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SCENIC

PERFORMANCE ANALYSIS OF IPNLMS FOR IDENTIFICATION OF TIME-VARYING SYSTEMS

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1. ABSTRACT

The tracking performance of adaptive filters is crucially important in practical applications involving time-varying systems. We present an analysis of the tracking performance for IPNLMS, one of the best known and best performing algorithms originally targeted at sparse system identification. We then validate our analytic results in practical simulations for echo cancellation for sparse and dispersive time-varying unknown echo path systems. These results show the analysis to be highly accurate in all the cases studied.

2. REVIEW OF IPNLMS

Notations:

$x(n) =$ far-end signal

$L =$ length of the room impulse response

$\mathbf{x}(n) = [x(n)\ x(n-1)\ \cdots\ x(n-L+1)]^T =$ input signal

$\mathbf{h}(n) = [h_1(n)\ h_2(n)\ \cdots\ h_L(n)]^T =$ unknown impulse response

$\hat{\mathbf{h}}(n) = [\hat{h}_1(n)\ \hat{h}_2(n)\ \cdots\ \hat{h}_L(n)]^T =$ estimated impulse response

$y(n) = \mathbf{h}^T(n)\mathbf{x}(n) + w(n) =$ echo and background noise

The improved-proportionate normalized-LMS algorithm (IPNLMS):

$$e(n) = y(n) - \hat{\mathbf{h}}^T(n)\mathbf{x}(n), \tag{1}$$

$$\hat{\mathbf{h}}(n+1) = \hat{\mathbf{h}}(n) + \frac{\mu\mathbf{Q}(n+1)\mathbf{x}(n)e(n)}{\mathbf{x}^T(n)\mathbf{Q}(n+1)\mathbf{x}(n)+\delta}, \tag{2}$$

$$\mathbf{Q}(n+1) = \mathrm{diag}\{q_1(n+1),\ q_2(n+1),\ \cdots,\ q_L(n+1)\}, \tag{3}$$

$$q_l(n+1) = \frac{1-\alpha}{2L} + \frac{(1+\alpha)|\hat{h}_l(n)|}{2\|\hat{\mathbf{h}}(n)\|_1 + \delta_{\mathrm{ip}}}, \quad 1 \le l \le L \tag{4}$$

3. CONVERGENCE ANALYSIS OF IPNLMS

3.1. Time-varying System Model

The modified first-order Markov model:

$$\mathbf{h}(n+1) = \varepsilon\mathbf{h}(n) + \sqrt{1-\varepsilon^2}\,\mathbf{s}(n) \tag{5}$$

- $\mathbf{s}(n)$ is a random sequence of length L with elements drawn from a normal (Gaussian) distribution with zero mean and variance σ_s^2 .
- ε ($0 \ll \varepsilon < 1$) controls the relative contributions to the instantaneous values of the "system memory" and "innovations".

3.2. Recursive Mean-square Error Analysis

- Weight deviation vector, $\mathbf{z}(n) = \mathbf{h}(n) - \hat{\mathbf{h}}(n)$ (6)
- The mean-square output error, MSE $(n) = \sigma_w^2 + \sigma_x^2 \sum_{l=1}^{L} E\{z_l^2(n)\}$. (7)

By substituting (2) and (5), the component-wise weight deviation is given by

$$z_l(n+1) = z_l(n) + (\varepsilon-1)h_l(n) + \sqrt{1-\varepsilon^2}\,s_l(n) - \left[\frac{\mu q_l(n+1)x_l(n)}{\mathbf{x}^T(n)\mathbf{Q}(n+1)\mathbf{x}(n)+\delta}\right]\left[w(n) + \sum_{j=1}^{L} x_j(n)z_j(n)\right]$$

Therefore,

$$\begin{aligned} z_l^2(n+1) = {} & z_l^2(n) + 2z_l(n)(\varepsilon-1)h_l(n) + (\varepsilon-1)^2h_l^2(n) + 2z_l(n)\sqrt{1-\varepsilon^2}\,s_l(n) + \\ & (1-\varepsilon^2)s_l^2(n) - \left[\frac{2\mu(\varepsilon-1)h_l(n)q_l(n+1)x_l(n)}{\mathbf{x}^T(n)\mathbf{Q}(n+1)\mathbf{x}(n)+\delta}\right]\left[w(n) + \sum_{j=1}^{L} x_j(n)z_j(n)\right] - \\ & \left[\frac{2\mu\sqrt{1-\varepsilon^2}\,s_l(n)q_l(n+1)x_l(n)}{\mathbf{x}^T(n)\mathbf{Q}(n+1)\mathbf{x}(n)+\delta}\right]\left[w(n) + \sum_{j=1}^{L} x_j(n)z_j(n)\right] + \\ & 2(\varepsilon-1)h_l(n)\sqrt{1-\varepsilon^2}\,s_l(n) - \left[\frac{2\mu z_l(n)q_l(n+1)x_l(n)}{\mathbf{x}^T(n)\mathbf{Q}(n+1)\mathbf{x}(n)+\delta}\right]\left[w(n) + \sum_{j=1}^{L} x_j(n)z_j(n)\right] + \\ & \left[\frac{\mu^2q_l^2(n+1)x_l^2(n)}{(\mathbf{x}^T(n)\mathbf{Q}(n+1)\mathbf{x}(n)+\delta)^2}\right]\left[w(n) + \sum_{j=1}^{L} x_j(n)z_j(n)\right]^2 \end{aligned} \tag{8}$$

Assumptions [1]:

- The step-size μ is chosen sufficiently small such that $z_l(n)$ changes slowly relative to $x_l(n)$.
- The length of the adaptive filter L is equivalent to that of the unknown system.
- $E\left\{\mathbf{x}^T(n)\mathbf{Q}(n+1)\mathbf{x}(n)+\delta\right\} = \sigma_x^2 + \delta$

  $E\left\{\left(\mathbf{x}^T(n)\mathbf{Q}(n+1)\mathbf{x}(n)+\delta\right)^2\right\} = \left(\sigma_x^2+\delta\right)^2$
- $E\left\{q_l^a(n)\right\} = E\left\{q_l(n)\right\}^a$

  $E\left\{q_l^a(n+1)z_l^b(n)\right\} = E\left\{q_l(n+1)\right\}^a E\left\{z_l^b(n)\right\}^a$
- The l^{th} component of the weight deviation at each iteration, $z_l(n)$, follows a normal distribution with $\bar{z}_l(n) \triangleq E\{z_l(n)\}$ and variance $\sigma_l^2(n)$. This implies that $\hat{h}_l(n) \sim N\left(m_l(n), \sigma_l^2(n)\right)$ where $m_l(n) = h_l(n) - \bar{z}_l(n)$, $\sigma_l^2(n) \triangleq E\left\{z_l^2(n)\right\} - E^2\left\{z_l(n)\right\}$. Therefore, the mean of $|\hat{h}_l(n)|$ is

$$E\left\{|\hat{h}_l(n)|\right\} = m_l(n)\ \mathrm{erf}\left(\frac{m_l(n)}{\sqrt{2\sigma_l^2(n)}}\right) + \sqrt{\frac{2}{\pi}}\sigma_l(n)e^{-\frac{m_l^2(n)}{2\sigma_l^2(n)}}. \tag{9}$$

3.3. Key Results

By employing these assumptions, the expectations of the component-wise weight deviations can be given by the following recursive forms:

$$E\{z_l(n+1)\} = E\{z_l(n)\} - \frac{\mu\sigma_x^2}{\sigma_x^2+\delta}E\{q_l(n+1)\}E\{z_l(n)\}, \tag{10}$$

$$\begin{aligned} E\{z_l^2(n+1)\} = {} & E\{z_l^2(n)\} + 2(1-\varepsilon)\sigma_s^2 - \frac{2\mu\sigma_x^2}{\sigma_x^2+\delta}E\{q_l(n+1)\}E\{z_l^2(n)\} + \\ & \frac{\mu^2\sigma_x^2\sigma_w^2}{(\sigma_x^2+\delta)^2}E\{q_l(n+1)\}^2 + \frac{\mu^2\sigma_x^4}{(\sigma_x^2+\delta)^2}E\{q_l(n+1)\}^2\sum_{j=1}^{L}E\{z_j^2(n)\} \end{aligned} \tag{11}$$

With $E\{q_l(n+1)\} = \dfrac{1-\alpha}{2L} + \dfrac{(1+\alpha)E\{|\hat{h}_l(n)|\}}{2\sum_{j=1}^{L}E\{|\hat{h}_j(n)|\} + \delta_{\mathrm{ip}}}$ (12)

Given (9)-(12), we can now recursively compute the MSE using (7).

4. SIMULATION RESULTS

The theoretical result derived in the previous section is confirmed with the Monte Carlo simulations with 100 independent trials, for different time-varying systems scenarios in the context of AEC. In all simulations, the adaptive filter length was set to $L = 1024$.

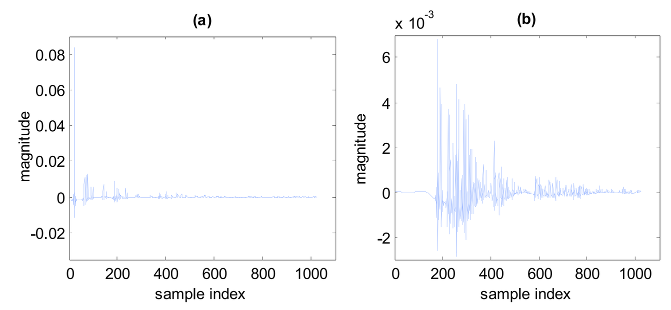

Fig.1. Acoustic impulse responses

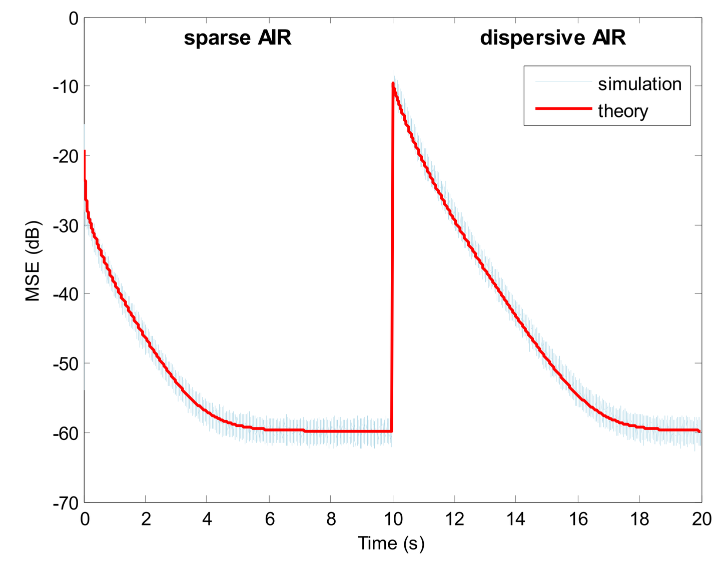

Fig.2. MSE of IPNLMS with a sudden echo path change at 10 s, from that shown from Fig. 1 (a) to (b) and $\mu = 0.7$, $\alpha =$ -0.75, $\sigma_x^2 = 10^{-2}, \sigma_w^2 = 10^{-6}$, $\delta = \delta_{ip} = 10^{-4}$. It can be seen that the predicted MSE corresponds very well with the simulated MSE, even during the echo path change.

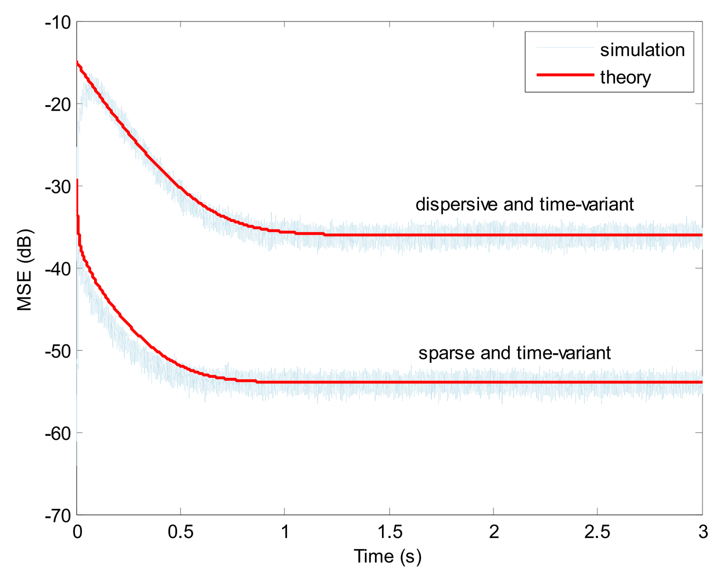

Fig.3. MSE of IPNLMS for a sparse and dispersive time-varying systems with $\mu = 0.7$, $\alpha =$ -0.75, $\sigma_x^2 = 10^{-3}$, $\sigma_w^2 = 10^{-6}$, $\sigma_s^2 = 1$, $\delta = \delta_{ip} = 10^{-4}$, ε = 1-10^{-9} (sparse), ε = 1-10^{-7} (dispersive). Due to the 4th assumption, the predicted MSE slightly deviates from the simulated MSE for the sparse time-varying system, during the initial stage.

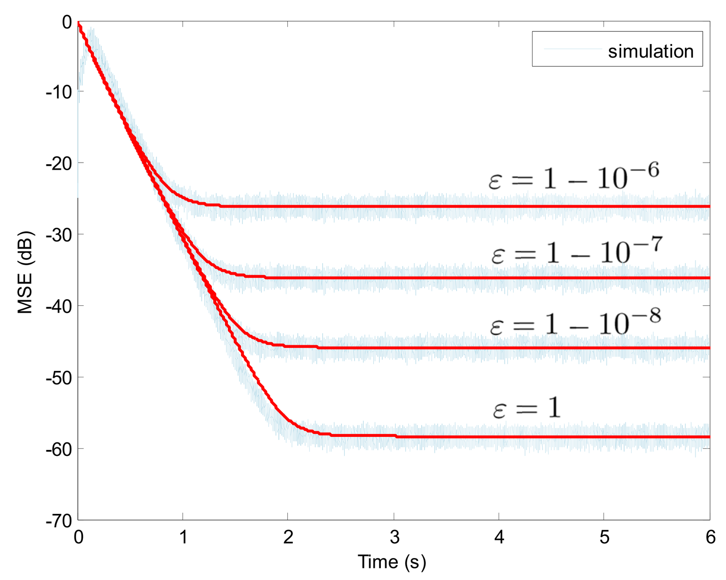

Fig.4. MSE of IPNLMS for varying ε with $\mu = 0.7$, $\alpha =$ -0.75, $\sigma_x^2 = 10^{-3}$, $\sigma_w^2 = 10^{-6}$, $\sigma_s^2 = 1$, $\delta = \delta_{ip} = 10^{-4}$. We notice that the steady-state MSE increases when ε decreases (i.e, the system becomes more time-variant).

5. CONCLUSION

A performance analysis has been presented for IPNLMS, one of the best known sparse adaptive filtering algorithms. The analysis considers the tracking case in which the unknown system to be identified is not only sparse and dispersive but also time-varying. The analysis has been validated against simulation results in the context of AEC and shown to be accurate. The cases of step-changes in the echo path as well as slowly time-varying echo paths have been included in the study with varying levels of sparseness.

6. REFERENCES

[1] K. Wagner and M. Doroslovacki, "Towards analytical convergence analysis of proportion-type NLMS algorithms," *in Proc. IEEE Int. Conf. Acoustics Speech Signal Processing*, 2008, pp. 3825–3828.

ACKNOWLEDGEMENTS

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