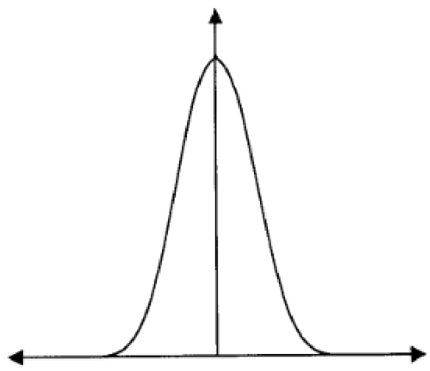
1.

(i) The new histogram will be the convolution of the 2 original histograms and therefore it will occupy a wider range of values. Therefore, the new image will be an image of higher contrast compared to the original.

(ii) By carrying out the proposed manipulation we see that most pixel values of pixels which belong to a constant or slowly varying area will turn into zeros. Furthermore, the resulting intensities will be both positive and negative. The resulting histogram will look as follows:



2.

Large spatial masks can be used within flat or slowly varying areas whereas small masks can be used within high activity areas. The idea behind this approach is that noise is more visible in the first type of areas. Local variance can classify each pixel into the right area.

3.

Bandpass filters are useful for removing background noise without completely eliminating the background information. A bandpass filter can be implemented by a spatial mask as follows. The original image $f(x,y)$ is first convolved with a spatial mask of size $N_1 \times N_1$ to produce an output $g_1(x,y)$. Then it is convolved with a spatial mask of size $N_2 \times N_2$ with $N_2 > N_1$ to produce an output $g_2(x,y)$. The final output is obtained as the difference $g_1(x,y) - g_2(x,y)$ and this is the bandpass filtered version of the image.

4.

It can easily be deducted that a pair of mask which detect edges at 45 and -45 degrees could be the following:

1	1	0
1	0	-1
0	-1	-1

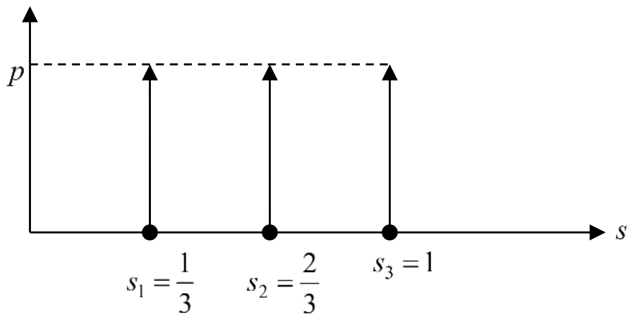
-1	-1	0
-1	0	1
0	1	1

5.

(i) $f(x, y)$ will be a dark image since the intensities are concentrated in the lower half of the intensity range. Moreover, it will consists of three intensities only with equal probabilities, and therefore, $p = \frac{1}{3}$.

(ii) We can use histogram equalization. By doing so, the three intensities are mapped to the following: $s_1 = T(r_1) = \frac{1}{3}$, $s_2 = T(r_2) = \frac{2}{3}$, $s_3 = T(r_3) = 1$

(iii)



(iv) For the original image we have:

Mean: $m_1 = \frac{1}{3}(r_1 + r_2 + r_3)$

Variance: $\sigma_1^2 = \frac{1}{3}(r_1^2 + r_2^2 + r_3^2) - \frac{1}{9}(r_1 + r_2 + r_3)^2$

For the equalised image we have:

Mean: $m_2 = \frac{1}{3}(s_1 + s_2 + s_3) = \frac{1}{3}(\frac{1}{3} + \frac{2}{3} + 1) = \frac{2}{3}$

Variance: $\sigma_2^2 = \frac{1}{3}(s_1^2 + s_2^2 + s_3^2) - \frac{4}{9} = \frac{1}{3}(\frac{1}{9} + \frac{4}{9} + 1) - \frac{4}{9} = \frac{2}{27}$

6.

Since dark areas are represented by small intensities we can calculate the local mean around each pixel in order to decide whether this belongs to a dark or to a bright area. We denote the mean around a pixel with co-ordinates (x, y) as $m_{(x,y),W}$. The subscript W denotes the neighborhood used in order to calculate this mean. If this is a rectangular window of size $(2W+1)(2W+1)$ symmetrically placed around the pixel of interest, then

$$m_{(x,y),W} = \frac{1}{(2W+1)^2} \sum_{i=-W}^{i=+W} \sum_{j=-W}^{j=+W} f(x+i, y+j)$$

We can calculate the local variance around each pixel in order to decide whether this belongs to a high detail or to a low detail area. We denote the variance around a pixel with co-ordinates (x, y) as $\sigma^2_{(x,y),W}$. The subscript W denotes the neighborhood used in order to calculate this variance. If this is a rectangular window of size $(2W+1)(2W+1)$ symmetrically placed around the pixel of interest, then

$$\sigma^2_{(x,y),W} = \frac{1}{(2W+1)} \sum_{i=-W}^{i=+W} \sum_{j=-W}^{j=+W} f(x+i, y+j)^2 - m^2_{(x,y),W}$$

Therefore,

If $m_{(x,y),W} > T_1$ or $\sigma^2_{(x,y),W} > T_2$ then use a small low-pass spatial mask for noise removal.

Otherwise, if $m_{(x,y),W} < T_1$ and $\sigma^2_{(x,y),W} < T_2$ then use a large low-pass spatial mask for noise removal.

7. (i)

0	10	10	10	0
11	12	11	12	11
11	12	12	12	11
0	12	12	11	0

(ii)

11	11	11	11	11
11	11	11	12	11
11	12	12	12	11
10	12	12	12	10

(iii) For an image containing rectangular objects the second method is better. The first method 'rounds off corners' because then the 3x3 window is centred at a corner pixel, 4 pixels within the window are on the object and 5 on the background, hence the output pixel is given the value of the background. For the second method with 1x3 and 3x1 filtering a when the window is centred a corner pixel a majority of pixels (2 out of 3) is on the object hence the background has no effect.

(iv) Now the first method is better. For a window centred on one of the black pixels, 4 out of 9 pixels within the window cover the black spot, whereas a majority, 5 out of 9 are on the background, hence the output value is the background. In contrast for the second method a majority of window pixels are one the black spot and so the output is the same as the input and the black spot is not removed.

(v) Check for instance whether the smallest value in the window is less than 20% of the median value, if it is apply median filtering and replace the centre pixel with the median value. Otherwise just copy the input value to the output value.

8.

$f(x,y)\} = f(x,y) * \delta(x,y)$ and mean$\{f(x,y)\} = f(x,y) * w(x,y)$. Therefore,

$$g(x,y) = \begin{cases} 1, & f(x,y) * (\delta(x,y) - w(x,y)) > T \\ 0, & \text{otherwise} \end{cases}$$

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

Figure: $\delta(x,y)$

1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25

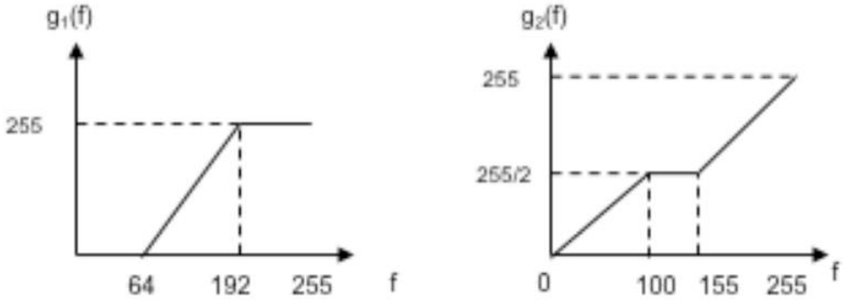
Figure: $w(x,y)$

-1/25	-1/25	-1/25	-1/25	-1/25
-1/25	-1/25	-1/25	-1/25	-1/25
-1/25	-1/25	24/25	-1/25	-1/25
-1/25	-1/25	-1/25	-1/25	-1/25
-1/25	-1/25	-1/25	-1/25	-1/25

Figure: $\delta(x,y) - w(x,y)$

9.

The transformation used for histogram equalisation is $s = T(r) = \int_0^r p_r(w)dw$. Based on that we get:



10.

$$P_F(f) = \int_o^f p_F(f)df = \int_o^f \frac{2}{255}(1 - \frac{f}{255})df = \frac{f}{255^2}(255 \times 2 - f)$$

$$P_G(g) = \int_o^g p_g(g)dg = \int_o^g \frac{2}{255^2} g dg = \frac{g^2}{255^2}$$

$$\frac{g^2}{255^2} = \frac{f}{255^2}(255 \times 2 - f) \Rightarrow g^2 = (510f - f^2) \Rightarrow g = \pm\sqrt{510f - f^2}$$

g should be between 0 and 255 and therefore $g(f) = \sqrt{510f - f^2}$

11.

Trivial. The answer is median. This can be demonstrated by simple example, i.e., median(0,1,0)+median(2,0,0)=0+0=0. This is not the same as the median of (2,1,0) which is 1.

12.

Book work. The answer is differentiation filter.