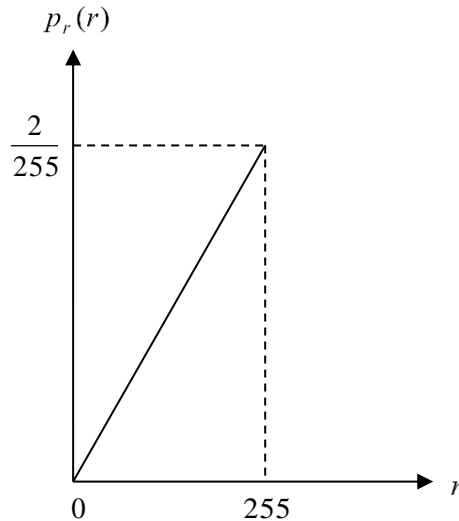# Image Compression Sample Exam Problems

1. Consider an image with intensity $f(x,y)$ that can be modeled as a sample obtained from the probability density function sketched below:

(i) Suppose three reconstruction levels are assigned to quantize the intensity $f(x,y)$. Determine these reconstruction levels using a uniform quantizer.

(ii) Determine the codeword to be assigned to each of the three reconstruction levels using Huffman coding. Specify what the reconstruction level is for each codeword. For your codeword assignment, determine the average number of bits required to represent r.

(iii) Determine the entropy, the redundancy and the coding efficiency of the Huffman code for this example. Comment on the efficiency of Huffman code for this particular set of symbols.

2. Consider a grey level image $f(x,y)$ with grey levels from 0 to 255. Assume that the image $f(x,y)$ has medium contrast. Furthermore, assume that the image $f(x,y)$ contains large areas of slowly varying intensity. Consider the image
$$g(x,y) = f(x,y) - 0.5f(x-1,y) - 0.5f(x,y-1).$$

(i) Sketch a possible histogram of the image $f(x,y)$.

(ii) Discuss the characteristics of the histogram of the image $g(x,y)$.

(iii) Explain which of the two images is more amenable to compression using Huffman code.

3. The following figure shows a 10×10 image with 3 different grey levels (black, grey, white).
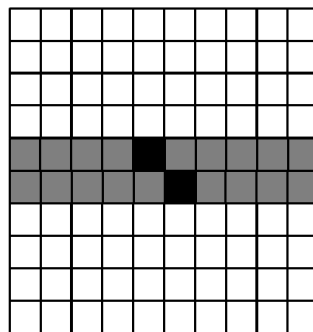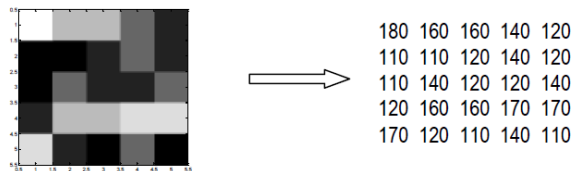
(i)   Derive the probability of appearance (that forms the histogram) for each intensity (grey) level. Calculate the entropy of this image.
(ii) Derive the Huffman code.
(iii) Calculate the average length of the fixed length code and that of the derived Huffman code.
(iv) Calculate the ratio of image size (in bits) between using the fixed length coding and Huffman coding. Calculate the relative coding redundancy.
(v)   Derive the extended-by-two Huffman code.
(vi) Calculate the ratio of image size (in bits) between using the fixed length coding and extended Huffman coding. Calculate the relative coding redundancy.
(vii) Comment on the efficiency of the extended Huffman code for this particular image.

4. (i)   Give the definition of a Discrete Memoryless Source (DMS).
    (ii) Consider a set of symbols generated from a DMS. Give the minimum number of bits per symbol that we can achieve if we use Huffman coding for the binary representation of the symbols. Explain what type of probabilities the symbols must possess in order to achieve the minimum number of bits per symbol using Huffman coding.
    (iii) Provide a scenario where Huffman coding would not reduce the number of bits per symbol from that achieved using fixed number of bits per symbol.

5. The following figure shows a list of 7 symbols and their probabilities. It is assumed that these symbols are generated by a Discrete Memoryless Source (DMS).

| Symbol	Probability
k	0.05
l	0.2
u	0.1
w	0.05
e	0.3
r	0.2
$?$	0.1

    (i)   Derive a Huffman code taking into consideration that in a particular transmission system, the probability of a 1 being transmitted as 0 is zero and the probability of a 0 being transmitted as a 1 is 0.05.
    (ii) Calculate the compression ratio.
    (iii) In the particular transmission system described in (i) above, find the probability of a codeword equal to 100 being transmitted wrongly.

6. (i)   Name three reasons why it might be a good idea to compress files.
    (ii) Discuss the characteristics of the histogram that an image must possess in order to be amenable to compression using Huffman code.
    (iii) Consider the 8×8 image $f(x, y), x, y = 0, \ldots, 7$ shown in figure below. The top left corner is the point $(x, y) = (0,0)$. Explain how *differential coding* can be used to compress this image if the prediction formula is $f(x, y) - f(x+1, y)$ for $x < 7$ and 0 for $x = 7$.

0	1	2	3	4	5	6	7
0	1	2	3	4	5	6	7
7	6	5	4	3	2	1	0
7	6	5	4	3	2	1	0
1	1	1	1	1	1	1	1
3	3	3	3	3	3	3	3
5	5	5	5	5	5	5	5
7	7	7	7	7	7	7	7

7. The following figure shows a 5×5 image with 5 different grey levels with values shown on the right figure.

$$180\ 160\ 160\ 140\ 120$$
$$110\ 110\ 120\ 140\ 120$$
$$110\ 140\ 120\ 120\ 140$$
$$120\ 160\ 160\ 170\ 170$$
$$170\ 120\ 110\ 140\ 110$$

   (i) Derive the probability of appearance (that forms the histogram) for each intensity (grey) level. Calculate the entropy of this image.

  (ii) Derive the Huffman code.

 (iii) Calculate the average length of the fixed length code and that of the derived Huffman coding.

 (iv) Calculate the compression ratio and the relative coding redundancy.

8. Consider an image with intensity $f(x,y)$ that can be modeled as a sample obtained from the probability density function sketched below:
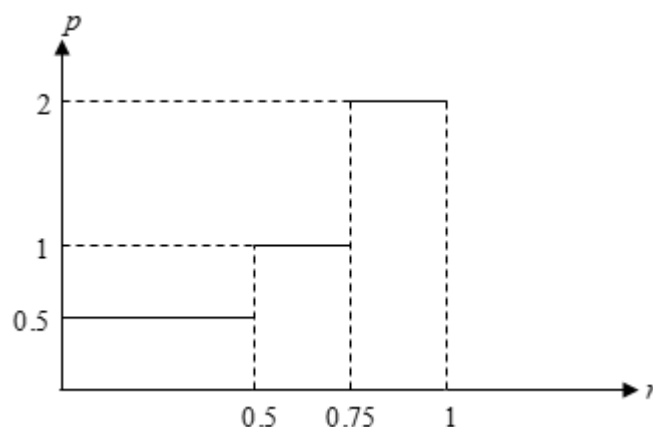
   (i) Suppose four reconstruction levels are assigned to quantize the intensity $f(x,y)$. Determine these reconstruction levels using a uniform quantizer.

  (ii) Explain briefly why uniform quantization of an image may not be optimal in terms of the mean squared error.

(iii) Determine the codeword to be assigned to each of the five reconstruction levels using Huffman coding. Specify what the reconstruction level is for each codeword. For your codeword assignment, determine the average number of bits required to represent r.

(iv) Determine the entropy, the redundancy and the coding efficiency of the Huffman code for this example.

9. Consider an image with intensity $f(x, y)$ that can be modeled as a sample obtained from the probability density function sketched below:
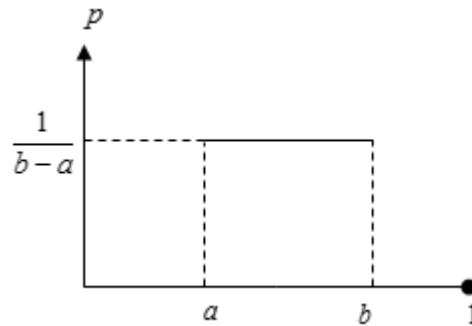

From $f(x, y)$ create an image

$$g(x, y) = f(x, y) - f(x-1, y-1).$$

Prove that applying symbol encoding to $g(x, y)$ is more efficient than applying symbol encoding directly to the original image $f(x, y)$. Use the property that adding uncorrelated images convolves their histograms.

10. Consider an image with intensity $f(x, y)$ that can be modeled as a sample obtained from the probability density function sketched in the figure below.
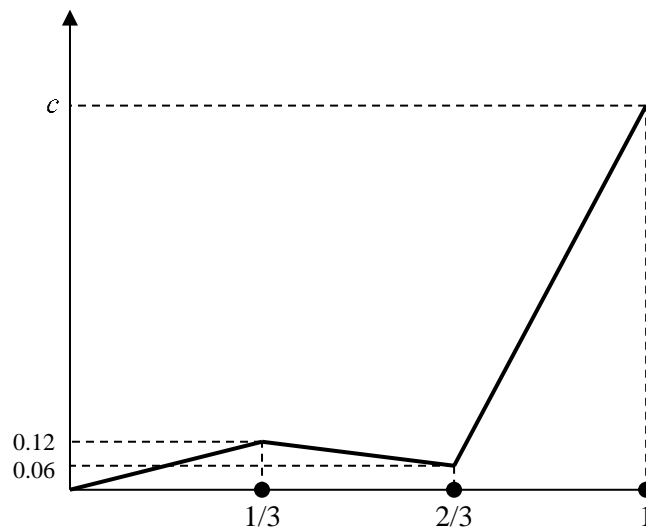
Figure

(i) Determine the constant c shown in figure above.

(ii) Suppose three reconstruction levels are assigned to quantize the intensity $f(x, y)$. Determine these reconstruction levels using a uniform quantizer.

(iii) Determine the codeword to be assigned to each of the three reconstruction levels (symbols) using Huffman coding. For your codeword assignment, determine the average number of bits required to represent the image intensity.

(iv) Determine the entropy, the redundancy and the coding efficiency of the Huffman code for this example. Comment on the efficiency of Huffman code for this particular set of symbols.

(v) In the above set of symbols apply the extended by two Huffman coding.
Explain the motivation for using extended Huffman code in the given set of symbols. Determine the redundancy and the coding efficiency of the extended by two Huffman code for this example. Comment on its efficiency.