Imperial College London

Signals and Systems

Tutorial Sheet 5 – More on Laplace Transforms

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Problem 1 (a)

Using Laplace transform solve the following differential equation:

(a) $(D^2 + 3D + 2)y(t) = Df(t)$ if $y(0^-) = \dot{y}(0^-) = 0$ and $f(t) = u(t)$.

Recall that $D = \frac{d}{dt}(\cdot)$.

We take the Laplace transform in both sides:

$\mathcal{L}\{(D^2 + 3D + 2)y(t)\} = \mathcal{L}\{Df(t)\} \Rightarrow$

$\mathcal{L}\{D^2y(t)\} + \mathcal{L}\{3Dy(t)\} + \mathcal{L}\{2y(t)\} = \mathcal{L}\{Df(t)\} \Rightarrow$

$s^2Y(s) - sy(0^-) - \dot{y}(0^-) + 3sY(s) - 3y(0^-) + 2Y(s) = sF(s) - f(0^-) \Rightarrow$

$s^2Y(s) + 3sY(s) + 2Y(s) = sF(s)$

$f(t) = u(t) \Rightarrow F(s) = \frac{1}{s}$

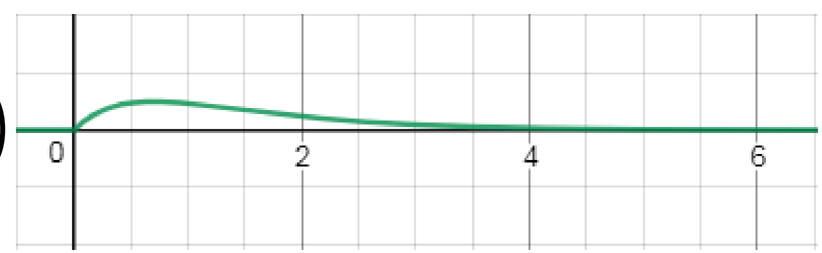
Therefore,

$s^2Y(s) + 3sY(s) + 2Y(s) = 1 \Rightarrow (s^2+3s + 2)Y(s) = 1 \Rightarrow$

$Y(s) = \frac{1}{(s+2)(s+1)} = \frac{1}{s+1} - \frac{1}{s+2}$

$y(t) = \mathcal{L}^{-1}(Y(s)) = \mathcal{L}^{-1}\left(\frac{1}{s+1}\right) - \mathcal{L}^{-1}\left(\frac{1}{s+2}\right)$

$= (e^{-t} - e^{-2t})u(t)$



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Problem 1 (b)

(b) $(D^2 + 4D + 4)y(t) = (D + 1)f(t)$ if $y(0^-) = 2, \dot{y}(0^-) = 1$ and

$f(t) = e^{-t}\, u(t) \Rightarrow F(s) = \frac{1}{s+1}$

We take the Laplace transform in both sides:

$\mathcal{L}\{(D^2 + 4D + 4)y(t)\} = \mathcal{L}\{(D + 1)f(t)\} \Rightarrow$

$\mathcal{L}\{D^2 y(t)\} + \mathcal{L}\{4Dy(t)\} + \mathcal{L}\{4y(t)\} = \mathcal{L}\{Df(t)\} + \mathcal{L}\{f(t)\} \Rightarrow$

$s^2Y(s) - sy(0^-) - \dot{y}(0^-) + 4sY(s) - 4y(0^-) + 4Y(s) =$

$sF(s) - f(0^-) + F(s) \Rightarrow$

$s^2Y(s) - 2s - 1 + 4sY(s) - 8 + 4Y(s) = sF(s) - f(0^-) + F(s) \Rightarrow$

$s^2Y(s) + 4sY(s) + 4Y(s) - 2s - 9 = \frac{s}{s+1} - 0 + \frac{1}{s+1} = 1 \Rightarrow$

$$s^2Y(s) + 4sY(s) + 4Y(s) = 2s + 9 + \frac{s+1}{s+1} = 2s + 10$$

Therefore,

$s^2Y(s) + 4sY(s) + 4Y(s) = 2s + 10 \Rightarrow (s + 2)^2 Y(s) = 2s + 10$

Problem 1 (b) cont.

(b) $Y(s) = \frac{2s+10}{(s+2)^2}$

In that case we have repeated roots, i.e., there is a factor $(s+2)^2$ in the denominator.

Recall: In general, if there is a factor $(s+a)^n$ in the denominator, the partial fraction expansion contains the term $\sum_{i=1}^{n} \frac{c_i}{(s+a)^i}$.

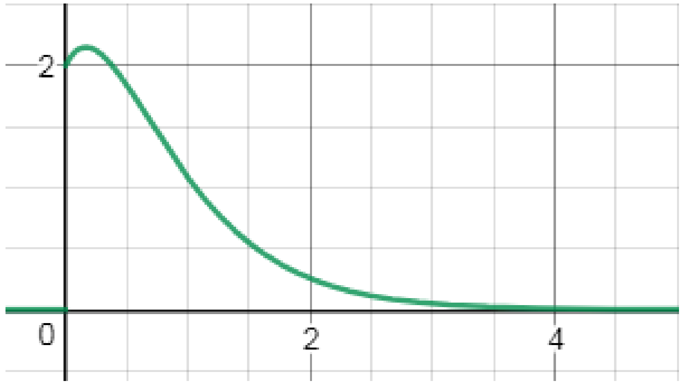
$$\frac{2s+10}{(s+2)^2} = \frac{A}{s+2} + \frac{B}{(s+2)^2} = \frac{A(s+2)+B}{(s+2)^2}$$

$A = 2, B = 6$

$$Y(s) = \frac{2s+10}{(s+2)^2} == \frac{2}{s+2} + \frac{6}{(s+2)^2}$$

$$y(t) = \mathcal{L}^{-1}(F(s)) = (2+6t)e^{-2t}u(t)$$

For the last term refer to Problem 1(b) Class 4.



Problem 1 (c)

Using Laplace transform solve the following differential equation:

(c) $(D^2 + 6D + 25)y(t) = (D + 2)f(t)$ if $y(0^-) = \dot{y}(0^-)$=1 and $f(t) = 25u(t)$.

We take the Laplace transform in both sides:

$\mathcal{L}\{(D^2 + 6D + 25)y(t)\} = \mathcal{L}\{(D + 2)f(t)\} \Rightarrow$

$\mathcal{L}\{D^2 y(t)\} + \mathcal{L}\{6Dy(t)\} + \mathcal{L}\{25y(t)\} = \mathcal{L}\{Df(t)\} + \mathcal{L}\{2f(t)\} \Rightarrow$

$s^2Y(s) - sy(0^-) - \dot{y}(0^-) + 6sY(s) - 6y(0^-) + 25Y(s)$

$= sF(s) - f(0^-) + 2F(s) \Rightarrow$

$s^2Y(s) - s - 1 + 6sY(s) - 6 + 25Y(s) = sF(s) + 2F(s)$

$f(t) = 25u(t) \Rightarrow F(s) = \frac{25}{s}$

Therefore,

$s^2Y(s) + 6sY(s) + 25Y(s) = s + 7 + 25 + \frac{50}{s} \Rightarrow$

$(s^2+6s + 25)Y(s) = \frac{s^2+32s+50}{s} \Rightarrow Y(s) = \frac{s^2+32s+50}{s(s^2+6s+25)}$

Problem 1 (c) cont.

$Y(s) = \frac{s^2+32s+50}{s(s^2+6s+25)}$

Partial fraction expansion in that case gives:

$$Y(s) = \frac{s^2+32s+50}{s(s^2+6s+25)} = \frac{A}{s} + \frac{Bs+C}{(s^2+6s+25)}$$

$A + B = 1$ (1)

$6A + C = 32$ (2)

$25A = 50 \Rightarrow A = 2$ (3)

From (1), (2) and (3) we have $B = -1, C = 20$.

$$Y(s) = \frac{s^2+32s+50}{s(s^2+6s+25)} = \frac{2}{s} + \frac{-s+20}{(s^2+6s+25)}$$

For the first term we have: $\mathcal{L}^{-1}\left\{\frac{2}{s}\right\} = 2u(t)$

For the second term we use the Laplace transform pair that we used in Problem 3(b) Class 4. Please see next slide.

Problem 1 (c) cont.

$\frac{-s+20}{s^2+6s+25}$. For this function we can use the Laplace transform pair:

$$\mathcal{L}\{re^{-at}\cos(bt+\theta)\,u(t)\} = \frac{As+B}{s^2+2as+c}$$

$$r = \sqrt{\frac{A^2c+B^2-2ABa}{c-a^2}},\ \ \theta = \tan^{-1}\left(\frac{Aa-B}{A\sqrt{c-a^2}}\right),\ \ b = \sqrt{c-a^2}$$

$A = -1, B = 20, a = 3, c = 25$, therefore,

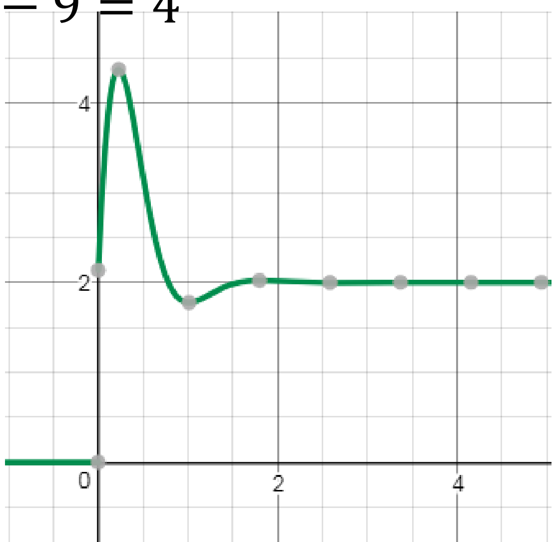
$$r = \sqrt{\frac{(-1)^2\cdot 25+20^2-2\cdot(-1)\cdot 20\cdot 3}{25-3^2}} = \sqrt{\frac{25+400+120}{16}} = \sqrt{34.0625} = 5.836$$

$$\theta = \tan^{-1}\left(\frac{-1\cdot 3-20}{-\sqrt{25-9}}\right) = \tan^{-1}(5.75) = 80.134^{o},\ \ b = \sqrt{25-9} = 4$$

$$\mathcal{L}^{-1}\left(\frac{-s+20}{s^2+6s+25}\right) = 5.836e^{-3t}\cos(4t+80.134^{o})\,u(t)$$

From the two previous functions we have:

$$y(t) = \mathcal{L}^{-1}(Y(s))$$
$$= (2+5.836e^{-3t}\cos(4t+80.134^{o}))u(t)$$



Problem 2 (a)

(a) For each of the system described by the following differential equation, find the system's transfer function.

$$\frac{d^2y(t)}{dt^2} + 11\frac{dy(t)}{dt} + 24y(t) = 5\frac{df(t)}{dt} + 3f(t)$$

We assume that the initial conditions are zero.

We take the Laplace transform in both sides:

$$\mathcal{L}\left\{\frac{d^2y(t)}{dt^2} + 11\frac{dy(t)}{dt} + 24y(t)\right\} = \mathcal{L}\left\{5\frac{df(t)}{dt} + 3f(t)\right\} \Rightarrow$$

$$\mathcal{L}\left\{\frac{d^2y(t)}{dt^2}\right\} + 11\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} + 24\mathcal{L}\{y(t)\} = 5\mathcal{L}\left\{\frac{df(t)}{dt}\right\} + 3\mathcal{L}\{f(t)\} \Rightarrow$$

$$s^2Y(s) + 11sY(s) + 24Y(s) = 5sF(s) + 3F(s) \Rightarrow$$

$$H(s) = \frac{Y(s)}{F(s)} = \frac{5s+3}{s^2+11s+24}$$

Note that we don't have to take this further, since the transfer function is the system's function in Laplace or any other frequency domain.

Problem 2 (b)

(b)

$$\frac{d^3y(t)}{dt^3}+6\frac{d^2y(t)}{dt^2}-11\frac{dy(t)}{dt}+6y(t)=3\frac{d^2f(t)}{dt^2}+7\frac{df(t)}{dt}+5f(t)$$

We assume that the initial conditions are zero.

We take the Laplace transform in both sides:

$$\mathcal{L}\left\{\frac{d^3y(t)}{dt^3}+6\frac{d^2y(t)}{dt^2}-11\frac{dy(t)}{dt}+6y(t)\right\}=\mathcal{L}\left\{3\frac{d^2f(t)}{dt^2}+7\frac{df(t)}{dt}+5f(t)\right\}$$

$\Rightarrow$

$$\mathcal{L}\left\{\frac{d^3y(t)}{dt^3}\right\}+6\mathcal{L}\left\{\frac{d^2y(t)}{dt^2}\right\}-11\mathcal{L}\left\{\frac{dy(t)}{dt}\right\}+6\mathcal{L}\{y(t)\}$$

$$=3\mathcal{L}\left\{\frac{d^2f(t)}{dt^2}\right\}+7\mathcal{L}\left\{\frac{df(t)}{dt}\right\}+5\mathcal{L}\{f(t)\}\ \Rightarrow$$

$$s^3Y(s)+6s^2Y(s)-11sY(s)+6Y(s)=3s^2F(s)+7sF(s)+5F(s)\Rightarrow$$

$$H(s)=\frac{Y(s)}{F(s)}=\frac{3s^2+7s+5}{s^3+6s^2-11s+6}$$

Problem 2 (c)

(c)

$$\frac{d^4y(t)}{dt^4} + 4\frac{dy(t)}{dt} = 3\frac{df(t)}{dt} + 2f(t)$$

We assume that the initial conditions are zero.

We take the Laplace transform in both sides:

$$\mathcal{L}\left\{\frac{d^4y(t)}{dt^4} + 4\frac{dy(t)}{dt}\right\} = \mathcal{L}\left\{3\frac{df(t)}{dt} + 2f(t)\right\} \Rightarrow$$

$$\mathcal{L}\left\{\frac{d^4y(t)}{dt^4}\right\} + 4\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} = 3\mathcal{L}\left\{\frac{df(t)}{dt}\right\} + 2\mathcal{L}\{f(t)\} \Rightarrow$$

$$s^4Y(s) + 4sY(s) = 3sF(s) + 2F(s) \Rightarrow$$

$$H(s) = \frac{Y(s)}{F(s)} = \frac{3s+2}{s^4+4s} = \frac{3s+2}{s(s^3+4)}$$

Problem 3 (a) (i)

(a) For a system with transfer function

(i) $H(s) = \frac{s+5}{s^2+5s+6}$

find the zero-state response if the input is $f(t) = e^{-4t}u(t)$.

$$F(s) = \frac{1}{s+4}$$

$$Y(s) = H(s)F(s) = \frac{s+5}{s^2+5s+6}\frac{1}{s+4} = \frac{s+5}{(s+2)(s+3)(s+4)} = \frac{A}{s+2} + \frac{B}{s+3} + \frac{C}{s+4}$$

$$A + B + C = 0$$

$$7A + 6B + 5C = 1 \Rightarrow -7B - 7C + 6B + 5C = -B - 2C = 1 \Rightarrow B + 2C = -1$$

$$12A + 8B + 6C = 5 \Rightarrow -12B - 12C + 8B + 6C = -4B - 6C = 5$$

$$\Rightarrow 4B + 6C = B + 3B + 6C = -5$$

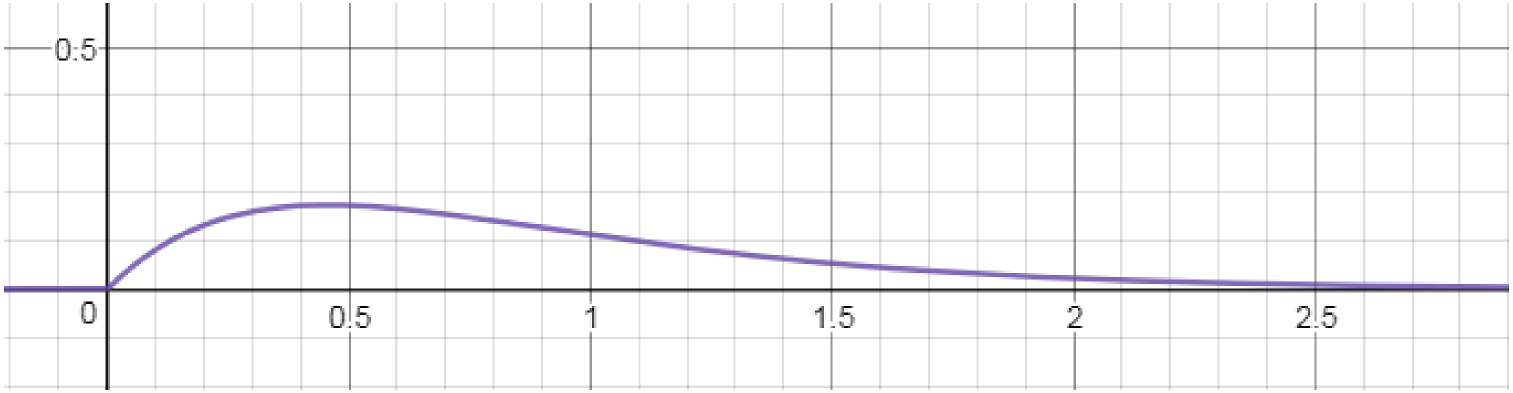
$$B + 2C = -1 \Rightarrow 3B + 6C = -3 \Rightarrow B + 3B + 6C = -5 \Rightarrow B - 3 = -5 \Rightarrow$$

$$B = -2,\ C = 1/2,\ A = 3/2.$$

$$Y(s) = \frac{3/2}{s+2} - \frac{2}{s+3} + \frac{1/2}{s+4} \Rightarrow y(t) = \left(\frac{3}{2}e^{-2t} - 2e^{-3t} + \frac{1}{2}e^{-4t}\right)u(t)$$

Problem 3 (a) (i) cont.

$$y(t) = (\frac{3}{2}e^{-2t} - 2e^{-3t} + \frac{1}{2}e^{-4t})u(t)$$



Problem 3 (a) (ii)

(ii) $H(s) = \frac{s+5}{s^2+5s+6}$

Find the zero-state response if the input is $f(t) = e^{-3t}u(t)$.

$$F(s) = \frac{1}{s+3}$$

$$Y(s) = H(s)F(s) = \frac{s+5}{s^2+5s+6}\frac{1}{s+3} = \frac{s+5}{(s+2)(s+3)^2} = \frac{A}{s+2} + \frac{B}{s+3} + \frac{C}{(s+3)^2}$$

$A + B = 0$

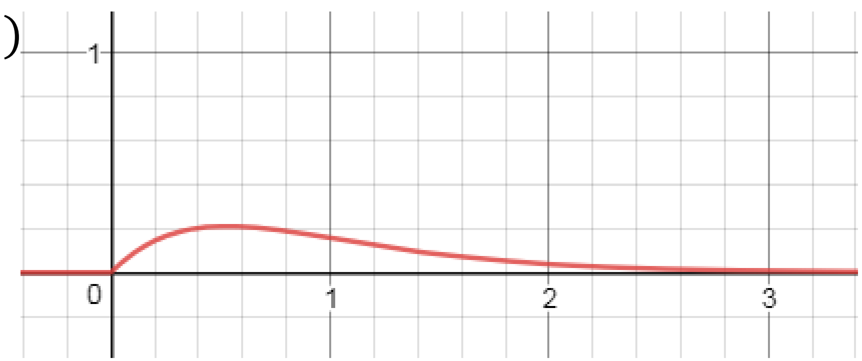
$6A + 5B + C = 1 \Rightarrow -6B + 5B + C = -B + C = 1$

$9A + 6B + 2C = 5 \Rightarrow -9B + 6B + 2C = -3B + 2C = 5$

$\Rightarrow -B - 2B + 2C = -B + 2 = 5 \Rightarrow B = -3, A = 3, C = -2$

$$Y(s) = H(s)F(s) = \frac{3}{s+2} - \frac{3}{s+3} - \frac{2}{(s+3)^2}$$

$y(t) = \mathcal{L}^{-1}(Y(s)) = (3e^{-2t} - 3e^{-3t} - 2te^{-3t})$



For the last term refer to Problem 1(b) Class 4.

Problem 3 (a) (iii)

(iii) $H(s) = \frac{s+5}{s^2+5s+6}$

Find the zero-state response if the input is $f(t) = e^{-4(t-5)}u(t-5)$.

If $x(t) = e^{-4t}u(t)$ with $X(s) = \frac{1}{s+4}$, then $f(t) = x(t-5)$ and therefore,

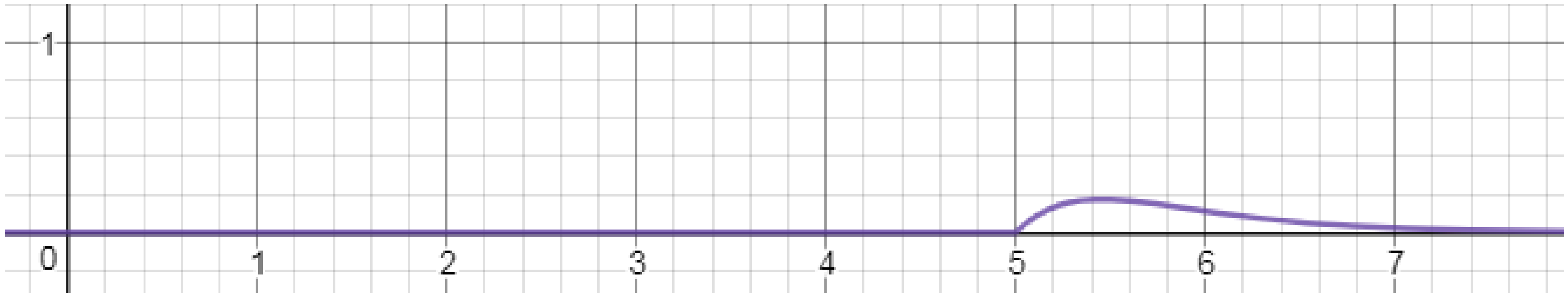
$$F(s) = e^{-5s}\frac{1}{s+4}$$

$$Y(s) = H(s)F(s) = \frac{s+5}{s^2+5s+6}e^{-5s}\frac{1}{s+4} = e^{-5s}\frac{s+5}{(s+2)(s+3)(s+4)} \Rightarrow$$

$$Y(s) = e^{-5s}\left(\frac{\frac{3}{2}}{s+2} - \frac{2}{s+3} + \frac{\frac{1}{2}}{s+4}\right)$$

We know that $\mathcal{L}^{-1}\left\{\frac{\frac{3}{2}}{s+2} - \frac{2}{s+3} + \frac{\frac{1}{2}}{s+4}\right\} = \left(\frac{3}{2}e^{-2t} - 2e^{-3t} + \frac{1}{2}e^{-4t}\right)u(t)$.

Therefore, $y(t) = \mathcal{L}^{-1}\{Y(s)\} = \left(\frac{3}{2}e^{-2(t-5)} - 2e^{-3(t-5)} + \frac{1}{2}e^{-4(t-5)}\right)u(t-5)$.



Problem 3 (b)

(b) For this system write the differential equation relating the output $y(t)$ to the input $f(t)$.

All initial conditions are assumed to be zero.

$$\frac{Y(s)}{F(s)} = \frac{s+5}{s^2+5s+6} \Rightarrow$$

$$s^2\,Y(s) + 5sY(s) + 6Y(s) = sF(s) + 5F(s) \Rightarrow$$

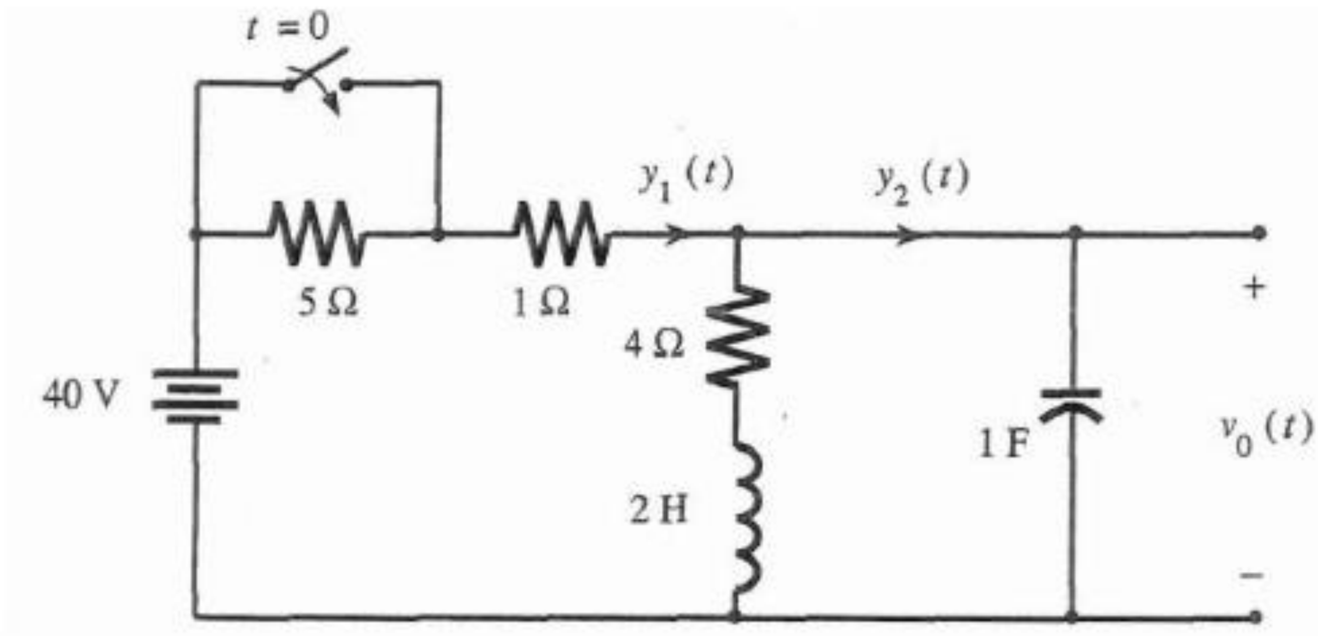
$$\mathcal{L}^{-1}\{s^2Y(s) + 5sY(s) + 6Y(s)\} = \mathcal{L}^{-1}\{sF(s) + 5F(s)\}$$

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{df(t)}{dt} + 5f(t)$$

Problem 4

- **Problem:** For the circuit shown in the figure below, the switch is in open position for a long time before $t = 0$, when it is closed instantaneously.

  Find the currents $y_1(t)$ and $y_2(t)$, $t \geq 0$.



Problem 4 cont.

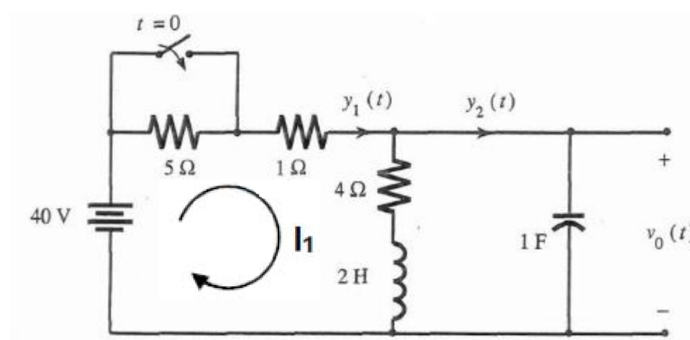
a) Let us examine what happens for $t < 0$.

We know that $i_C(t) = C\frac{dv_C(t)}{dt}$ (or $v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(\tau)\, d\tau$) and $v_L(t) = L\frac{di_L(t)}{dt}$.

- If the system is in steady state, the current $y_2(t)$ across the capacitor is 0 since the voltage across the capacitor is constant. Therefore, $y_2(0^-) = 0\ A$.
- Furthermore, the current $y_1(t)$ which flows through the left loop (Loop 1) is constant and therefore, the voltage across the inductor is 0.
- Based on the above two points, the voltage across the capacitor $v_0(t)$ is the same as the voltage across the $4\ \Omega$ resistor, i.e.,$y_1(t) \cdot 4\ V$.

In that case for the left loop (Loop 1) we have:

$y_1(t) \cdot (5 + 1 + 4) = 40V \Rightarrow y_1(t) = 4\ A, t < 0$

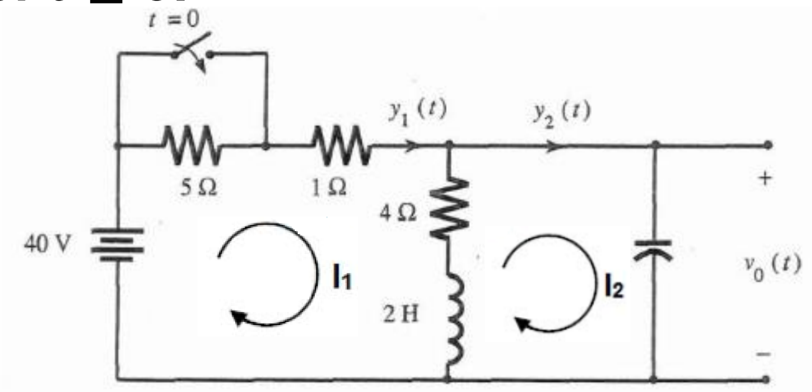


$v_0(t) = y_1(t) \cdot 4\ V = 16V, t < 0$

Initial conditions: $y_1(0^-) = 4\ A, y_2(0^-) = 0\ A,\ v_0(0^-) = 16\ V$

Problem 4 cont.

a) Write loop equations in time domain for $t \geq 0$.



Loop 1:

$$L\frac{di_L(t)}{dt} + 4 \cdot i_L(t) + 1 \cdot y_1(t) = 40u(t)$$

$$i_L(t) = y_1(t) - y_2(t)$$

$$L\left(\frac{dy_1(t)}{dt} - \frac{dy_2(t)}{dt}\right) + 4(y_1(t) - y_2(t)) + 1 \cdot y_1(t) = 40u(t)$$

$$2\frac{dy_1(t)}{dt} - 2\frac{dy_2(t)}{dt} + 5y_1(t) - 4y_2(t) = 40u(t)$$

Loop 2:

$$L\left(\frac{dy_1(t)}{dt} - \frac{dy_2(t)}{dt}\right) + 4\big(y_1(t) - y_2(t)\big) = \frac{1}{C}\int_{-\infty}^{t} i_C(\tau)\,d\tau = \frac{1}{C}\int_{-\infty}^{t} y_2(\tau)\,d\tau$$

Problem 4 cont.

b) Loop 1 equation in time and Laplace domain for $t \geq 0$:

Loop 1:

$$2\frac{dy_1(t)}{dt} - 2\frac{dy_2(t)}{dt} + 5y_1(t) - 4y_2(t) = 40\,u(t)$$

$$y_1(0^-) = 4\,A$$

$$y_2(0^-) = 0\,A$$

$$2[sY_1(s) - y_1(0^-)] - 2[sY_2(s) - y_2(0^-)] + 5Y_1(s) - 4Y_2(s) = 40/s \Rightarrow$$

$$(2s + 5)Y_1(s) - (2s + 4)\,Y_2(s) = 8 + 40/s$$

Problem 4 cont.

b) Loop 2 equation in time and Laplace domain for $t \geq 0$:

Loop 2:

$$L\left(\frac{dy_1(t)}{dt} - \frac{dy_2(t)}{dt}\right) + 4\left(y_1(t) - y_2(t)\right) = \frac{1}{C}\int_{-\infty}^{t} y_2(\tau)\,d\tau$$

$$y_1(0^-) = 4\ A$$
$$y_2(0^-) = 0\ A$$
$$v_0(0^-) = 16\ V$$

$$\int_{-\infty}^{t} x(\tau)\,d\tau \Leftrightarrow \frac{X(s)}{s} + \frac{1}{s}\int_{-\infty}^{0^-} x(t)\,dt$$

$$\mathcal{L}\left\{\frac{1}{C}\int_{-\infty}^{t} y_2(\tau)\,d\tau\right\} = \frac{1}{C}\frac{Y_2(s)}{s} + \frac{1}{s}\frac{1}{C}\int_{-\infty}^{0^-} y_2(t)\,dt = \frac{Y_2(s)}{s} + \frac{1}{s}v_0(0^-)\ (C = 1).$$

$$2[sY_1(s) - y_1(0^-)] - 2[sY_2(s) - y_2(0^-)] + 4Y_1(s) - 4Y_2(s) = \frac{Y_2(s)}{s} + \frac{16}{s}$$

$$-(2s + 4)Y_1(s) + \left(2s + 4 + \frac{1}{s}\right)Y_2(s) = -8 - \frac{16}{s}$$

Problem 4 cont.

b) Merge equations for Loops 1 and 2 in a matrix form $t \geq 0$

$$\begin{bmatrix} 2s+5 & -(2s+4) \\ -(2s+4) & 2s+4+\frac{1}{s} \end{bmatrix} \begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} 8+\frac{40}{s} \\ -8-\frac{16}{s} \end{bmatrix}$$

By solving the above system we obtain:

$$Y_1(s) = \frac{4(6s^2+13s+5)}{s(s^2+3s+2.5)} = \frac{8}{s} + \frac{16s+28}{(s^2+3s+2.5)}$$

$$Y_2(s) = \frac{20(s+2)}{(s^2+3s+2.5)}$$

Problem 4 cont.

b) Find $y_1(t), t \geq 0$

$$Y_1(s) = \frac{4(6s^2+13s+5)}{s(s^2+3s+2.5)} = \frac{8}{s} + \frac{16s+28}{s^2+3s+2.5}$$

We use Property 10c from Laplace Properties tables.

$$re^{-at}\cos(bt+\theta)\,u(t) \Leftrightarrow \frac{As+B}{s^2+2as+c}$$

$$r = \sqrt{\frac{A^2c + B^2 - 2ABa}{c - a^2}}$$

$$\theta = \tan^{-1}\left(\frac{Aa - B}{A\sqrt{c - a^2}}\right)$$

$$b = \sqrt{c - a^2}$$

$A = 16, B = 28, a = 1.5, c = 2.5$

$$y_1(t) = [8 + 17.89e^{-1.5t}\cos(0.5t - 26.56^{o})]\,u(t)$$

Problem 4 cont.

b) Find $y_2(t)$, $t \geq 0$

$$Y_2(s) = \frac{20(s+2)}{(s^2+3s+2.5)} = \frac{20s+40}{(s^2+3s+2.5)}$$

Property 10c

$$re^{-at}\cos(bt+\theta)\,u(t) \Leftrightarrow \frac{As+B}{s^2+2as+c}$$

$$r = \sqrt{\frac{A^2c + B^2 - 2ABa}{c-a^2}}$$

$$\theta = tan^{-1}\left(\frac{Aa-B}{A\sqrt{c-a^2}}\right)$$

$$b = \sqrt{c-a^2}$$

$$A = 20, B = 40, a = 1.5, c = 2.5$$

$$y_2(t) = 20\sqrt{2}e^{-1.5t}\cos(0.5t - 45^{o})\,u(t)$$

Problem 5 (a)

Using the initial and final value theorems, find the initial and final values of the zero-state response of a system with the transfer function

$$H(s) = \frac{6s^2 + 3s + 10}{2s^2 + 6s + 5}$$

and the input is

(a) $x(t) = u(t)$

$$Y(s) = H(s)X(s) = \frac{6s^2 + 3s + 10}{2s^2 + 6s + 5} \cdot \frac{1}{s}$$

- **Initial Value**

$$y(0^+) = \lim_{s\to\infty} sY(s) = \lim_{s\to\infty} s\,\frac{6s^2+3s+10}{2s^2+6s+5} \cdot \frac{1}{s} = \lim_{s\to\infty} \frac{6s^2+3s+10}{2s^2+6s+5} = 3$$

- **Final Value**

$$y(\infty) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} s\,\frac{6s^2+3s+10}{2s^2+6s+5} \cdot \frac{1}{s} = \lim_{s\to 0} \frac{6s^2+3s+10}{2s^2+6s+5} = 2$$

Problem 5 (b)

Using the initial and final value theorems, find the initial and final values of the zero-state response of a system with the transfer function

$$H(s) = \frac{6s^2 + 3s + 10}{2s^2 + 6s + 5}$$

and the input is

(b) $x(t) = e^{-t}u(t)$

$$Y(s) = H(s)X(s) = \frac{6s^2 + 3s + 10}{2s^2 + 6s + 5} \cdot \frac{1}{s+1}$$

- **Initial Value**

$$y(0^+) = \lim_{s\to\infty} sY(s) = \lim_{s\to\infty} s\frac{6s^2+3s+10}{2s^2+6s+5} \cdot \frac{1}{s+1}$$

$$= \lim_{s\to\infty} \frac{6s^3+3s^2+10s}{2s^3+6s^2+5s+2s^2+6s+5} = \lim_{s\to\infty} \frac{6s^3+3s^2+10s}{2s^3+8s^2+11s+5} = 3$$

- **Final Value**

$$y(\infty) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} \frac{6s^3 + 3s^2 + 10s}{2s^3 + 8s^2 + 11s + 5} = 0$$