Image Restoration Sample Exam Problems

1. We are given the noisy version $g(x,y)$ of an image $f(x,y)$. We wish to denoise $g(x,y)$ using a spatially adaptive image denoising mask with coefficients $h(m,n)$. The mask is estimated at each pixel based on the local signal variance $\sigma_g^2(x,y)$. The local signal variance is estimated from the degraded image $g(x,y)$ using a local neighborhood around each pixel. The mask coefficients are obtained using the following equation:

$$h_{(x,y)}(m,n) = k_1(x,y)e^{-k_2(x,y)(m^2+n^2)}w(m,n)$$

where $w(m,n)$ is a 5×5-point rectangular window placed symmetrically around the origin, i.e.,

$$w(m,n) = \begin{cases} 1, & -2 \le m \le 2, -2 \le n \le 2 \\ 0, & \text{otherwise} \end{cases}$$

We require that

$$\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} h_{(x,y)}(m,n) = 1$$

(i) Give the form of $h_{(x,y)}(m,n)$ for a very small k_2 (close to 0).

(ii) Give the form of $h_{(x,y)}(m,n)$ for a very large k_2 (close to ∞).

(iii) Explain why random noise is typically less visible to human viewers in image regions of high detail, such as edge regions, than in image regions of low detail, such as uniform background regions.

(iv) Give one reasonable choice of $k_2(x,y)$ as a function of $\sigma_g^2(x,y)$. Justify your answer. For your choice of $k_2(x,y)$ determine $k_1(x,y)$.

(v) The image restoration system discussed here can exploit the observation stated in (iii). The system, however, cannot exploit the observation that random noise is typically less visible to human viewers in bright areas than in dark areas. How would you modify the image restoration system so that this additional piece of information can be exploited?

2. We are given the degraded version g of an image f such that in lexicographic ordering

$$g = Hf + n$$

where H is the degradation matrix which is assumed to be block-circulant and n is the noise term which is assumed to be zero-mean, white and independent of the image f. All images involved have size $N \times N$ after extension and zero-padding.

(i) Consider the Inverse Filtering image restoration technique. Give the general expressions for both the Inverse Filtering estimator and the restored image in both spatial and frequency domains and explain all symbols used.

(ii) In a particular scenario the degradation process can be modelled as a linear filter with the two-dimensional impulse response given below:

$$h(x,y) = \begin{cases} 1 & -1 \le x \le 1, y = 0 \\ 0 & \text{elsewhere} \end{cases}$$

Estimate the frequency pairs for which Inverse Filtering cannot be applied.

3. (i) Consider the Constrained Least Squares (CLS) filtering image restoration technique. Give the general expressions for both the CLS filter estimator and the restored image in both spatial and frequency domains and explain all symbols used.

(ii) In a particular scenario, the degradation process can be modelled as a linear filter with the transfer function given below:

$$H(u,v) = \sqrt{2\pi}\sigma(u^2+v^2)e^{-j2\pi^2\sigma^2(u^2+v^2)}$$

In the above formulation σ is a constant parameter. Generate the expression of the CLS filter in frequency domain by assuming that the high pass filter used in CLS is a Laplacian filter.

4. Let $g(x,y)$ be a degraded only by noise image that can be expressed as

$$g(x,y) = f(x,y) + n(x,y)$$

where $f(x,y)$ is the original image and $n(x,y)$ is background noise. In Wiener filtering we assume that the Discrete Space Fourier Transform $S_{ff}(u,v)$ of the autocorrelation function of the original image is available.

One method of estimating $S_{ff}(u,v)$ is to model the autocorrelation function as follows

$$R_{ff}(k,l) = \underset{(x,y)}{E}\{f(x,y)f(x+k,y+l\} = \rho^{|k|+|l|}$$

with ρ an unknown parameter with $0<\rho<1$.

(i) Assuming $n(x,y)$ is a zero mean, white noise with unknown variance σ_n^2 and independent of $f(x,y)$, write down without proof the expressions for the Wiener filter estimator and the restored image in the frequency domain as functions of ρ and σ_n^2.

(ii) Develop a method of estimating ρ using the autocorrelation function samples of $g(x,y)$.

Hints:

1. The Discrete Space Fourier Transform $S_{ff}(u,v)$ of the autocorrelation function is defined as

$$S_{ff}(u,v) = \sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} R_{ff}(k,l)e^{-j(uk+vl)}.$$

2. The following result holds: $\sum_{k=0}^{\infty} a^x = \frac{1}{1-a}, |a|<1$.

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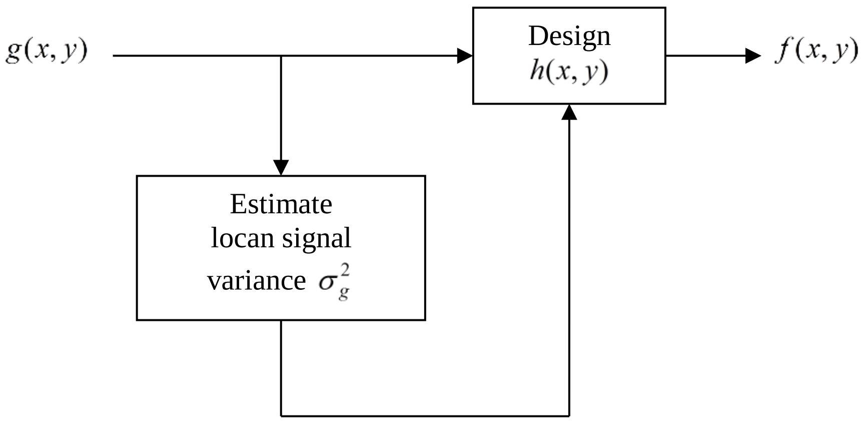
5. We are given the noisy version $g(x,y)$ of an image $f(x,y)$. Consider the adaptive image restoration system sketched in the following figure.



In this system, $h(x,y)$ is designed at each pixel based on the local signal variance σ_g^2 estimated from the degraded image $g(x,y)$. The filter $h(x,y)$ is assumed to have the form

$$h(x,y) = k_1 e^{-k_2(x^2+y^2)} w(x,y)$$

where $w(x,y)$ is a 5×5-point rectangular window placed symmetrically around the origin, i.e.,

$$w(x,y) = \begin{cases} 1, & -2 \le x \le 2,\ -2 \le y \le 2 \\ 0, & \text{otherwise} \end{cases}$$

We require that

$$\sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} h(x,y) = 1$$

(i) Sketch $h(x,y)$ for a very small k_2 (close to 0).

(ii) Sketch $h(x,y)$ for a very large k_2 (close to ∞).

(iii) Based on the observation that random noise is typically less visible to human viewers in image regions of high detail, such as edge regions, than in image regions of low detail, such as uniform background regions, sketch one reasonable choice of k_2 as a function of σ_g^2.

(iv) Denote your choice in (iii) as $k_2(\sigma_g^2)$. Determine k_1.

(v) The image restoration system discussed here can exploit the observation stated in (iii). The system, however, cannot exploit the observation that random noise is typically less visible to human viewers in bright areas than in dark areas. How would you modify the image restoration system so that this additional piece of information can be exploited?

6. We are given the degraded version g of an image f such that in lexicographic ordering

$$g = Hf + n$$

where H is the degradation matrix which is assumed to be block-circulant, and n is the noise term which is assumed to be zero mean, independent and white. The images have size $N \times N$ with N even. In a particular scenario, the image under consideration is blurred to relative motion between the image and the camera. The pixel of the image g at location (x,y) is related to the corresponding pixel of image f through the following relationship:

$$g(x,y) = f(x,y) + 2f(x,y-1) + f(x,y-2) + n(x,y)$$

(i) Consider the Inverse Filtering image restoration technique. Find the expressions for both the Inverse filter estimator and the restored image in the frequency domain.

(ii) Find the specific frequencies for which the restored image cannot be estimated. Consider the Constrained Least Squares image restoration technique. The two-dimensional high pass filtering operator used in the regularization term is given by the function below:

$$c(x,y) = \begin{cases} 1 & x = 0, y = 0 \\ -2 & x = 0, y = 1 \\ 1 & x = 0, y = 2 \\ 0 & \text{elsewhere} \end{cases}$$

Find the expressions for both the Constrained Least Squares filter estimator and the restored image in the frequency domain.

(iii) Find whether there are any specific frequencies for which the restored image cannot be estimated.