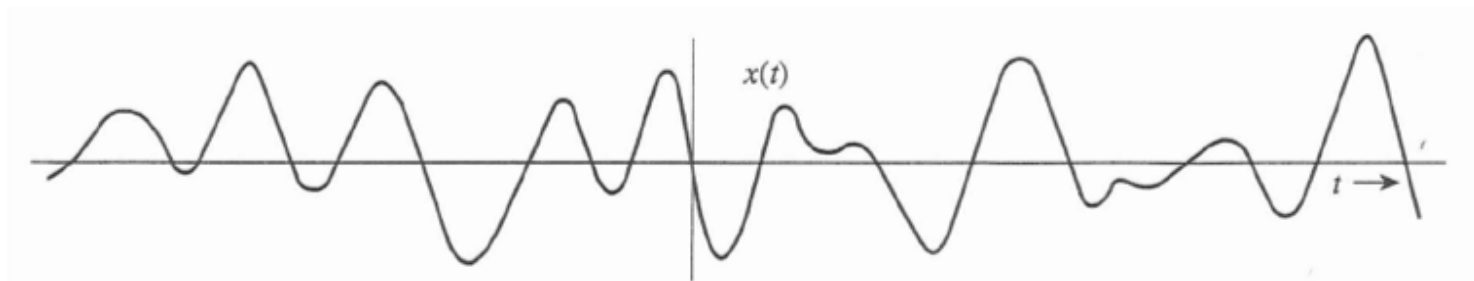- For a complex – valued signal the above relationship becomes:

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x^2(t)|dt$$

Energy and power cont.

- Example of a signal with finite energy, which implies that the power of the signal is zero.
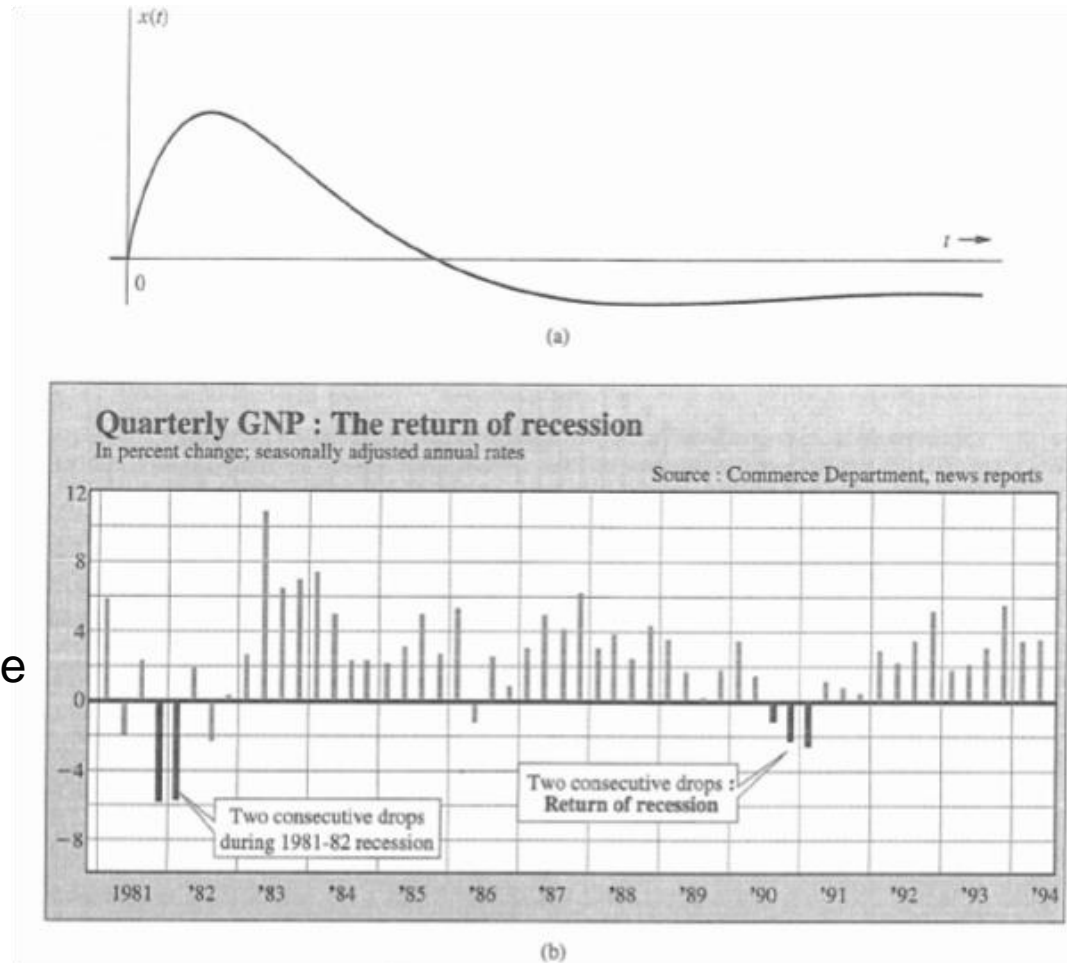

- Example of a signal with finite power, which implies that the energy of the signal is infinite.

- Example of a continuous – time signal. This is defined for any value of time.

- Example of a discrete – time signal.
  - This is defined for certain instants of time only.
  - It arises from a continuous – time signal if only its values at these certain time instants are kept.
 - This process is called **sampling**.

(a)

Quarterly GNP : The return of recession
In percent change; seasonally adjusted annual rates

Source : Commerce Department, news reports

Two consecutive drops during 1981-82 recession

Two consecutive drops : Return of recession

(b)

Analog and Digital Signals
This classification refers to the value of the signal $x(t)$

- An analog signal can take infinite number of values.

- A digital signal can take only a finite number of values.
  - It arises from an analog signal if all values are replaced with a finite number of values.
  - The total error arising from this process must be as small as possible.
  - This process is called **quantization**.

**Imperial College
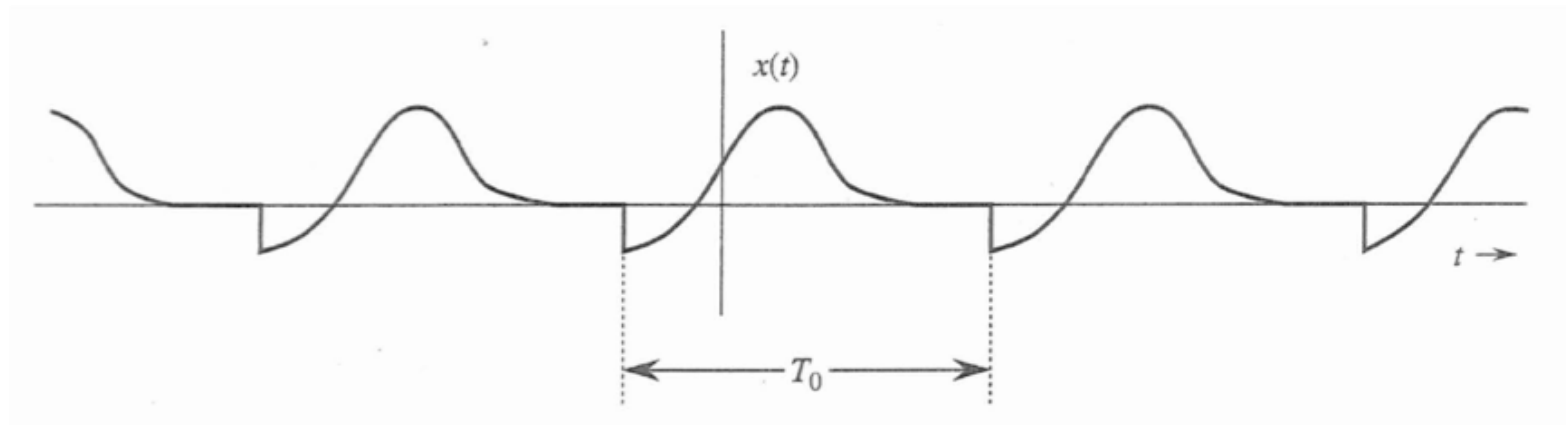London**
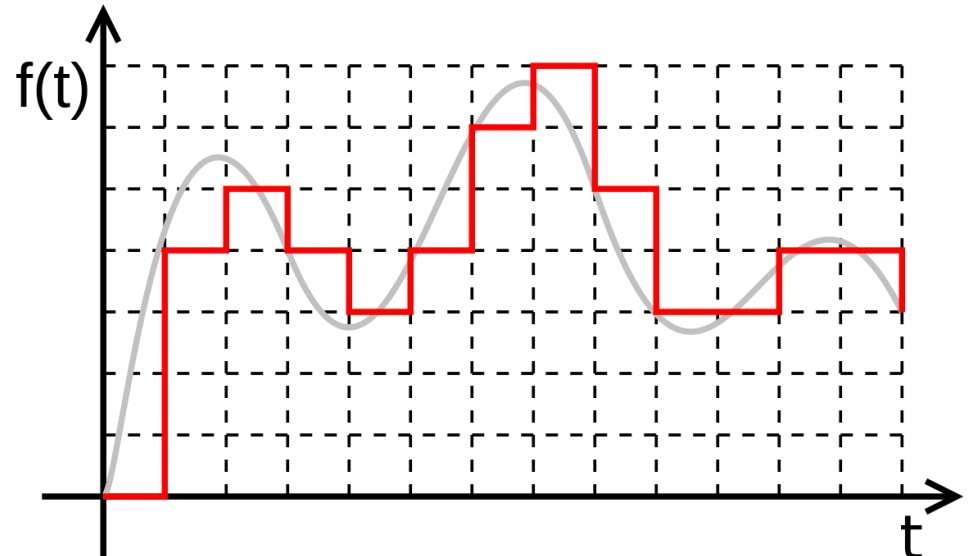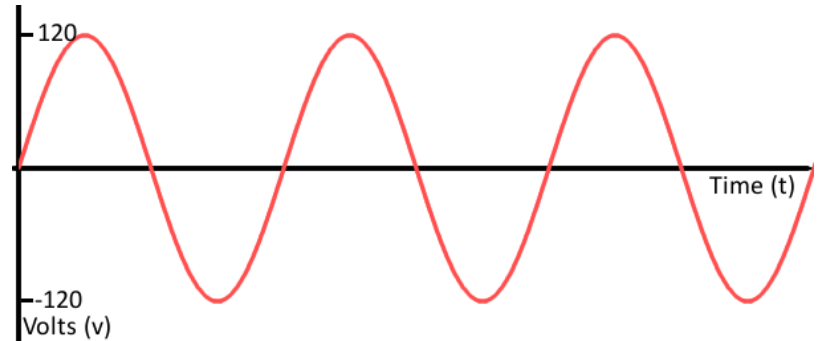
We now have 4 types of signals
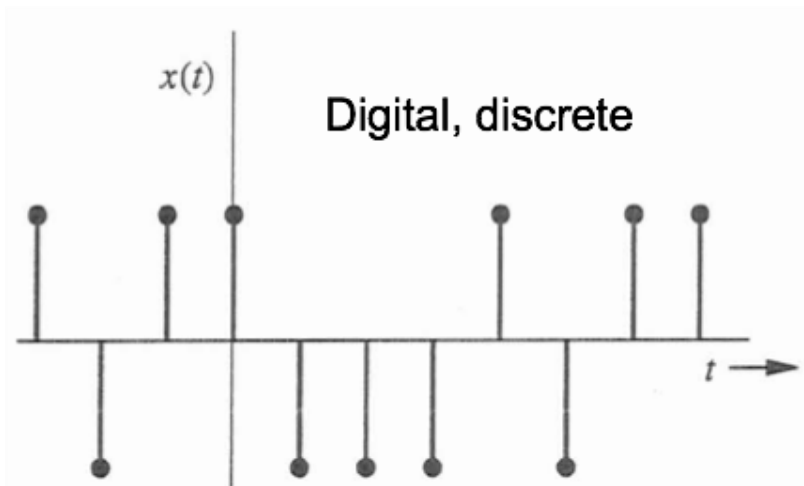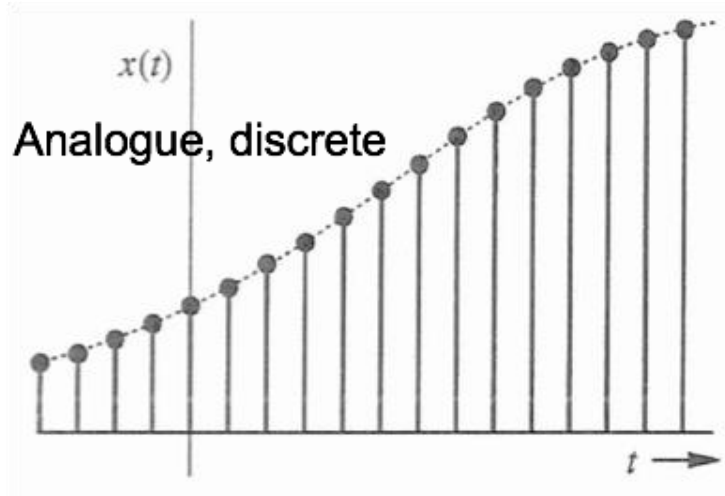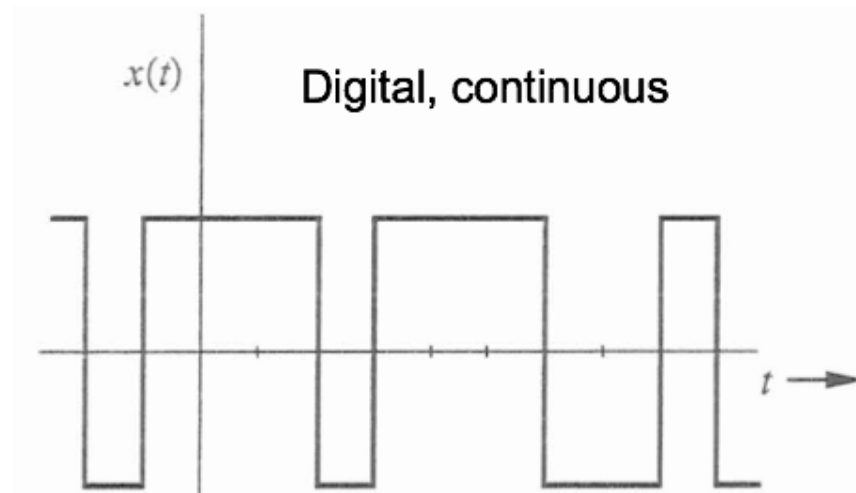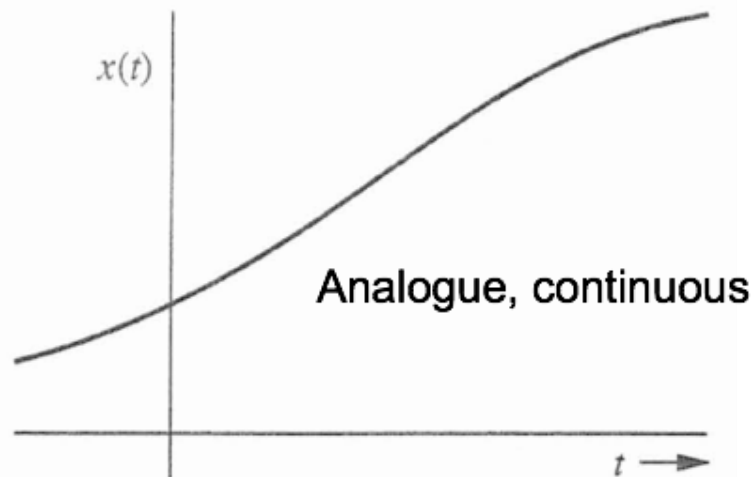
Periodic and Aperiodic Signals

- A signal is said to be periodic if for some constant T_0 the following condition holds:
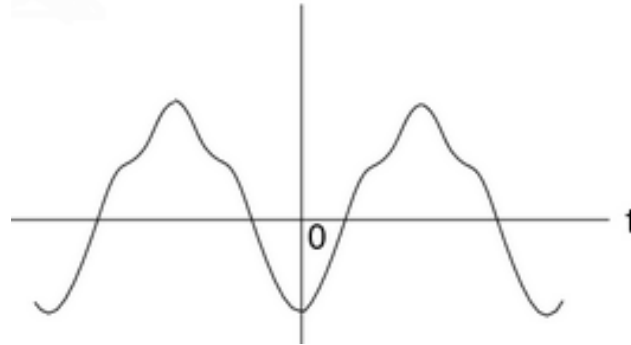
$$x(t + T_0) = x(t), \forall t$$

- The smallest T_0 that satisfies the above relationship is called the fundamental period of the signal.
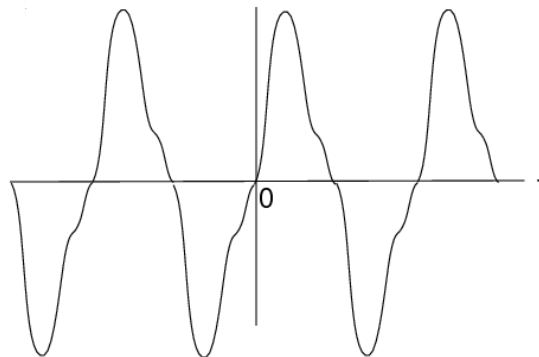
Even and Odd Signals

- An even signal remains the same if you rotate it along the vertical axis. In mathematical terms this property is defined as $x(-t) = x(t)$.

- An odd signal gets reflected along the horizontal axis if you rotate it along the vertical axis. In mathematical terms this property is defined as $x(-t) = -x(t)$.
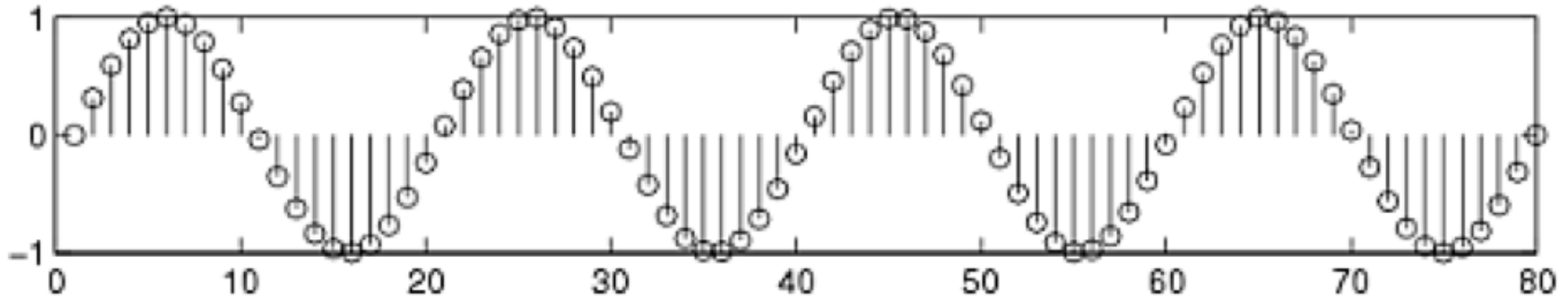
Easy problems

- Verify that the signal $x(t) + x(-t)$ is always even.
- Verify that the signal $x(t) - x(-t)$ is always odd.

Based on the above, any signal can be written as a the sum of an even and an odd signal as follows:

$$x(t) = \frac{1}{2}[x(t) + x(-t)] + \frac{1}{2}[x(t) - x(-t)]$$

Deterministic and Stochastic Signals

- The values of a deterministic signal can be obtained from a closed – form mathematical expression. For example, $x(t) = \sin(t)$.

- The values of a stochastic signal can only be given as the outputs of a probabilistic model.

  Experiment. Drop a coin 1000 times and create the following 1000 – sample digital signal: Every time you get **head** the value of your signal will be $+1$, every time you get **tails** the value of your signal will be -1.

  - Can you tell what is the value of the signal at time instant n?
  - Can you tell the probability of the value being $+1$ or -1?
  - What is approximately the mean (average value) of the signal you created?

Unit step function

- The unit step function is defined as follows:

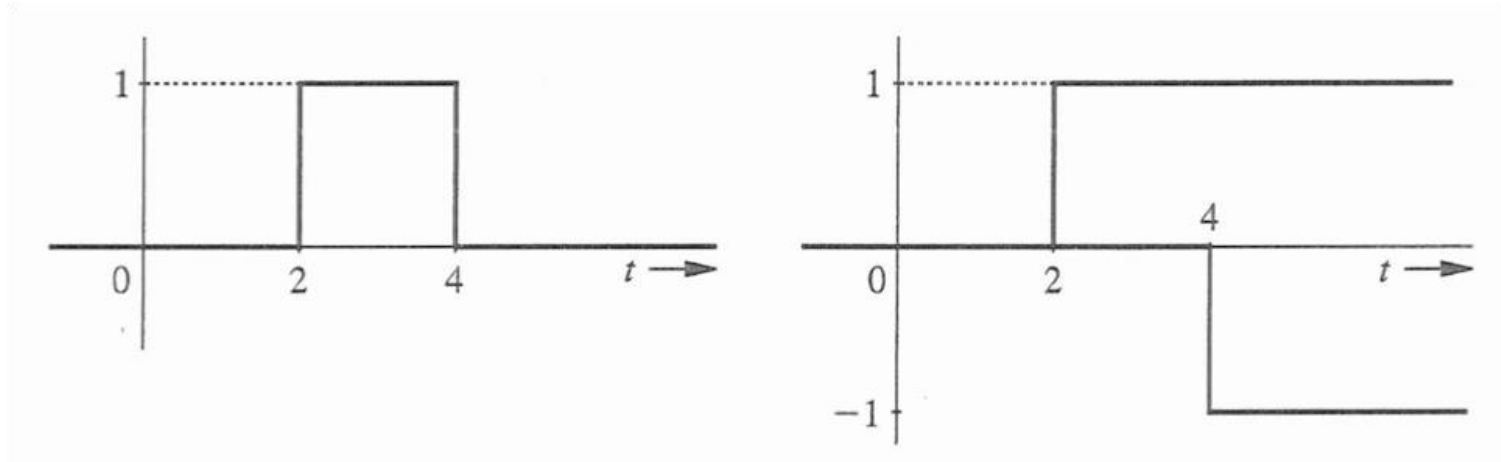$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

- The unit step function is often used to describe a signal that starts at $t = 0$.

- For example, consider the everlasting exponential signal $x(t) = e^{-t}$.

- The causal form of the above signal is $e^{-t}u(t)$.
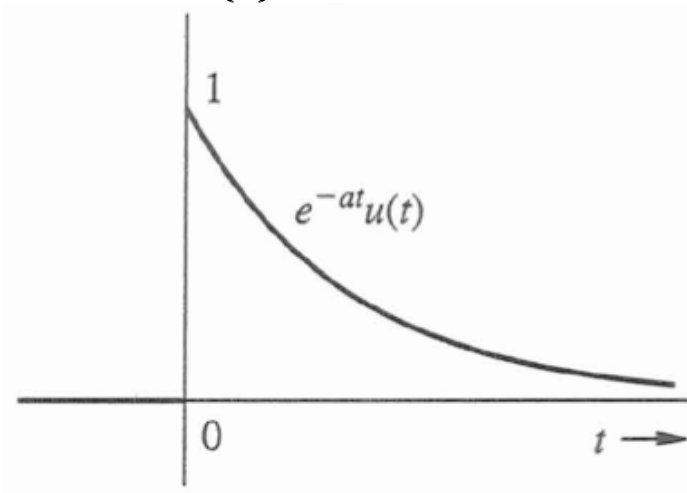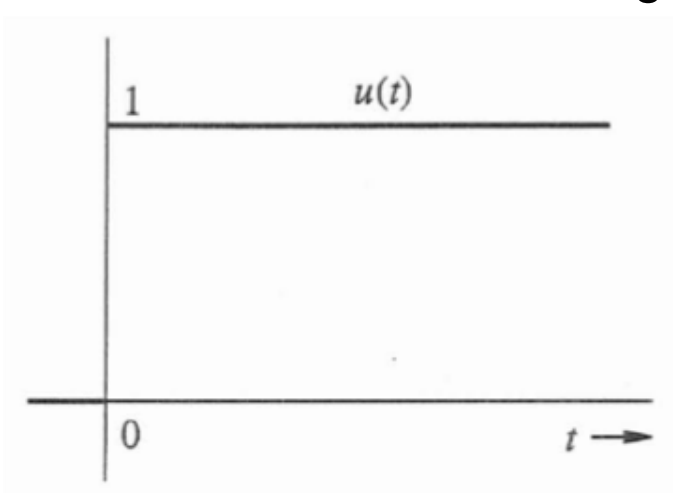
Pulse signals

- A pulse signal can be generated using two step functions, as for example:
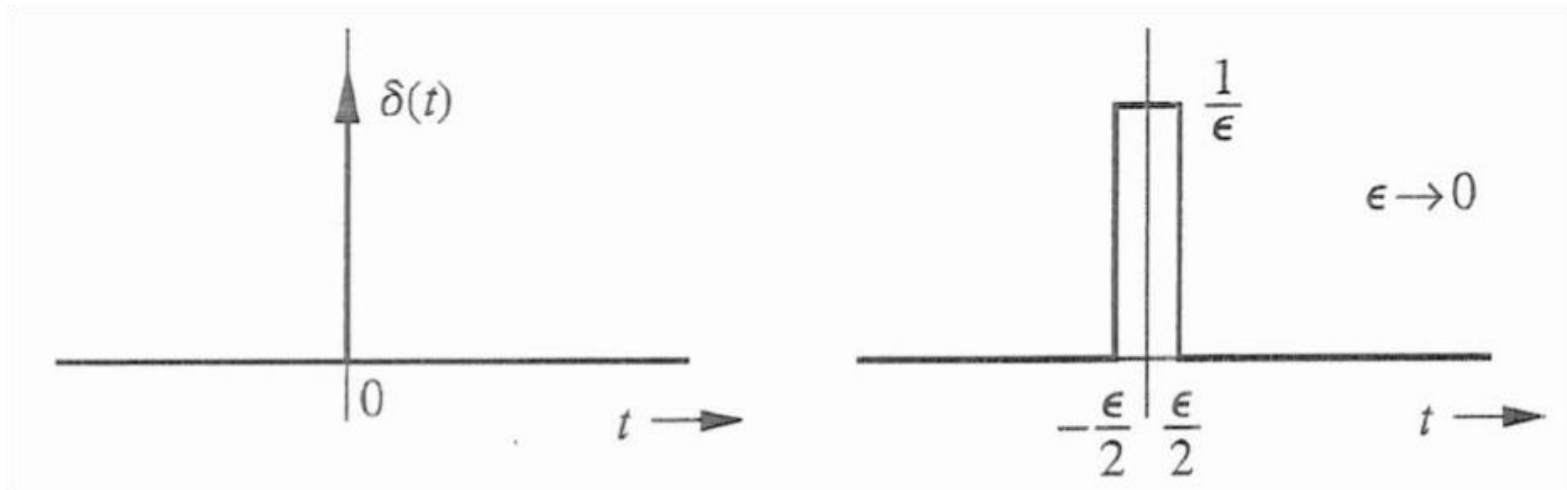
$$x(t) = u(t-2) - u(t-4)$$

Unit impulse (Dirac) function

- The Dirac function is defined as follows:

$$\delta(t) = 0, t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

- The Dirac function is the limit of a family of functions $\delta_\epsilon(t)$ when $\epsilon \rightarrow 0$. The simplest of these functions is the rectangular pulse shown in the figure below right. The width and height depend on ϵ but the entire area under the function is equal to 1.

Sampling property of the unit impulse function

- Since the unit impulse function is non – zero only at $t = 0$, for any function $\phi(t)$ we have

$$\phi(t)\delta(t) = \phi(0)\delta(t)$$
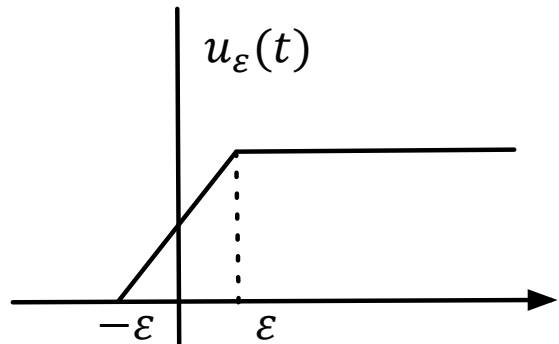
- From the above we get:

$$\int_{-\infty}^{\infty} \phi(t)\delta(t)dt = \int_{-\infty}^{\infty} \phi(0)\delta(t)dt = \phi(0)\int_{-\infty}^{\infty} \delta(t)dt = \phi(0)$$
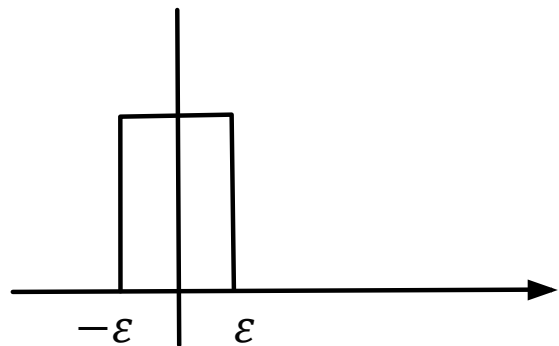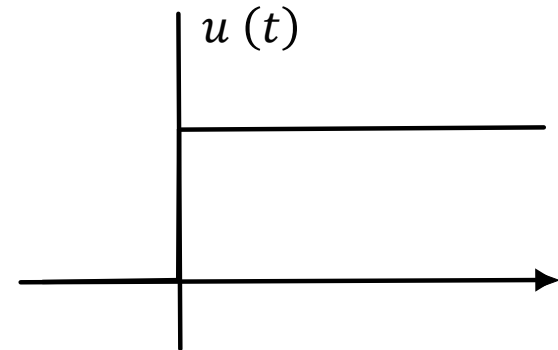
- Furthermore, we have:

$$\int_{-\infty}^{\infty} \phi(t)\delta(t - T)dt = \int_{-\infty}^{\infty} \phi(\tau + T)\delta(\tau)d\tau = \phi(T)$$
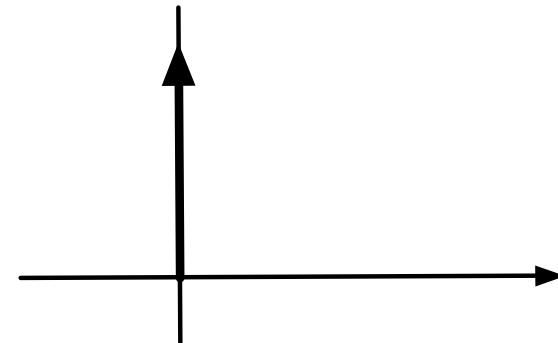
Unit Step and Unit Impulse function

- It is proven that $\frac{du(t)}{dt} = \delta(t)$

The Exponential Function e^{st}

- The exponential function is very important in science and engineering.
- The parameter s is a complex variable given by $s = \sigma + j\omega$.
- Therefore, $e^{st} = e^{(\sigma+j\omega)t} = e^{\sigma t}\, e^{j\omega t} = e^{\sigma t}(\cos\omega t + j\sin\omega t)$
- If s^* is the complex conjugate of s then $s = \sigma - j\omega$

$$e^{s^*t} = e^{(\sigma-j\omega)t} = e^{\sigma t}\, e^{-j\omega t} = e^{\sigma t}(\cos\omega t - j\sin\omega t)$$

$$e^{\sigma t}\cos\omega t = \frac{1}{2}\left(e^{st} + e^{s^*t}\right)$$

$$e^{\sigma t}\sin\omega t = \frac{1}{2j}\left(e^{st} - e^{s^*t}\right)$$

- The exponential function can be used to model a large class of signals.
  - A constant $k = ke^{0t}, s = 0$
  - A monotonic real exponential $e^{\sigma t}, s = \sigma,\ \omega = 0$
  - A complex sinusoid $\cos\omega t \pm j\sin\omega t,\ \sigma = 0, s = \pm j\omega$
  - An exponentially varying real sinusoid $e^{\sigma t}\cos\omega t = \mathrm{Re}\{e^{st}\}, s = \sigma \pm j\omega$

Discrete-Time Exponential γ^n

- A continuous $-$ time exponential e^{st} can be expressed in alternate form as $e^{st} = \gamma^t$ with $\gamma = e^s$.
- Similarly for discrete time exponentials we have $e^{\lambda n} = \gamma^n$.
- When $\mathrm{Re}\{\lambda\} < 0$ then $\left| e^\lambda \right| = |\gamma| < 1$ and the exponential decays.
- When $\mathrm{Re}\{\lambda\} > 0$ then $\left| e^\lambda \right| = |\gamma| > 1$ and the exponential grows.
- When $\mathrm{Re}\{\lambda\} = 0$ then $\left| e^\lambda \right| = |\gamma| = 1$. The exponential is of constant-amplitude and oscillates.
- If we depict λ and γ in the two $-$ dimensional plane, we can see the mapping between the two variables (look at next slide).

Mapping from λ to γ
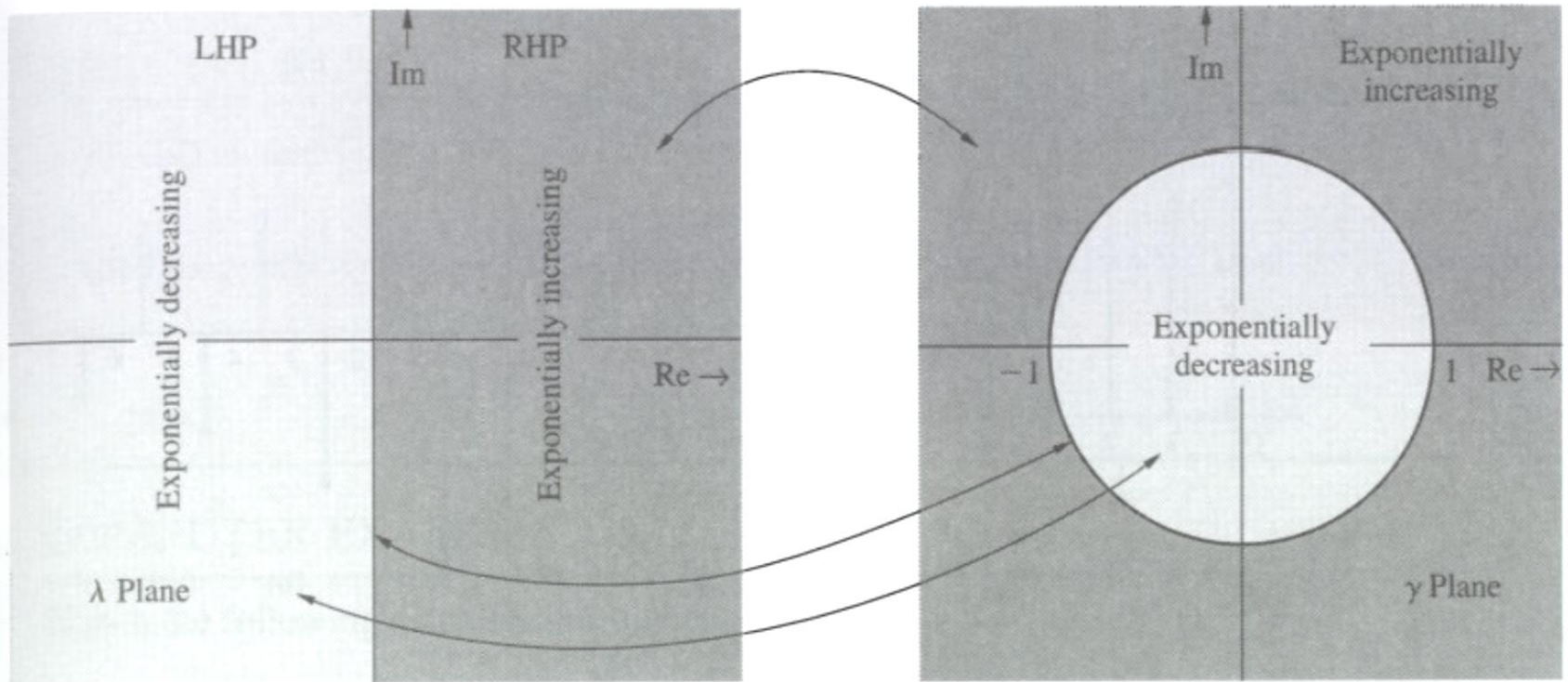
I hope you enjoyed this lecture.

Thank you for attending and see you again in the next class!