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PATH UNCERTAINTY ROBUST BEAMFORMING

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ABSTRACT

- Phase differences between received microphone signals are not a deterministic function of the array geometry.
- They are subject to random variations arising from position uncertainties and fluctuations in the sound velocity in the propagation channel.
- They degrade beamformer performance [1].
- We present a framework for modelling these uncertainties and show that improved beamformers are obtained when they are taken into account.

BEAMFORMER WEIGHTS DESIGN

- Beamformer weights are based on the covariance of the array data, $\mathbf{x}$.
- Array data is modelled as propagation coefficients, $\mathbf{D}$, multiplied by the sources, $\mathbf{s}$, with some additive noise.
- We redefine these propagation coefficients with an additional random propagation time, $\mathbf{T}$:

$$\left\langle \mathbf{x}\mathbf{x}^H \right\rangle \qquad \mathbf{x} = \mathbf{D}\mathbf{s} + \mathbf{v} \qquad \mathbf{D} = \overline{\mathbf{D}} \odot \exp(j\omega_k \mathbf{T})$$

- Which gives us a new model for array covariance:

$$\left\langle \mathbf{x}\mathbf{x}^H \right\rangle = \underbrace{\left\langle \left(\overline{\mathbf{D}} \odot \exp(j\omega_k \mathbf{T})\right) \mathbf{S} \left(\overline{\mathbf{D}} \odot \exp(j\omega_k \mathbf{T})\right)^H \right\rangle}_{\mathbf{A}} + \left\langle \mathbf{v}\mathbf{v}^H \right\rangle$$

- We can expand the propagation term as follows:

$$a_{i,j} = \sum_{p=1}^{P} \bar{d}_{ip} \bar{d}^*_{jp} \left\langle s_p^2 \right\rangle \times \exp\left(-\frac{\omega_k^2}{2} \left(\left\langle t_{ip}^2 \right\rangle + \left\langle t_{jp}^2 \right\rangle - 2 \left\langle t_{ip} t_{jp} \right\rangle \right) \right)$$

- We need the covariance in the additional random propagation times, $\mathbf{T}$:

$$\langle t_{ip} t_{jp} \rangle = \langle t_{ip} t_{jp} \rangle_S + \langle t_{ip} t_{jp} \rangle_C$$

ELEMENT POSITION UNCERTAINTIES

- Assume the positions of all elements are not exact.
- Source and microphone positions are given as: $\mathbf{p}_p$ and $\mathbf{m}_i$.
- They deviate from their nominal positions through a zero-mean Gaussian distribution ($\bar{\mathbf{p}}_p$ and $\bar{\mathbf{m}}_i$).
- The covariance in path length changes between microphones i and j is:

$$\langle \delta(i,p)\delta(j,p) \rangle = \frac{(\mathbf{m}_i - \mathbf{p}_p)^T \left(\left\langle \bar{\mathbf{m}}_i \bar{\mathbf{m}}_j^T \right\rangle + \left\langle \bar{\mathbf{p}}_p \bar{\mathbf{p}}_p^T \right\rangle \right) (\mathbf{m}_j - \mathbf{p}_p)}{|\mathbf{m}_i - \mathbf{p}_p| \, |\mathbf{m}_j - \mathbf{p}_p|}$$

- The resulting time covariance is:

$$\langle t_{ip} t_{jp} \rangle_S = \frac{1}{c^2} \langle \delta(i,p)\delta(i,p) \rangle$$

CHANNEL UNCERTAINTIES

- It is commonly assumed that the channel propagation speed between a source and two different microphones is identical.
- This does not apply when the arrays are far apart [2].
- Variations can become large enough to cause phase differences that degrade the performance of the beamformer [3].
- These channel uncertainty cause correlated errors terms in $\left\langle \mathbf{x}\mathbf{x}^H \right\rangle$.
- Propagation is modelled using the inverse velocity as a function of position $\mathbf{z}$, where $g(\mathbf{z})$ is a zero-mean Gaussian variable:

$$\frac{1}{c(\mathbf{z})} = \frac{1}{c_0} + g(\mathbf{z})$$

- The total path delay from $\mathbf{z}_a$ to $\mathbf{z}_b$ is given by:

$$q\left(\mathbf{z}_a, \mathbf{z}_b\right) = \frac{|\mathbf{z}_b - \mathbf{z}_a|}{c_0} + |\mathbf{z}_b - \mathbf{z}_a| \int_{t=0}^{1} g\left(\mathbf{z}_a + (\mathbf{z}_b - \mathbf{z}_a)\, t\right) dt$$

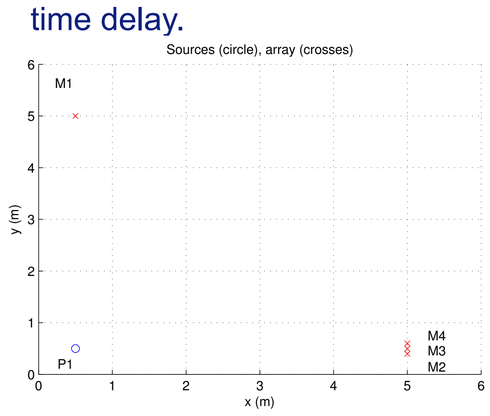
- The time difference covariance matrix is given by:

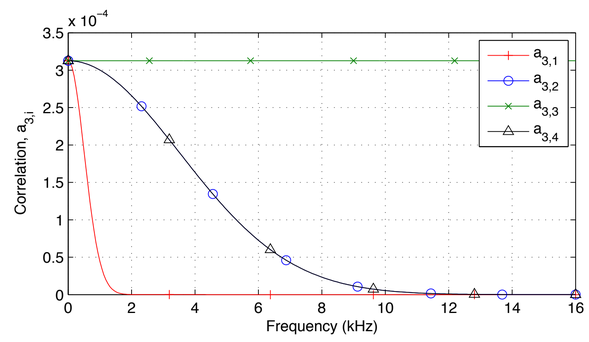
$$\langle t_{ip} t_{jp} \rangle_C = \left\langle \left((q(0, \mathbf{m}_i - \mathbf{p}_p) - \frac{|\mathbf{m}_i - \mathbf{p}_p|}{c_0} \right) \left((q(0, \mathbf{m}_j - \mathbf{p}_p) - \frac{|\mathbf{m}_j - \mathbf{p}_p|}{c_0} \right)^* \right\rangle$$

SIMULATIONS

SINGLE BEAMFORMER

- We applied the above model to the array geometry shown on the left.
- The correlation, $a_{3,j}$, of M3 with the other microphones is on the right.
- Neighbouring propagation paths have a higher correlation in propagation time delay.



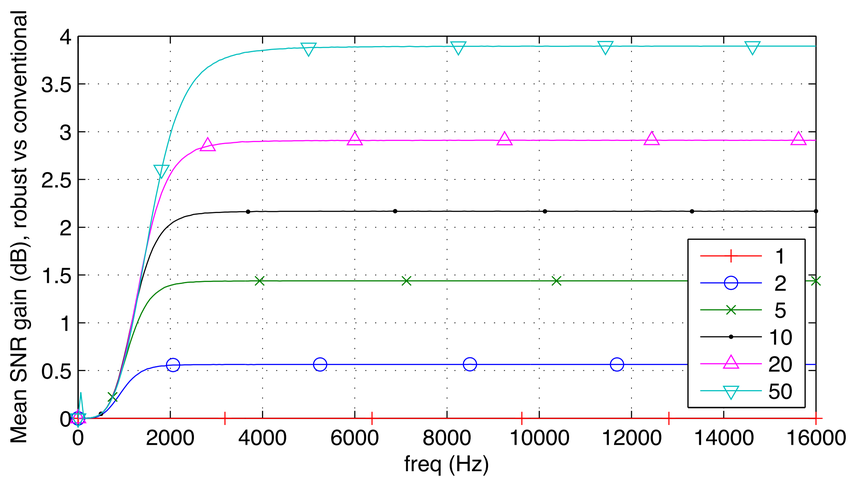


GENERAL TESTS

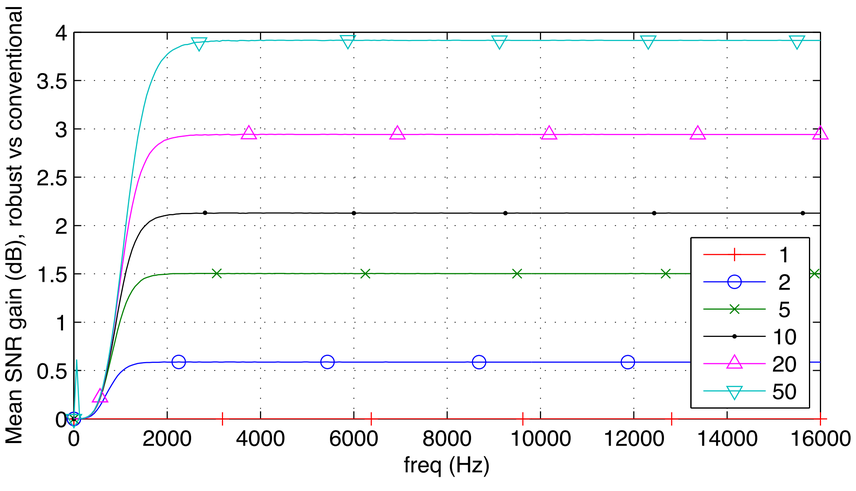
- Model applied to 300 random beamformer geometries.
- We compared SNR optimal beamformer weights [4] with (1) and without (2) considering random deviations:

$$\mathbf{D} = \begin{cases} \overline{\mathbf{D}} & (1) \\ \overline{\mathbf{D}} \odot \exp(j\omega_k \mathbf{T}) & (2) \end{cases}$$

- In the presence of Gaussian sensor noise and spatially diffuse noise.
- Measured resulting SNR in each case.
- Results are averaged over all 300 beamformers.
- Considering **only channel uncertainties** we observe the following difference in SNR between (1) and (2).



- As the correlation in the channels decays, the SNR of (1) degrades.
- Whereas (2) is able to utilise the most reliable microphones to maintain higher SNR.
- The more microphones in the array, the larger the SNR gains.
- Considering **channel uncertainties and positional uncertainties** with standard deviations of 5cm in each element, we obtain the following:



- This further improved the performance of the robust beamformer compared to the conventional.

REFERENCES

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