**Questions and Problems - Part 1**

**Discrete Fourier Transform**

**Questions**

1. Explain why it is common to work with the transform of an image instead of the image itself.
2. Explain why the Fourier transform amplitude of an image alone often does not capture the intelligibility of the image.
3. Explain why the Fourier transform phase of an image alone often captures most of the intelligibility of the image.
4. In a specific experiment it is observed that the amplitude response of an image exhibits energy concentration along a straight line in the 2-D frequency plane. What are the implications of this observation as far as the original image is concerned?
5. In a specific experiment it is observed that the amplitude of the Fourier transform of an image exhibits high values only very close to the origin and takes very small values within the rest of the two-dimensional frequency plane. State the implications of this observation as far as the original image is concerned.

**Problems**

1. Consider an -pixel image  which is zero outside  and . In transform coding, we discard the transform coefficients with small magnitudes and code only those with large magnitudes. Let  denote the -point Discrete Fourier Transform (DFT) of . Let  denote  modified by



Let



We reconstruct an image  by computing the -point inverse DFT of . Express  in terms of .

1. Consider an -pixel gray level image  which is zero outside  and . The image intensity is given by the following relationship



where  is a constant value between 0 and 255 and  is a constant value between 0 and .

1. Plot the image intensity.
2. Find the -point Discrete Fourier Transform (DFT) of . Plot its amplitude response.
3. Compare the plots found in (i) and (ii) above.

The following result holds: .

**Discrete Hadamard Transform**

**Questions**

Let  denote an -point 2-D sequence that is zero outside , , where  and  are integer powers of 2. In implementing the standard Hadamard Transform of , we relate  to a new -point sequence .

* + - 1. Define the sequence  in terms of .
      2. Define the concept of sequency in Hadamard transform.
      3. Define the Ordered Hadamard Transform without giving any mathematical equations.
      4. Comment on the energy compaction property of the Ordered Hadamard transform as compared to that of the standard Hadamard Transform and as a result of your answer explain which from the two transforms is more commonly used in image processing.

**Problems**

Let  denote the following constant  digital image that is zero outside , , with  a constant value.



Give the standard Hadamard Transform of  without carrying out any mathematical manipulations.

Comment on the energy compaction property of the standard Hadamard Transform.

**Karhunen Loeve Transform**

**Questions**

Consider the population of random vectors  of the form

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Each component  represents an image. The population arises from their formation across the entire collection of pixels. Suppose that , i.e. you have at least three images.

Consider now a population of random vectors of the form



where the vectors  are the Karhunen-Loeve transforms of the vectors .

* + - 1. Write down the relationship between  and .
      2. It is known that the covariance matrix of  is diagonal. Explain the relationship between these diagonal elements and the covariance matrix of .
      3. Suppose some elements of the diagonal are very small. Comment on the significance of this in relation to processing the images.
      4. Suppose that a credible job could be done of reconstructing approximations to the  original images by using only the two principal component images associated with the largest eigenvalues. What would be the mean square error incurred in doing so? Express your answer as a percentage of the maximum possible error.
      5. Suppose that a credible job could be done of reconstructing approximations to the  original images by using only the two principal component images associated with the largest eigenvalues. Find a lower bound for the mean square error incurred in doing, as a function of , where  is the covariance matrix of .
      6. Suppose that some elements of the covariance matrix of  are very small. Discuss the effectiveness of the KL transform in that case.
      7. Suppose that the elements of the covariance matrix of  are very similar. Discuss the effectiveness of the KL transform in that case.
      8. Suppose that a credible job could be done of reconstructing approximations to the  original images by using only the half of the principal component images associated with the largest eigenvalues. What would be the mean square error incurred in doing so? Express your answer as a percentage of both the maximum and the minimum possible error.
      9. Suppose that the covariance matrix of  turns out to be the identity matrix. Is the Karhunen-Loeve transform useful in that case? Justify your answer.

**Problems**

The covariance matrix of the population  calculated as part of the transform is



1. Suppose that a credible job could be done of reconstructing approximations to the three original images by using one principal component image. What would be the mean square error incurred in doing so, if it is known that ?
2. Suppose that a credible job could be done of reconstructing approximations to the three original images by using two principal component images. What would be the mean square error incurred in doing so, if it is known that ?

The covariance matrix of the population  calculated as part of the transform is



Suppose that a credible job could be done of reconstructing approximations to the three original images by using one or two principal component images. What would be the mean square error incurred in doing so in each case?