Histogram Processing Sample Exam Problems

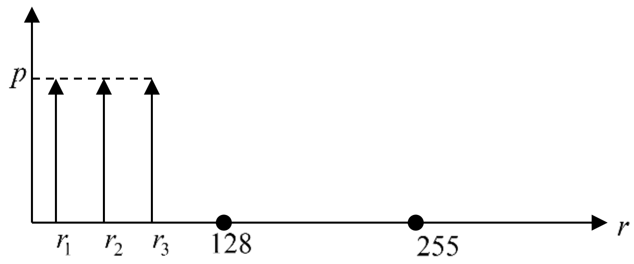
1. (i) Knowing that adding uncorrelated images convolves their histograms, how would you expect the contrast of the sum of two uncorrelated images to compare with the contrast of its component images? Justify your answer.

   (ii) Consider an $N \times N$ image $f(x,y)$. From $f(x,y)$ create an image

   $$g(x,y) = -2f(x,y) + f(x,y-1) + f(x,y+1).$$

   Comment on the histogram of $g(x,y)$ in relation to the histogram of $f(x,y)$.

2. Consider a grey-level image $f(x,y)$ with histogram sketched below.



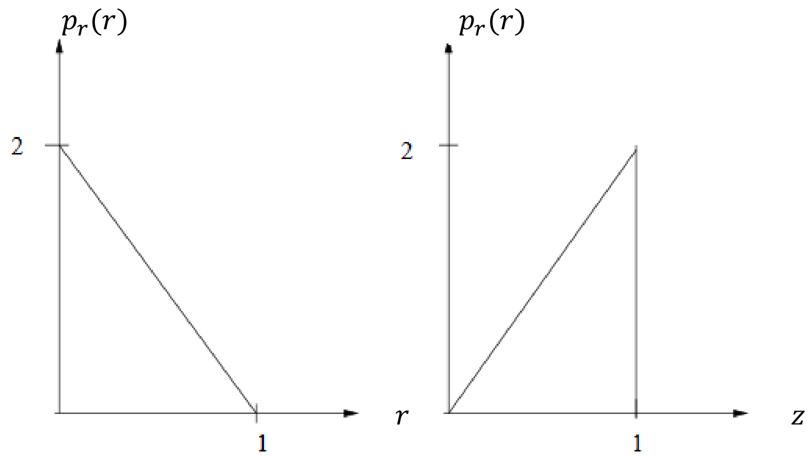
   (i) What can we say about $f(x,y)$?

   (ii) Propose an intensity transformation function which will improve the contrast of the image when it is used to modify the intensity of the image.

   (iii) Sketch the histogram of the transformed intensity.

   (iv) Calculate the mean and the variance of the two images.

3. An image has the grey level probability density function (or histogram normalized by the number of pixels) $p_r(r)$ shown below left.



   (i) Find the pixel transformation $y = g(r)$ such that after transformation the image has a flat PDF, i.e. which accomplishes histogram equalisation. Assume continuous variables r, y.

   (ii) It is desired to find a transformation $z = f(r)$ such that the transformed image will have the PDF of $p_r(r)$ shown above right. Assume continuous quantities and determine the transformation function $z = f(r)$.

4. Two images have the same histogram. Which of the following properties must they have in common? Justify your answer.
   (i) Same total power (sum of squares of pixel values)
   (ii) Same Entropy (sum of I ln I over all pixel values)
   (iii) Same degree of pixel to pixel correlation?

5. Consider a grey level image $f(x,y)$ of size 256×256 with $1\le x,y\le256$, which has the following intensities:

$$f(x,y)=\begin{cases} r+1 & 1\le x\le12 \text{ and } 1\le y\le12 \\ r & 13\le x\le16, 1\le y\le16 \text{ and } 1\le x\le12, 13\le y\le16 \\ r+2 & \text{elsewhere} \end{cases}$$

with $0\le r\le253$.
   (i) Sketch the image $f(x,y)$ and comment on its visual appearance. Justify your answer.
   (ii) Apply global histogram equalisation on the above image. Comment on the visual appearance of the resulting equalised image.
   (iii) Apply local histogram equalisation on the above image using non-overlapping image patches of size 16×16. Comment on the visual appearance of the resulting locally equalised image.
   (iv) Based on the above observations, which of the two types of equalisation processes would you choose for the visual improvement of the particular image? Justify your answer.

6. Consider the 3-grey-level digital image $f(x,y)$ of size 256×256 shown below in **Figure 1** where $0\le x,y\le255$. The intensity of this image is constant and equal to r_1 for most of the pixel locations. Inside the image there is a pattern of two small rectangular areas. The smallest area is of intensity r_2 and occupies the pixels at locations $144\le x,y\le175$. This is placed within a slightly larger square area which occupies the pixels at locations $128\le x,y\le191$. The intensities of this area are r_3 everywhere apart from the locations that form the smallest square placed in the middle.

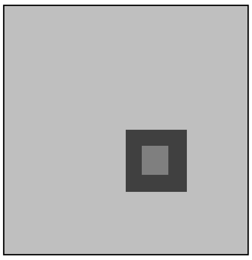


Figure 1

Intensities r_1,r_2,r_3 lie within 0 and 255, such that $r_3\le r_2\le r_1$ and $r_2=r_3+1$.
   (i) Apply global histogram equalization to the image $f(x,y)$. Let $h_{\text{out}}(s)$ denote the resulting (equalized) histogram of pixel values s taking values in [0, 255]. Sketch the plot of $h_{\text{out}}(s)$, and label the plot axes. Sketch the resulting histogram equalised image.
   (ii) Apply local histogram equalization to the image $f(x,y)$ by diving the image in non-overlapping patches of size 64×64. Sketch the resulting histogram equalised image.
   (iii) Discuss which of the two transforms, i.e., the global or the local histogram equalization is more beneficial for the given image.

[Note: the dark line that appears around the image is used to signify the boundary of the image, but is not part of it.]

7. The two images shown below in **Figure 2** are quite different, but their histograms are identical. Both images have size 8 × 8, with black and white pixels. Suppose that both images are blurred with a 3×3 smoothing mask. Would the resultant histograms still be the same? Draw approximately the two histograms and explain your answer.
[Note: the dark lines that appear around the two images are used to signify the boundaries of the images but are not part of them.]

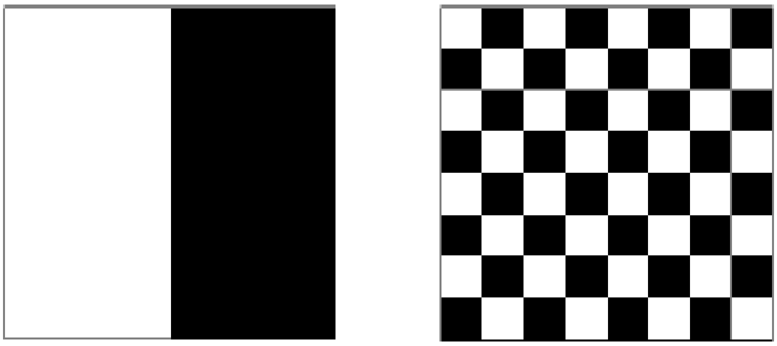


Figure 2

8. The histograms of two images are illustrated below. Sketch a transformation function for each image that will make the image have a better contrast. Use the axis provided below to sketch your transformation functions.