DCT-DHT Sample Exam Problems

1. Let $f(x,y)$ denote a digital image of size 256×256. In order to compress this image, we take its Discrete Cosine Transform $C(u,v), u,v = 0,\dots,255$ and keep only the Discrete Cosine Transform coefficients for $u,v = 0,\dots,n$ with $0 \le n < 255$. The percentage of total energy of the original image that is preserved in that case is given by the formula $an + b + 85$ with a,b constants. Furthermore, the energy that is preserved if $n = 0$ is 85%. Find the constants a,b.

2. Let $f(x,y)$ denote a digital image of size $M \times N$ pixels that is zero outside $0 \le x \le M-1$, $0 \le y \le N-1$, where M and N are integers and powers of 2. In implementing the standard Discrete Hadamard Transform of $f(x,y)$, we relate $f(x,y)$ to a new $M \times N$ point sequence $H(u,v)$.
   (i) State the main disadvantage of the Discrete Hadamard Transform.
   (ii) In the case of $M = N = 2$ and $f(x,y) = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ calculate the Hadamard transform coefficients of $f(x,y)$.

3. Let $f(x,y)$ denote the following constant 4×4 digital image that is zero outside $0 \le x \le 3$, $0 \le y \le 3$, with r a constant value.

$$\begin{bmatrix} r & r & r & r \\ r & r & r & r \\ r & r & r & r \\ r & r & r & r \end{bmatrix}$$

   (i) Give the standard Hadamard Transform of $f(x,y)$ without carrying out any mathematical manipulations.
   (ii) Comment on the energy compaction property of the standard Hadamard Transform.

4. Let $f(x,y)$ denote the following constant 4×4 digital image that is zero outside $0 \le x \le 3$, $0 \le y \le 3$, with r a constant value.

$$\begin{bmatrix} r & r & r & r \\ r & r & r & r \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

   Give the standard Hadamard Transform of $f(x,y)$.

   Hint: Use the recursive relationship of the Hadamard matrix and the separability property of the Hadamard Transform.