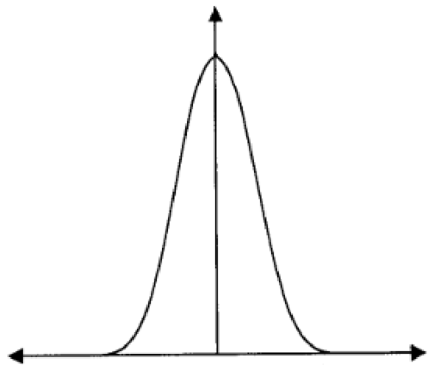
1.

(i) The new histogram will be the convolution of the 2 original histograms and therefore it will occupy a wider range of values. Therefore, the new image will be an image of higher contrast compared to the original.

(ii) By carrying out the proposed manipulation we see that most pixel values of pixels which belong to a constant or slowly varying area will turn into zeros. Furthermore, the resulting intensities will be both positive and negative. The resulting histogram will look as follows:

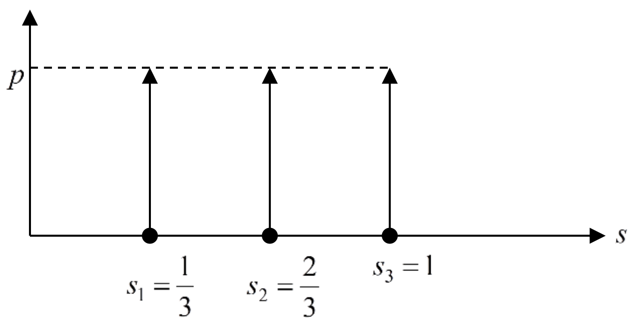


2.

(i) $f(x, y)$ will be a dark image since the intensities are concentrated in the lower half of the intensity range. Moreover, it will consists of three intensities only with equal probabilities, and therefore, $p = \frac{1}{3}$.

(ii) We can use histogram equalization. By doing so, the three intensities are mapped to the following: $s_1 = T(r_1) = \frac{1}{3}$, $s_2 = T(r_2) = \frac{2}{3}$, $s_3 = T(r_3) = 1$

(iii)



(iv) For the original image we have:
Mean: $m_1 = \frac{1}{3}(r_1 + r_2 + r_3)$
Variance: $\sigma_1^2 = \frac{1}{3}(r_1^2 + r_2^2 + r_3^2) - \frac{1}{9}(r_1 + r_2 + r_3)^2$
For the equalised image we have:
Mean: $m_2 = \frac{1}{3}(s_1 + s_2 + s_3) = \frac{1}{3}(\frac{1}{3} + \frac{2}{3} + 1) = \frac{2}{3}$
Variance: $\sigma_2^2 = \frac{1}{3}(s_1^2 + s_2^2 + s_3^2) - \frac{4}{9} = \frac{1}{3}(\frac{1}{9} + \frac{4}{9} + 1) - \frac{4}{9} = \frac{2}{27}$

3.

(i) $p_r(r) = 2 - 2r$

$T(r) = 2r - r^2$

(ii) $p_z(z) = 2z$

$T(z) = z^2$

$z = \sqrt{2r - r^2}$

4.

The first two properties, total power (sum of square of pixel values) and the entropy must be the same. Both depend only on the pixel values not the order they are arranged in the image and can be expressed as

$$\text{Power} = \sum_I H(I) I^2$$

$$\text{Entropy} = \sum_I -H(I) I \ln_2 I$$

where $H(I)$ is the histogram, and I is the gray level. The inter-pixel covariance function however is not necessarily the same. One could take the pixels in an image of a face (high inter-pixel covariance or similarity between adjacent pixels) and move them randomly around, the image histogram would be the same but the covariance between pixels would be very small.

5.

(i)

The three intensities are very close to each other so their differences are not large enough to be perceived by the human eye. Therefore, the above image should appear constant with a grey level around r .

(ii)

$$p(r) = \frac{256-144}{256 \cdot 256} = \frac{112}{256 \cdot 256} = \frac{7}{16 \cdot 256}$$

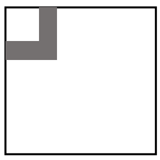
$$p(r+1) = \frac{144}{256 \cdot 256} = \frac{9}{16 \cdot 256}$$

$$p(r+2) = \frac{256 \cdot 256 - 256}{256 \cdot 256} = \frac{255}{256}$$

Therefore, we get:

$$T(r) = p(r) = \frac{7}{16 \cdot 256}$$
$$T(r+1) = \frac{7+9}{16 \cdot 256} = \frac{1}{256}$$
$$T(r+2) = 1$$
By multiplying with 255 we get the new image intensities as follows:
$$r \to \frac{7}{16}\frac{255}{256} = 0$$
$$(r+1) \to \frac{255}{256} = 1$$
$$(r+2) \to 255$$
In the resulting image we will still not be able to distinguish the two new intensities which arise from r and $r+1$.

(iii) Apply local histogram equalisation on the above image using non-overlapping image patches of size 16×16. Comment on the visual appearance of the resulting equalised image.
For the top left part of the image of size 16×16 we will get:
$$p(r) = \frac{256-144}{256} = \frac{112}{256} = \frac{7}{16}$$
$$p(r+1) = 1$$
By multiplying with 255 we get the new image intensities as follows:
$$r \to \frac{7}{16} 255 = 112$$
$$(r+1) \to 255$$
The rest of the image will turn white, i.e., it will be of intensity 255.
Therefore, the locally equalised will look as below:



(iv) Obviously local histogram equalisation is able to extract the local pattern on the top left part and therefore, it is preferable.

6.

(i)

The intensities of the two inner squares are very similar and therefore, the inner pattern is not visible by the human eye. It basically looks like a single square instead of the following pattern:

The probabilities of the three intensities are as follows:
$$p(r_2) = \frac{32 \times 32}{256 \times 256} = \frac{1}{64}$$
$$p(r_3) = \frac{64 \times 64 - 32 \times 32}{256 \times 256} = \frac{3}{64}$$
$$p(r_1) = \frac{60}{64} = \frac{15}{16}$$
After histogram equalisation we obtain the following mapping:

$s_3 = T(r_3) = p(r_3) = \frac{3}{64}$ and with normalisation we get $\frac{3}{64} \times 255 \approx 12$.

$s_2 = T(r_2) = p(r_2) + p(r_3) = \frac{4}{16}$ and with normalisation we get $\frac{4}{64} \times 255 \approx 16$.

$s_1 = T(r_1) = 1$ and with normalization we get $1 \times 255 \approx 255$.

The intensities 12 and 16 are quite close and therefore, the inner pattern will still not be clearly visible to the human eye.

(ii)

In case we opt for local histogram equalisation the inner patch with the pattern will perfectly fit in a scanning patch. For that patch we have the following intensity transformations:

$$p(r_2) = \frac{32 \times 32}{64 \times 64} = \frac{1}{4}$$

$$p(r_3) = \frac{64 \times 64 - 32 \times 32}{64 \times 64} = \frac{3}{4}$$

After histogram equalisation we obtain the following mapping:

$s_3 = T(r_3) = p(r_3) = \frac{3}{4}$ and with normalisation we get $\frac{3}{4} \times 255 \approx 191$.

$s_2 = T(r_2) = p(r_3) + p(r_2) = 1$ and with normalization we get $1 \times 255 \approx 255$.

The difference between the new intensities is quite substantial and therefore, the inner pattern is now clearly visible.

The rest of the image will turn white after histogram equalisation.

(iii)

Based on the above analysis local histogram equalisation is more beneficial.

7.

We assume that the images are extended by zeros.
The responses of the various pixels to a smoothing mask are as follows.

For the left image we have:
Response of black corners (2 on total): 0
Response of white corners (2 on total): 4/9
White non-border pixels next to the image’s edge (6 on total): 6/9=2/3
Black non-border pixels next to the image’s edge (6 on total): 6/9=2/3
White border pixels next to the image’s edge (2 on total): 4/9
Black border pixels next to the image’s edge (2 on total): 4/9
Rest of white border pixels (10 on total): 6/9=2/3
Rest of black border pixels (10 on total): 0
Rest of white (inner) pixels (12 on total): 1
Rest of black (inner) pixels (12 on total): 0

For the right image we have:
Response of black corners (2 on total): 2/9
Response of white corners (2 on total): 2/9
Rest of white border pixels (12 on total): 3/9=2/3
Rest of black border pixels (12 on total): 2/3
Rest of white (inner) pixels (18 on total): 5/9
Rest of black (inner) pixels (18 on total): 4/9

It is straightforward to see that the two histograms are different.

8.

The transformation used for histogram equalisation is $s = T(r) = \int_0^r p_r(w)dw$. Based on that we get: