EE1 and ISE1 Communications I

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Lecture two

Lecture Aims

- To introduce signals,
- Classifications of signals,
- Some particular signals.

Signals

- A signal is a set of information or data.

- Examples
  - a telephone or television signal,
  - monthly sales of a corporation,
  - the daily closing prices of a stock market.

- We deal exclusively with signals that are functions of time.

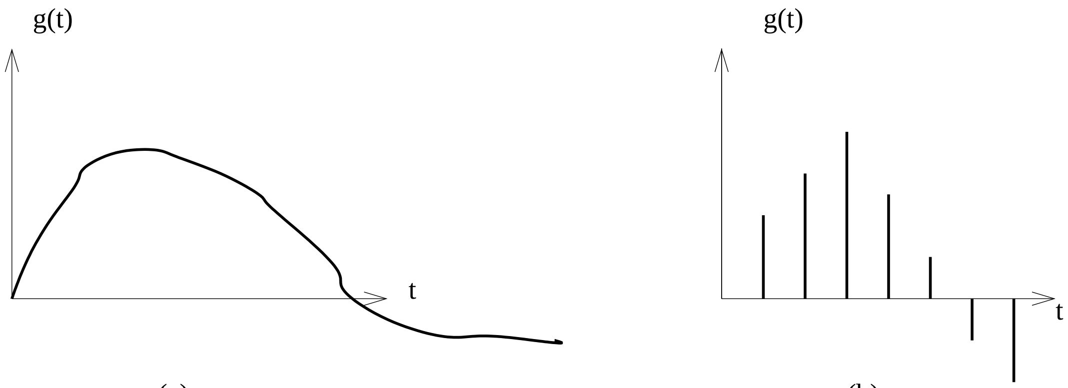
How can we measure a signal?
How can we distinguish two different signals?

Classifications of Signals

- Continuous-time and discrete-time signals
- Analog and digital signals
- Periodic and aperiodic signals
- Energy and power signals
- Deterministic and probabilistic signals

Continuous-time and discrete-time signals

- A signal that is specified for every value of time t is a continuous-time signal.
- A signal that is specified only at discrete values of t is a discrete-time signals.



(a) (b)

Continuous-time and discrete-time signals (continued)

- A discrete-time signal can be obtained by **sampling** a continuous-time signal.
- In some cases, it is possible to 'undo' the sampling operation. That is, it is possible to get back the continuous-time signal from the discrete-time signal.

Sampling Theorem

The sampling theorem states that if the highest frequency in the signal spectrum is B, the signal can be reconstructed from its samples taken at a rate not less than $2B$ sample per second.

Analog and digital signals

- A signal whose amplitude can take on any value in a continuous range is an analog signal.
- The concept of analog and digital signals is different from the concept of continuous-time and discrete-time signals.
- For example, we can have a digital and continuous-time signal, or a analog and discrete-time signal.

Analog and digital signals (continued)

- One can obtain a digital signal from an analog one using a *quantizer*.
- The amplitude of the analog signal is partitioned into L intervals. Each sample is approximated to the midpoint of the interval in which the original value falls.
- Quantization is a **lossy** operation.

Notice that:
One can obtain a digital discrete-time signal by sampling and quantizing an analog continuous-time signal.

Periodic and aperiodic signals

- A signal $g(t)$ is said to be periodic if for some positive constant T_0,

$$g(t) = g(t + T_0) \text{ for all } t.$$

- A signal is aperiodic if it is **not** periodic.

Same *famous* periodic signals:

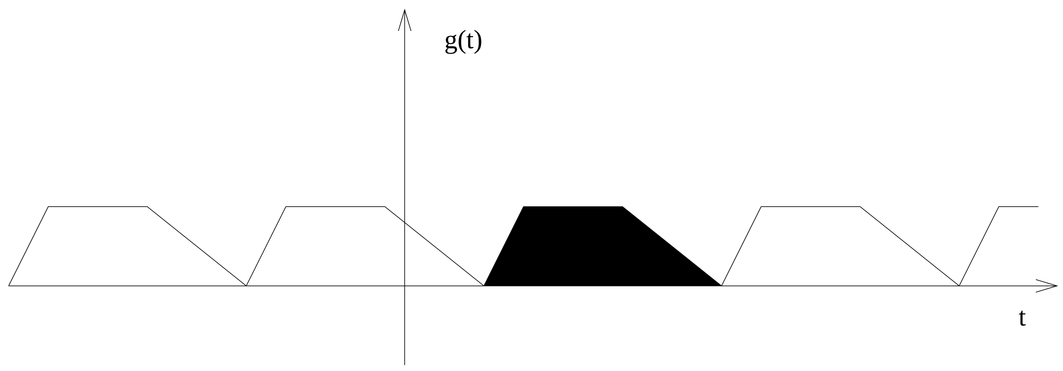
$$\sin \omega_0 t, \quad \cos \omega_0 t, \quad e^{j\omega_0 t},$$

where $\omega_0 = 2\pi/T_0$ and T_0 is the period of the function.

(Recall that $e^{j\omega_0 t} = \cos \omega_0 t + j \sin \omega_0 t$).

Periodic Signal

A periodic signal $g(t)$ can be generated by periodic extension of any segment of $g(t)$ of duration T_0.



Energy and power signal

First, define energy.

- The signal energy E_g of $g(t)$ is defined (for a real signal) as

$$E_g = \int_{-\infty}^{\infty} g^2(t)dt.$$

- In the case of a complex valued signal $g(t)$, the energy is given by

$$E_g = \int_{-\infty}^{\infty} g^*(t)g(t)dt = \int_{-\infty}^{\infty} |g(t)|^2 dt.$$

A signal $g(t)$ is an energy signal if $E_g < \infty$.

Power

A necessary condition for the energy to be finite is that the signal amplitude goes to zero as time tends to infinity.
In case of signals with infinite energy (e.g., periodic signals), a more meaningful measure is the signal power.

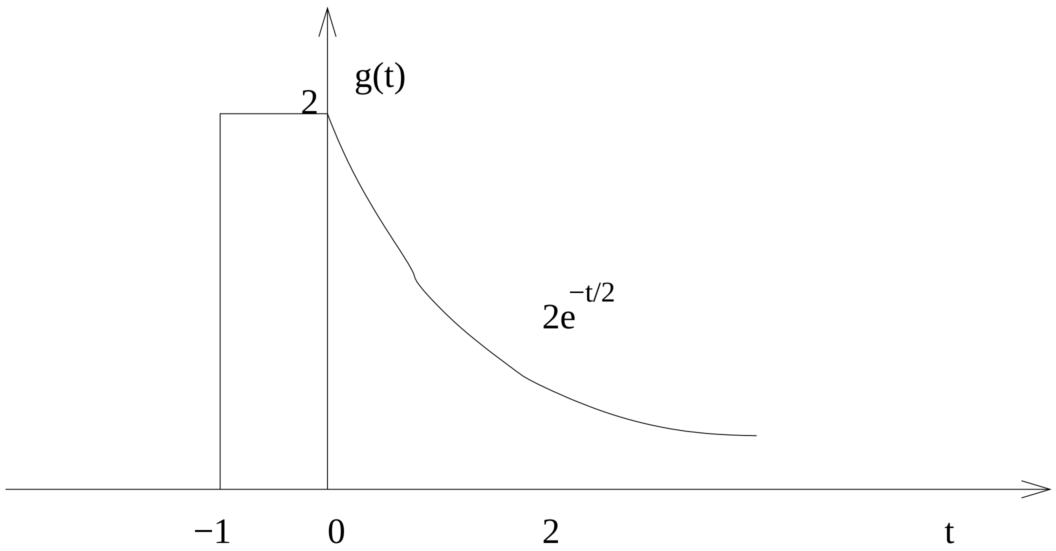
$$P_g = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt$$

A signal is a power signal if

$$0 < \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt < \infty$$

A signal cannot be an energy and a power signal at the same time.

Energy signal example



Signal Energy calculation

$$E_g = \int_{-\infty}^{\infty} g^2(t)dt = \int_{-1}^{0} (2)^2 dt + \int_{0}^{\infty} 4e^{-t} dt = 4 + 4 = 8.$$

Power signal example

Assume $g(t) = Acos(\omega_0 t + \theta)$, its power is given by

$$\begin{aligned} P_g &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2 cos^2(\omega_0 t + \theta) dt \\ &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{A^2}{2} [1 + cos(2\omega_0 t + 2\theta)] dt \\ &= \lim_{T\to\infty} \frac{A^2}{2T} \int_{-T/2}^{T/2} dt + \lim_{T\to\infty} \frac{A^2}{2T} \int_{-T/2}^{T/2} cos(2\omega_0 t + 2\theta)] dt \\ &= A^2/2 \end{aligned}$$

Power of Periodic Signals

Show that the power of a periodic signal $g(t)$ with period T_0 is

$$P_g = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |g(t)|^2 dt$$

Another important parameter of a signal is the **time average**:

$$g_{average} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} g(t) dt.$$

Deterministic and probabilistic signals

- A signal whose physical description is known completely is a deterministic signal.
- A signal known only in terms of probabilistic descriptions is a random signal.

Summary

- Signal classification
- Power of a periodic signal of period T_0

$$P_g = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |g(t)|^2 dt$$

- Time average

$$g_{average} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} g(t) dt.$$

- Power of a sinusoid $A \cos(2\pi f_0 t + \theta)$ is $\frac{A^2}{2}$