Complex-Valued Least Squares Frequency Estimation for Unbalanced Power Systems

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***Abstract*— A class of fundamental frequency estimation algorithms for unbalanced three-phase power systems is proposed. This is achieved in the complex domain by establishing and exploiting three time-series relationships among equidistantly spaced Clarke's transformed system voltage samples. To enable meaningful frequency estimation with high immunity to noise and higher order harmonic pollution, complex-valued least squares (CLS) framework is employed in conjunction with those relationships. Theoretical bias and variance analysis are further conducted to yield the optimal distance (phase interval) between the voltage samples within the models considered. It is shown that although all the CLS approaches, with their own optimal phase intervals, ideally have identical minimum mean square estimation errors, the CLS approaches based on four-point relationships are favorable in practical applications, since the optimal phase interval attains statistically unbiased frequency estimation in the presence of both noise and harmonic pollution, which is impossible when using their three-point counterpart. Simulations over a range of unbalanced system conditions and in the presence of noise and higher order harmonics, as well as for real-world measurements, support the analysis.**

***Index Terms*— Complex-valued least squares (CLS) estimation, frequency estimation, minimum mean square error (MMSE) analysis, optimal phase interval analysis, unbalanced power systems.**

I. INTRODUCTION

FREQUENCY is an important power quality parameter and is only allowed to vary within a small predefined range; its variations are a consequence of a dynamic imbalance between the generation and the load [1]. Accurate frequency estimation is essential, as maintaining the nominal frequency value is a prerequisite for both the stability of the grid and for normal operation of electrical devices [2]. Frequency is also a key parameter in the control of distributed grids where it can be used, for instance, to determine the harmonic contents of currents drawn by nonlinear loads.

Traditionally, the frequency is estimated by the time between two zero crossings as well as the calculation of the number of cycles [3]. However, this method is relatively sensitive for distorted signals under harmonics and noise pollution. To suppress this drawback, many fast and accurate frequency estimation methods have been developed during recent years. Some are developed based on the analysis of the voltage frequency spectrum. These methods usually estimate the frequency by searching for the maximum in the spectrum by using, e.g., discrete Fourier transform (DFT) [4] or minimum variance distortionless response (MVDR) [5], and advanced windowed functions and interpolation schemes have been applied to compensate for the effects of leakage caused by incoherent sampling and finite frequency resolution problems, respectively [6]–[8]. Another large class of efficient frequency estimation approaches is based on either the phase-locked loop (PLL) techniques or the adaptive notch filters (ANFs) [9]–[13]. The principal idea of phase locking is to actively generate a signal whose phase angle is adaptively tracking variations of that of a given signal by means of a control loop, whereas the ANF is a concept similar to the PLL in the sense that it passively extracts its phase angle output from the given signal. Both linear and nonlinear state model-based algorithms have also attracted widespread attention, as they accurately estimate the amplitude, phase, and frequency of a signal buried under noise and harmonics, including extended Kalman filtering [14], [15], unscented Kalman filtering [16], [17], and resonated-based observers [18], [19]. Frequency estimation using identification theory is typically based on the time-series relationships among consecutive pure sinusoidal/exponential voltage samples, such as the method of least squares (LS) and its variants [20]–[22], least mean square-based adaptive filtering [23], and recursive methods [24], [25]. Other interesting frequency estimation algorithms are based on statistics, including Taylor series expansion-based signal approximation, higher order signal integration, and so on [26]–[28]. For real time use, most of the aforementioned methods offer a tradeoff between accuracy and speed. Among these, due to their linear structures, the time-series relationships among consecutive pure sinusoidal voltage samples are widely employed, as they admit both closed-form expressions for frequency estimation, and recursive operation in an online manner. Although it is widely accepted that such estimation methods are susceptible to heavy noise and higher order harmonic disturbances, so that the desirable noise/harmonic suppression can only be achieved with signal prefiltering, advantages have recently been reported whereby the time-series relationships among consecutive sinusoidal samples were generalized for equidistantly spaced samples with an arbitrary distance

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(phase interval). This also facilitates optimal phase interval search based on the minimization of the estimation bias in heavy noise/harmonic pollution [29], [30]. Another advantage is that the LS approach can be used in its original form without the necessity for signal preprocessing.

To characterize power system frequency from multiphase measurements, it is convenient to exploit the complex-valued Clarke's transformed system voltage, which allows for a simultaneous modeling of all the available information among the three-phase reference channels [31]. Most complex-valued frequency estimation algorithms applied to the transformed voltage work well under normal balanced power system conditions, and also operate well in noisy environments and in the presence of frequency deviation, however, vast majority suffer from performance degradation under unbalanced system conditions, including unequal voltage magnitudes at the fundamental system frequency (under-voltages and over-voltages), together with fundamental phase angle deviation and unequal levels of harmonics among the phases, a long standing problem in distributed power systems [32], [33]. Such unbalanced system problems cause the Clarke's transformed system voltage to no longer represent a single complex exponential (positive sequence) that rotates anticlockwise at the fundamental system frequency in the complex plane, but an orthogonal sum of a positive sequence and a negative sequence, a complex exponential rotating clockwise at the system frequency. Standard phase angle calculation techniques employed by frequency estimation algorithms [5], [23], [34] are designed only for the positive sequence, yet it is the negative sequence that introduces unavoidable frequency estimation bias and estimation oscillations at twice the system frequency, resulting in suboptimal performance [35].

One solution for robust frequency estimation for unbalanced power systems is to mitigate the effect of the negative sequence by employing PLL systems enhanced by pq theory or discrete filters to extract the positive sequence component from the unbalanced complex-valued system voltage [36]–[39], so that the standard phase angle extraction algorithms designed for balanced power systems can be directly applied. Other attempts include the use of proportional-integral-derivative controllers to iteratively correct the phase angle of the Clarke's transformed voltage under unbalanced power conditions [40], and applying in parallel adaptive notch filtering or weighted LS along the three-phase voltage [13], [41]. A recent effort, which benefits from advances in complex-valued second-order statistics [42], [43], utilizes the widely linear (WL) estimation model, incorporating both the complex-valued system voltage and its complex conjugate, to exploit the noncircular statistical nature of the unbalanced system voltage and extend standard frequency estimation algorithms into a more general form which takes into account the information contained in the negative sequence in order to achieve unbiased frequency estimates [44]–[47].

In this paper, a new and computationally efficient frequency estimation scheme, without the necessity for a signal preprocessing of positive sequence extraction, is proposed for unbiased frequency estimation of unbalanced three-phase power systems. Motivated by the recent work in [29] and [30], three time-series relationships among equidistantly spaced unbalanced system voltages with an arbitrary phase interval are established in the complex domain, to cater for the complex-valued Clarke's transformed system voltage. In order to enable meaningful frequency estimation with high immunity to noise and higher order harmonic pollution, the complex-valued least squares (CLS) framework is built upon those relationships and its fundamental properties are investigated via statistical bias analysis on the optimal distance (phase interval) search among equidistantly spaced voltage samples. This allows us to minimize the estimation bias, as shown in our mean square error analysis which establishes the achievable minimum frequency estimation variance by the CLS framework. Unlike WL estimation algorithms [44]–[47], which consider both the system voltages and their complex conjugates within the estimation model so as to remove the effect of the negative sequence on the frequency estimates, the underlying time series relationships of the proposed CLS framework are strictly linear in the sense that only the system voltages themselves are considered in both balanced and unbalanced conditions, hence reducing the computational complexity and facilitating the theoretical performance analysis. Numerical simulations on unbalanced power systems in the presence of noise and higher order harmonics, as well as for real-world measurements, are conducted to verify the performance of the proposed CLS frequency estimation algorithms. Performance advantages over two existing WL frequency estimation algorithms based on the conventional LS and a recently developed MVDR spectrum [5], [23] can also be observed.

II. Unbalanced Three-Phase Power Systems

The three-phase voltages of a power system in a noise/harmonic-free environment can be represented in a discrete time form as

$$\begin{aligned} v_a(k) &= V_a\cos(k\Omega\triangle T + \phi) \\ v_b(k) &= V_b\cos\left(k\Omega\triangle T + \phi - \frac{2\pi}{3}\right) \\ v_c(k) &= V_c\cos\left(k\Omega\triangle T + \phi + \frac{2\pi}{3}\right) \end{aligned} \tag{1}$$

where V_a, V_b, and V_c are the peak values of each fundamental voltage component at a time instant k, $\triangle T = 1/f_s$ is the sampling interval, f_s is the sampling frequency, ϕ is the initial phase, and $\Omega = 2\pi f_o$ is the angular frequency of the voltage signal, with f_o being the system frequency. The three-phase voltage is routinely transformed by the orthogonal $\alpha\beta 0$ transformation matrix (Clarke's transform) into a zero-sequence v_0 and the direct and quadrature-axis components, v_α and v_β, as [31]

$$\begin{bmatrix} v_0(k) \\ v_\alpha(k) \\ v_\beta(k) \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_a(k) \\ v_b(k) \\ v_c(k) \end{bmatrix}. \tag{2}$$

The factor $(2/3)^{1/2}$ ensures that the system power is invariant under this transformation. In balanced system conditions, V_a, V_b, and V_c are identical, giving $v_0(k) = 0$, $v_\alpha(k) = A\cos(\Omega k\Delta T + \phi)$, and $v_\beta(k) = A\sin(\Omega k\Delta T + \phi)$.

In practice, normally only the v_α and v_β parts are used to form the complex system voltage $v(k)$, given by [31]

$$v(k) = v_\alpha(k) + \jmath v_\beta(k) = Ae^{j(\Omega k\Delta T+\phi)} = Ae^{j(\omega_o k+\phi)} \quad (3)$$

where $\omega_o = \Omega\Delta T$ is the normalized angular frequency, and $v_\alpha(k)$ and $v_\beta(k)$ represent the orthogonal coordinates of a point whose position is time variant at a rate proportional to the system frequency. For a constant f_s, the scatter diagram of the positive sequence $v(k)$ is rotation invariant, since both v and $ve^{\jmath\theta}$ have the same distribution for any real θ. This means that the complex-valued $v(k)$ is second-order circular (proper), while its distribution in the complex plane is a circle with constant radius [42]–[45]. This facilitates standard phase angle calculation employed by traditional frequency estimation algorithms designed for balanced power systems [35]. However, when the three-phase power system deviates from its nominal condition, such as under different levels of dips or transients, V_a, V_b, and V_c are not identical (voltage sag), and the complex voltage from (3) becomes [44]–[47]

$$\begin{aligned} v(k) &= Ae^{jk\omega_o} + Be^{-jk\omega_o} \\ k &= 0, 1, \ldots, K-1 \end{aligned} \quad (4)$$

where

$$\begin{aligned} A &= \frac{\sqrt{6}(V_a + V_b + V_c)}{6} e^{\jmath\phi} = |A|e^{\jmath\phi_A} \\ B &= \left(\frac{\sqrt{6}(2V_a - V_b - V_c)}{12} - \jmath\frac{\sqrt{2}(V_b - V_c)}{4}\right) \\ e^{-\jmath\phi} &= |B|e^{\jmath\phi_B} \end{aligned} \quad (5)$$

and K is the number of observations. Note that this expression is theoretically accurate for both the balanced and unbalanced system conditions. For balanced system condition, $V_a = V_b = V_c$ and thus $B = 0$, whereas for unbalanced conditions, $B \neq 0$, and the so introduced negative sequence $Be^{-jk\omega_o}$ causes the samples of $V(k)$ generated from (5) to deviate from the circle with constant radius, making the distribution of $v(k)$ rotation dependent (noncircular) [42], [43]. This, in turn, results in the failure of standard phase angle calculation algorithms designed for balanced power systems, represented by the unavoidable estimation bias and oscillations at double the system frequency [13], [35], [41], [44], [45], [47].

III. Time-Series Relationships Among Voltage Samples in Unbalanced Conditions

Motivated by the recent work in [29] and [30], we next investigate the time-series relationships among unbalanced complex-valued system voltage samples, making it possible to design unbiased frequency estimators for unbalanced power systems. To that end, we shall first consider the difference between Clarke's voltage samples $v(k-m)$ and $v(k-2m)$, for an arbitrary distance m, to give

$$\begin{aligned} v(k-m) - v(k-2m) &= \left(e^{j(k-m)\omega_o} - e^{j(k-2m)\omega_o}\right)A \\ &\quad + \left(e^{-j(k-m)\omega_o} - e^{-j(k-2m)\omega_o}\right)B \\ &= \left(e^{-\jmath m\omega_o} - e^{-\jmath 2m\omega_o}\right)Ae^{jk\omega_o} \\ &\quad + \left(e^{\jmath m\omega_o} - e^{\jmath 2m\omega_o}\right)Be^{-jk\omega_o}. \end{aligned} \quad (6)$$

Note that

$$e^{-\jmath m\omega_o} - e^{-\jmath 2m\omega_o} = 2\jmath e^{-\jmath\frac{3m\omega_o}{2}}\sin\left(\frac{m\omega_o}{2}\right) \quad (7)$$

which allows us to simplify (6) as

$$\begin{aligned} v(k-m) - v(k-2m) = 2\jmath\big(&e^{-j\frac{3m\omega_o}{2}}Ae^{jk\omega_o} \\ &- e^{j\frac{3m\omega_o}{2}}Be^{-jk\omega_o}\big)\sin\left(\frac{m\omega_o}{2}\right). \end{aligned} \quad (8)$$

In a similar way, for the four consecutive time intervals in the unit of time distance m, we have

$$\begin{aligned} &v(k) - v(k-m) + v(k-2m) - v(k-3m) \\ &= 4\jmath\left(e^{-j\frac{3m\omega_o}{2}}Ae^{jk\omega_o} - e^{j\frac{3m\omega_o}{2}}Be^{-jk\omega_o}\right) \\ &\quad \times \sin\left(\frac{m\omega_o}{2}\right)\cos(m\omega_o). \end{aligned} \quad (9)$$

We can now obtain a frequency estimator, referred to as FE1, by dividing (9) by (8) to give [29], [30]

$$\cos(m\hat{\omega}) = \frac{v(k) - v(k-m) + v(k-2m) - v(k-3m)}{2\big(v(k-m) - v(k-2m)\big)} \quad (10)$$

where $\hat{\omega}$ is the estimate of ω_o. Observe that the estimation in (10) is of the same form as its real-valued counterpart built upon the pure sinusoidal voltage samples [29], [30]. In a similar way, frequency estimators can be developed for complex-valued unbalanced voltage samples based on the other two well established time-series relationships [22], [41]. They are, respectively, referred to as FE2 and FE3 and are summarized in Table I using a unifying framework where

$$\cos(m\hat{\omega}) = \frac{b(k)}{a(k)} \quad (11)$$

from which we obtain the estimated frequency $\hat{f}$ as

$$\hat{f} = \frac{f_s}{2\pi m}\cos^{-1}\left(\frac{b(k)}{a(k)}\right). \quad (12)$$

Although these time-series relationships are theoretically correct, using the scalar-based frequency estimator in its bare form as in (12) is unrealistic even in noise-free environments, since for some voltage samples, the value of the denominator $a(k)$ is close to zero, resulting in unstable and incorrect frequency estimates [22]. An even more critical problem appears when noise/harmonics contaminated voltage samples are considered, since it is very unlikely to obtain real-valued estimates $\cos(m\hat{\omega})$ using (11), which in turn renders frequency estimates $\hat{f}$ meaningless.

IV. Frequency Estimation Based on CLS

To solve the above problems encountered by current scalar frequency estimators (FE1, FE2, and FE3) and at the same time to enable meaningful frequency estimation with high immunity and insensitivity to noise and higher order harmonic pollution, we propose a CLS framework based on the time-series relationships among unbalanced system voltage samples with arbitrary phase interval, established in Section III.

Based on (11), we shall define the estimation error $e(k)$ as

$$e(k) = a(k)\cos(m\hat{\omega}) - b(k) \quad (13)$$

TABLE I
EXPRESSIONS OF A CLASS OF FREQUENCY ESTIMATORS BASED ON TIME-SERIES VOLTAGE SAMPLE RELATIONSHIPS

	a(k)	b(k)
FE1	$v(k-m)-v(k-2m)$	$\frac{1}{2}(v(k)-v(k-m)+v(k-2m)-v(k-3m))$
FE2	$v(k-m)-v(k-3m)$	$\frac{1}{2}(v(k)-v(k-4m))$
FE3	$v(k-m)$	$\frac{1}{2}(v(k)+v(k-2m))$

and its complex conjugate

$$e^*(k) = a^*(k)\cos(m\hat{\omega}) - b^*(k) \tag{14}$$

where $(\cdot)^*$ is the complex conjugation operator; we shall further make use of the fact that $\cos(m\hat{\omega})$ is real-valued, giving $\cos^*(m\hat{\omega}) = \cos(m\hat{\omega})$.

The CLS framework aims to find an optimal $\hat{\omega}$ which minimizes the total mean square error $J(\hat{\omega})$ over a number of available observations [43], [48]. For the illustration purpose, the time-series relationship among the four voltage samples used in FE1 is employed as the underlying estimation model, giving

$$\begin{aligned} J(\hat{\omega}) &= \sum_{k=3m}^{K-1} e(k)e^*(k) \\ &= \sum_{k=3m}^{K-1} \big(|a(k)|^2\cos^2(m\hat{\omega}) \\ &\qquad -2\Re(a(k)b^*(k))\cos(m\hat{\omega}) + |b(k)|^2\big) \end{aligned} \tag{15}$$

where $\Re(\cdot)$ is the real part operator.

To find the optimal value $\hat{\omega}$ which minimizes $J(\hat{\omega})$, we take the partial derivative $\frac{\partial J(\hat{\omega})}{\partial \hat{\omega}}$ as

$$\begin{aligned} \frac{\partial J(\hat{\omega})}{\partial \hat{\omega}} = -2m\ \sin(m\hat{\omega})\Bigg(&\cos(m\hat{\omega})\sum_{k=3m}^{K-1}|a(k)|^2 \\ &- \sum_{k=3m}^{K-1}\Re(a(k)b^*(k))\Bigg). \end{aligned} \tag{16}$$

Upon setting $\frac{\partial J(\hat{\omega})}{\partial \hat{\omega}} = 0$, a closed-form solution, referred to as the CLS1 approach, is obtained as

$$\cos(m\hat{\omega}) = \frac{\sum_{k=3m}^{K-1}\Re(a(k)b^*(k))}{\sum_{k=3m}^{K-1}|a(k)|^2} \tag{17}$$

which yields the frequency estimate in the form

$$\hat{\omega} = \frac{1}{m}\cos^{-1}\left(\frac{\sum_{k=3m}^{K-1}\Re(a(k)b^*(k))}{\sum_{k=3m}^{K-1}|a(k)|^2}\right) \text{ and } \hat{f} = \frac{f_s}{2\pi}\hat{\omega}. \tag{18}$$

Substituting (17) back into (15), the total minimum mean square error (MMSE) J_{Total} of the CLS1 solution is obtained as

$$J_{\text{Total}} = \sum_{k=3m}^{K-1}|b(k)|^2 - \frac{\left(\sum_{k=3m}^{K-1}\Re(a(k)b^*(k))\right)^2}{\sum_{k=3m}^{K-1}|a(k)|^2} \tag{19}$$

where $a(k) = v(k-m) - v(k-2m)$ and $b(k) = 1/2(v(k) - v(k-m) + v(k-2m) - v(k-3m))$.

In this way, as shown in (17) and (18), meaningful frequency estimates (real-valued) can be always guaranteed by the CLS1 frequency estimator; in addition, based on the least-squares framework, CLS1 provides more reliable frequency estimates than its scalar version, FE1. Note that the above CLS framework (15)–(22) is also applicable to the time-series relationships behind FE2 and FE3, underpinning the CLS2 and CLS3 frequency estimation algorithms, respectively. However, the main difference lies in the starting points of the observations within the summation operation: within CLS2, the time instant k starts from $4m$, whereas in CLS3, it starts from $2m$, giving meaningful time instants $(k-4m)$ and $(k-2m)$, respectively, as shown in Table I.

V. PERFORMANCE ANALYSIS OF CLS FREQUENCY ESTIMATION APPROACHES UNDER NOISE AND HARMONICS

Note that the time-series relationships among unbalanced voltage samples underpinning CLS-based algorithms hold for any arbitrary integer m. This gives us the opportunity optimize for the value of m, in order to minimize the estimation bias and variance statistically, even under noise/harmonics pollution conditions. To this end, we first adopt a reasonable assumption that when the number of observations K is large enough (for instance within the CLS1 approach), the total mean square error in (15) can be approximated as

$$J(\hat{\omega}) \approx \sum_{k=3m}^{K-1} E[e(k)e^*(k)] \tag{20}$$

where $E[\cdot]$ is the statistical expectation operator. In such a way, the estimates $\cos(m\hat{\omega})$ in (17) can be approximated as

$$\cos(m\hat{\omega}) \approx \frac{\sum_{k=3m}^{K-1}\Re\big(E[a(k)b^*(k)]\big)}{\sum_{k=3m}^{K-1}E[|a(k)|^2]} \tag{21}$$

and therefore

$$J_{\text{Total}} \approx \sum_{k=3m}^{K-1} E[|b(k)|^2] - \frac{\left(\sum_{k=3m}^{K-1} \Re(E[a(k)b^*(k)])\right)^2}{\sum_{k=3m}^{K-1} E[|a(k)|^2]}. \quad (22)$$

A. Effects of Noise on the Estimation Accuracy of CLS Approaches

To examine the effects of noise on the performance of the proposed CLS approaches, consider the case when the complex-valued Clarke's transformed system voltage $v(k)$ in (4) is contaminated by noise, so that

$$\tilde{v}(k) = Ae^{jk\omega_o} + Be^{-jk\omega_o} + n(k) = v(k) + n(k) \quad k = 0, 1, \ldots, K-1 \quad (23)$$

where $n = n_r + jn_i$ is assumed to be a complex-valued zero mean doubly white Gaussian noise $\sim \mathcal{N}(0, \sigma^2)$, $\sigma_r^2 = \sigma_i^2 = \sigma^2/2$, and[1] $n_r \perp n_i$. This assumption yields $E[n(k)n(k)] = 0$ and $E[n(k)n^*(k-m)] = 0$ for $m \neq 0$, which will simplify our statistical analysis.

By considering the independence between the voltage and the noise, that is, $E[v(k-m)n^*(k-n)] = 0$ for any n and m, and upon substituting (23) into the denominator on the right hand side (RHS) of (21), we obtain

$$\begin{aligned} E[|a(k)|^2] &= E\big[|\tilde{v}(k-m) - \tilde{v}(k-2m)|^2\big] \\ &= E\big[|v(k-m)|^2\big] + E\big[|v(k-2m)|^2\big] \\ &\quad -2\Re\big(E[v(k-2m)v^*(k-m)]\big) \\ &\quad +E\big[|n(k-m)|^2\big] + E\big[|n(k-2m)|^2\big]. \end{aligned}$$

Note that

$$\begin{aligned} E[|v(k-m)|^2] &= |A|^2 + |B|^2 + 2\Re\big(E\big[AB^*e^{j2(k-m)\omega_o}\big]\big) \\ E[|v(k-2m)|^2] &= |A|^2 + |B|^2 + 2\Re\big(E\big[AB^*e^{j2(k-2m)\omega_o}\big]\big). \end{aligned} \quad (24)$$

Using further realistic assumptions that a large enough number of observations K covers multiple voltage periods and the sampling frequency f_s is fixed to generate equidistantly spaced voltage samples [29], [30], we have $E[e^{jk\omega_o}] = 0$ for any k, and hence (24) becomes

$$E\big[|v(k-m)|^2\big] = E\big[|v(k-2m)|^2\big] = |A|^2 + |B|^2 \quad (25)$$

and also

$$\Re\big(E[v(k-2m)v^*(k-m)]\big) = \big(|A|^2 + |B|^2\big)\cos(m\omega_o). \quad (26)$$

Since $E[|n(k-m)|^2] = E[|n(k-2m)|^2] = \sigma^2$, the expression for $E[|a(k)|^2]$ becomes

$$E\big[|a(k)|^2\big] = 2\big(|A|^2 + |B|^2\big)\big(1 - \cos(m\omega_o)\big) + 2\sigma^2. \quad (27)$$

We shall now expand the term $\Re(E[a(k)b^*(k)])$ in the numerator of (21) as

$$\begin{aligned} \Re\big(E\big[a(k)b^*(k)\big]\big) &= \frac{1}{2}\Re\big(E\big[v(k-m)\big(v^*(k) - v^*(k-m) \\ &\quad + v^*(k-2m) - v^*(k-3m)\big)\big]\big) \\ &\quad - \frac{1}{2}\Re\big(E\big[|n(k-m)|^2\big]\big) \\ &\quad - \frac{1}{2}\Re\big(E\big[v(k-2m)\big(v^*(k) - v^*(k-m) \\ &\quad + v^*(k-2m) - v^*(k-3m)\big)\big]\big) \\ &\quad - \frac{1}{2}\Re\big(E\big[|n(k-2m)|^2\big]\big). \end{aligned} \quad (28)$$

Note that under the assumptions that K covers multiple signal periods and f_s is constant, we have

$$\begin{aligned} \Re\big(E[v(k-m)v^*(k)]\big) &= \Re\big(E\big[v(k-2m)v^*(k-m)\big]\big) \\ &= \Re\big(E\big[v(k-m)v^*(k-2m)\big]\big) \\ &= \Re\big(E\big[v(k-2m)v^*(k-3m)\big]\big) \\ &= (|A|^2 + |B|^2)\cos(m\omega_o) \end{aligned} \quad (29)$$

and

$$\begin{aligned} \Re(E[v(k-m)v^*(k-3m)]) &= \Re(E[v(k-2m)v^*(k)]) \\ &= (|A|^2 + |B|^2)\cos(2m\omega_o) \end{aligned} \quad (30)$$

so that (28) becomes

$$\begin{aligned} \Re\big(E[a(k)b^*(k)]\big) &= 2\big(|A|^2 + |B|^2\big)\cos(m\omega_o) \\ &\quad \times(1 - \cos(m\omega_o)) - \sigma^2. \end{aligned} \quad (31)$$

Therefore, by considering (27) and (31), the estimates $\cos(m\hat{\omega})$ obtained by CLS1 in (21), can be evaluated as

$$\begin{aligned} \cos(m\hat{\omega}) &= \frac{2\big(|A|^2 + |B|^2\big)(1 - \cos(m\omega_o))\cos(m\omega_o) - \sigma^2}{2\big((|A|^2 + |B|^2)(1 - \cos(m\omega_o)) + \sigma^2\big)} \\ &= \cos(m\omega_o) - \frac{1 + 2\cos(m\omega_o)}{2(1 - \cos(m\omega_o)) \cdot \text{SNR} + 2} \end{aligned} \quad (32)$$

where

$$\text{SNR} = \frac{E[|v(k)|^2]}{E[|n(k)|^2]} = \frac{|A|^2 + |B|^2}{\sigma^2} \quad (33)$$

is the signal-to-noise ratio (SNR).

Remark 1: Expression (32) implies that for a finite SNR, the estimated frequency $\hat{\omega}$ cannot coincide with the actual system frequency ω_o, i.e., $\cos(m\hat{\omega}) \neq \cos(m\omega_o)$, except for an optimal value of m for which $\cos(m\omega_o) = -1/2$. This condition implies that $m\hat{\omega} = 2\pi/3$ is the minimum optimal phase interval needed to achieve statistically unbiased estimates within CLS1 in the noisy environment.[2]

To analyze the minimum estimation variance that can be obtained by CLS1, consider the above assumption that a large enough observation number K covers multiple signal periods, and using the noise contaminated voltage samples $\tilde{v}(k)$ instead

[1]Note that the statistical independence between n_r and n_i and their equal powers can be achieved by assuming the three-phase channels contaminated by independent and identically distributed zero-mean Gaussian noises with variance $\sigma^2/2$ [47].

[2]In general, any $m\omega_o = 2n\pi + 2\pi/3$ or $m\omega_o = 2n\pi + 4\pi/3$ would be the optimal phase interval for the CLS1 algorithm to achieve unbiased estimates statistically. However, as expected, a larger m would result in an increased response time of the algorithm. Therefore, in this paper, we focus on the minimum optimal phase interval only.

of $v(k)$ in Table I, after a few algebraic manipulations, we can expand $E[b(k)b^*(k)]$ as

$$E[|b(k)|^2] = 2(|A|^2 + |B|^2)\cos^2(m\omega_o)(1 - \cos(m\omega_o)) + \sigma^2. \quad (34)$$

By combining (22), (27), (31), and (34) together, the MMSE of CLS1, defined as $J_{\text{MMSE}} = J_{\text{Total}}/(N - 3m)$, becomes

$$J^{\text{CLS1}}_{\text{MMSE}} = \frac{J_{\text{Total}}}{N - 3m} = \frac{\big(1 + \cos(m\omega_o) - 2\cos^3(m\omega_o)\big)\sigma^2 \cdot \text{SNR} + \frac{\sigma^2}{2}}{\big(1 - \cos(m\omega_o)\big) \cdot \text{SNR} + 1}. \quad (35)$$

Remark 2: The optimal phase interval $\cos(m\omega_o) = -1/2$ ($m\hat{\omega} = 2\pi/3$), which minimizes the estimation bias in (32), also in turn enables us to achieve the optimal estimation variance $J^{\text{CLS1}}_{\text{MMSE}} = \sigma^2/2$. In this sense, the optimal estimation variance produced by the CLS1 algorithm attains the *irreducible* minimum value at $\sigma^2/2$, independent on the voltage signal itself, and the proposed estimator is consistent and optimal.

Under the above assumption that the observation number K is large enough so that it covers multiple signal periods, the frequency estimates by CLS2 using the time-series relationship employed by FE2 shown in Table I, can be evaluated as

$$\cos(m\hat{\omega}) = \frac{\cos(m\omega_o)}{1 + \frac{1}{2\sin^2(m\omega_o)\cdot \text{SNR}}} \quad (36)$$

so that the associated MMSE is given by

$$J^{\text{CLS2}}_{\text{MMSE}} = \frac{\sigma^2}{2} + \frac{4\sin^2(m\omega_o)\cos^2(m\omega_o)}{2\sin^2(m\omega_o) \cdot \text{SNR} + 1}. \quad (37)$$

Similarly, for CLS3, we have

$$\cos(m\hat{\omega}) = \frac{\cos(m\omega_o)}{1 + \frac{1}{\text{SNR}}} \quad (38)$$

and

$$J^{\text{CLS3}}_{\text{MMSE}} = \frac{\sigma^2}{2} + \frac{\cos^2(m\omega_o)}{\text{SNR} + 1}. \quad (39)$$

Remark 3: It is now straightforward to show that the optimal phase interval, $m\omega_o = \pi/2$ ($\cos(m\omega_o) = 0$), enables both the CLS2 and CLS3 algorithms to achieve statistically unbiased frequency estimates in (36) and (38), together with the irreducible estimation variance $J^{\text{CLS2}}_{\text{MMSE}} = J^{\text{CLS3}}_{\text{MMSE}} = \sigma^2/2$ which is identical to that of CLS1. In other words, the CLS framework, when in its optimal setting, is independent on the underlying time-series relationships among equidistantly spaced voltage samples of unbalanced power systems.

B. Effects of Harmonics on the Estimation Accuracy of CLS Approaches

This section examines the effects of harmonic pollution on the frequency estimation performance of CLS-based algorithms. We also address the optimal phase intervals which minimize the estimation error introduced by harmonics. Consider the case when the Clarke's transformed system voltage under unbalanced conditions is contaminated by the nth harmonic, that is

$$v(k) = Ae^{jk\omega_o} + Be^{-jk\omega_o} + A_n e^{jkn\omega_o} + B_n e^{-jkn\omega_o} \quad (40)$$

where $A_n = |A_n|e^{j\phi_{A_n}}$, $B_n = |B_n|e^{j\phi_{B_n}}$, and $|A_n|$ and $|B_n|$ are, respectively, the magnitude of the positive sequence and the negative sequence of the nth harmonic, for which the phases are, respectively, ϕ_{A_n} and ϕ_{B_n}.

Under the earlier assumptions that a large K is a multiple of the signal period and f_s is a constant, we have

$$E[|v(k-2m)|^2] = E[|v(k-m)|^2] = |A|^2 + |B|^2 + |A_n|^2 + |B_n|^2 \quad (41)$$

$$E[v(k-2m)v^*(k-m)] = |A|^2 e^{-jm\omega_o} + |B|^2 e^{jm\omega_o} + |A_n|^2 e^{-jmn\omega_o} + |B_n|^2 e^{jmn\omega_o} \quad (42)$$

and hence

$$\Re\big(E[v(k-2m)v^*(k-m)]\big) = (|A|^2 + |B|^2)\cos(m\omega_o) + (|A_n|^2 + |B_n|^2)\cos(mn\omega_o). \quad (43)$$

Therefore, $E[|a(k)|^2]$ in (27) now becomes

$$E[|a(k)|^2] = 4(|A|^2 + |B|^2)\sin^2\Big(\frac{m\omega_o}{2}\Big) + 4(|A_n|^2 + |B_n|^2)\sin^2\Big(\frac{mn\omega_o}{2}\Big). \quad (44)$$

In a similar way, we have

$$\Re\big(E[a(k)b^*(k)]\big) = 4(|A|^2 + |B|^2)\sin^2\Big(\frac{m\omega_o}{2}\Big)\cos(m\omega_o) + 4(|A_n|^2 + |B_n|^2)\sin^2\Big(\frac{mn\omega_o}{2}\Big) \times \cos(mn\omega_o). \quad (45)$$

By defining the signal-to-harmonics ratio (SHR) as $\text{SHR} = (|A_n|^2 + |B_n|^2)/(|A|^2 + |B|^2)$, the frequency estimates $\cos(m\hat{\omega})$ by CLS1 under harmonic pollution can be evaluated as

$$\begin{aligned}\cos(m\hat{\omega}) &= \frac{\sum_{k=3m}^{K-1} \Re\big(E[a(k)b^*(k)]\big)}{\sum_{k=3m}^{K-1} E[|a(k)|^2]} \\ &= \frac{\sin^2\big(\frac{m\omega_o}{2}\big)\cos(m\omega_o) + \sin^2\big(\frac{mn\omega_o}{2}\big)\cos(mn\omega_o)\cdot \text{SHR}}{\sin^2\big(\frac{m\omega_o}{2}\big) + \sin^2\big(\frac{mn\omega_o}{2}\big)\cdot \text{SHR}} \\ &= \cos(m\omega_o) - \frac{\sin^2\big(\frac{mn\omega_o}{2}\big)(\cos(m\omega_o - \cos(mn\omega_o))\cdot \text{SHR}}{\sin^2\big(\frac{m\omega_o}{2}\big) + \sin^2\big(\frac{mn\omega_o}{2}\big)\cdot \text{SHR}}\end{aligned} \quad (46)$$

and from (46), with a finite value of SHR, the statistically unbiased frequency estimates under harmonic pollution, that is $\cos(m\hat{\omega}) = \cos(m\omega_o)$, can be achieved if either the condition $\sin^2(mn\omega_o/2) = 0$ or $\cos(m\omega_o) = \cos(mn\omega_o)$ is satisfied.

Remark 4: Recall from Section V-A that the optimal phase interval for the CLS1 algorithm under the noise environment is $m\omega_o = 2\pi/3$; this also becomes a solution to the first condition $\sin^2(mn\omega_o/2) = 0$, if n is an integer multiple of three. When n is not an integer multiple of three, i.e., $m = 3k + 1$

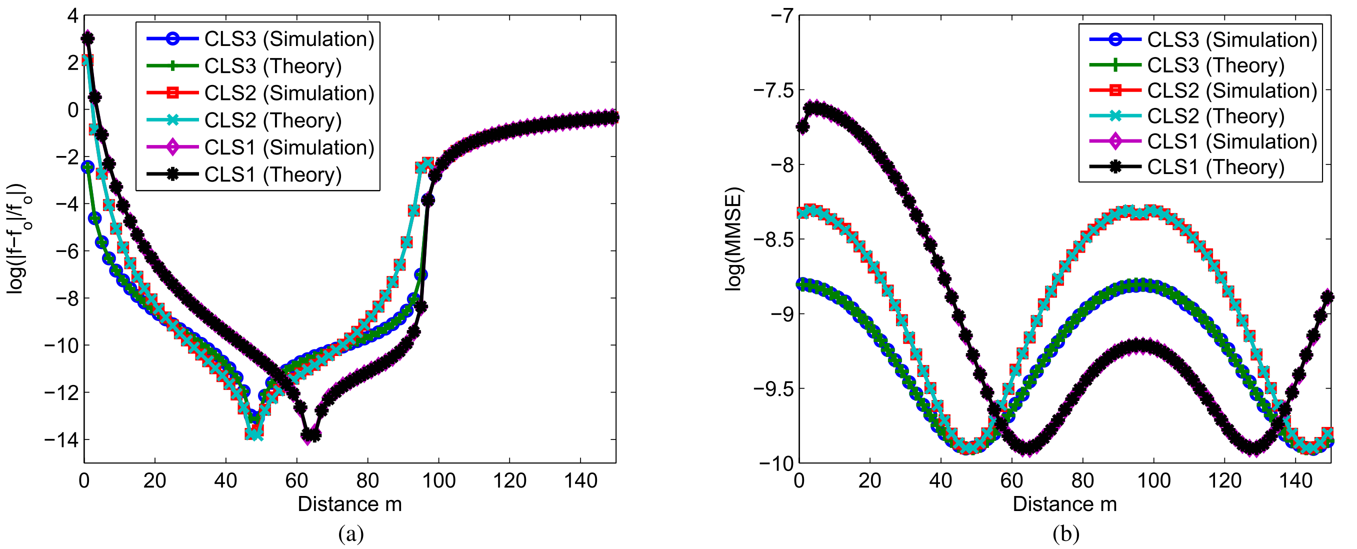


Fig. 1. Comparison of the estimation bias and variance between the numerical simulation results and the theory of the proposed CLS-based algorithms against the sample distance m, with $K = 100\,000$ in a noisy environment. (a) Estimation bias. (b) Estimation variance.

or $m = 3k+2$, with the same phase interval $m\omega_o = 2\pi/3$, we either have $\cos(mn\omega_o) = \cos(2\pi/3(3k+1)) = \cos(2\pi/3)$ or $\cos(mn\omega_o) = \cos(2\pi/3(3k+2)) = \cos(4\pi/3) = \cos(2\pi/3)$, satisfying the second condition that $\cos(m\omega_o) = \cos(mn\omega_o)$. In this way, the value $m\omega_o = 2\pi/3$ is the general solution to remove the effects of noise and harmonics simultaneously, and to achieve statistically unbiased frequency estimates for CLS1 algorithm.

In a similar way, frequency estimation by the CLS2 algorithm can be evaluated as

$$\cos(m\hat{\omega}) = \cos(m\omega_o) - \frac{\sin^2(mn\omega_o)(\cos(m\omega_o) - \cos(mn\omega_o)) \cdot \text{SHR}}{\sin^2(m\omega_o) + \sin^2(mn\omega_o) \cdot \text{SHR}}. \quad (47)$$

Remark 5: Consider the optimal phase interval $m\omega_o = \pi/2$ in noisy environments. Then, for even harmonics ($n = 2k$), we have $\sin(mn\omega_o) = \sin(k\pi) = 0$, whereas for odd harmonics ($n = 2k+1$), we have $\cos(mn\omega_o) = \cos(k\pi + \pi/2) = \cos(\pi/2) = \cos(n\omega_o)$. Either condition makes the second term on the RHS of (47) vanish, thus attaining statistically unbiased frequency estimates. Therefore, $m\omega_o = \pi/2$ is a general solution which simultaneously removes the effects of noise and harmonics in the CLS2 algorithm.

The frequency estimates by the CLS3 algorithm can also be evaluated in a similar way as

$$\cos(m\hat{\omega}) = \cos(m\omega_o) + \frac{(\cos(m\omega_o) - \cos(mn\omega_o)) \cdot \text{SHR}}{1 + \text{SHR}}. \quad (48)$$

Remark 6: Consider the optimal phase interval $m\omega_o = \pi/2$, in noisy environments. It can only remove the effects of odd harmonics, as for instance, for $n = 2k+1$, we have $\cos(mn\omega_o) = \cos(k\pi + \pi/2) = \cos(\pi/2) = \cos(n\omega_o)$, whereas for even harmonics ($n = 2k$), there is no unique solution to $m\omega_o$.

Remark 7: When the total number of voltage observations K is large enough and is equal to an integer multiple of the voltage period, the two four-point relationship-based CLS approaches, the CLS1 and CLS2, are more favorable in practice, since the same optimal phase interval can achieve statistically unbiased frequency estimation in both noisy and harmonically polluted environments, which otherwise cannot be obtained by their three-point relationship-based counterpart CLS3.

VI. Simulations

To verify the theoretical analysis developed in Sections II–V, numerical simulations were conducted in the MATLAB programming environment at a sampling frequency $f_s = 9.6$ kHz. The system frequency of the unbalanced three-phase systems, f_o, was fixed at $f_o = 50$ Hz, giving the normalized angular frequency $\omega_o = 2\pi f_o/f_s = \pi/96$.

To validate our bias and variance analysis of the proposed CLS approaches in noisy environments, we first generated the unbalanced Clarke's transformed system voltage using (23), where $|A| = 1$, $\phi_A = 0.2\pi$, $|B| = 0.2$, $\phi_B = -0.1\pi$, $\sigma = 0.01$, and the number of observations $K = 100\,000$. The frequency estimation performance of the proposed CLS algorithms obtained by both the numerical simulation and theoretical evaluation versus a range of sample distance m is shown in Fig. 1. As shown in Fig. 1(a), the numerically simulated estimation bias, represented in terms of the normalized absolute error, matched very well the theoretical bias analysis of all the algorithms [using (32), (36), and (38), respectively] for a large value of K. The estimation accuracy of all the considered algorithms was very poor when the arbitrary distance m was small, indicating that the conventional frequency estimators based on the relationships among consecutive voltage samples, i.e., $m = 1$, are very biased. In addition, the best frequency estimation by the CLS1 algorithm was achieved when $m = 64$, corresponding to its theoretically optimal phase interval $m\omega_o = 2\pi/3$, whereas the minimum frequency estimation bias of CLS2 and CLS3 was achieved both at $m = 48$ ($m\omega_o = \pi/2$), conforming with the analysis in Section V-A. As desired, the proposed CLS-based

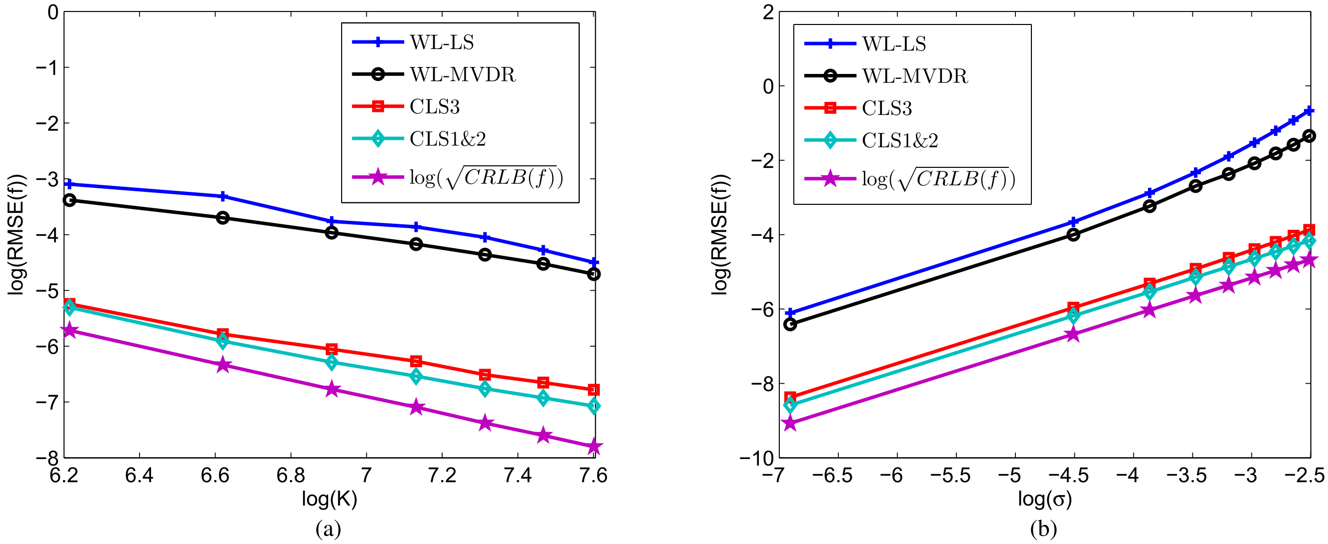


Fig. 2. Comparison of the estimation variance, in terms of RMSE, of the proposed CLS-based frequency estimation algorithms with their optimal phase intervals and two existing WL frequency estimation algorithms, that is, WL-LS and WL-MVDR, against the CRLB. The results were obtained by averaging $L = 50\,000$ trials. (a) K varies from 500 to 2000 with $\sigma = 0.01$. (b) σ varies from 0.001 to 0.1 with $K = 1000$.

algorithms with their own optimal phase intervals arrived at the optimal MMSE $\sigma^2/2$, as shown in Fig. 1(b), and the good agreement between the simulated and theoretical frequency estimation variance, in terms of MMSE, of all the algorithms against sample distance m was also observed.

To further investigate the estimation accuracy of the CLS-based algorithms in a noisy environment, we next performed the variance analysis by assessing the proposed algorithms against the theoretical Cramer–Rao lower bound (CRLB), which characterizes the variance of an unbiased optimal frequency estimator based on the unbalanced system voltage model in (23) and is derived in [47, eq. (57)].

The root mean square of the estimated frequency error RMSE(f) was employed to quantify the estimation variance of the proposed algorithms, defined as

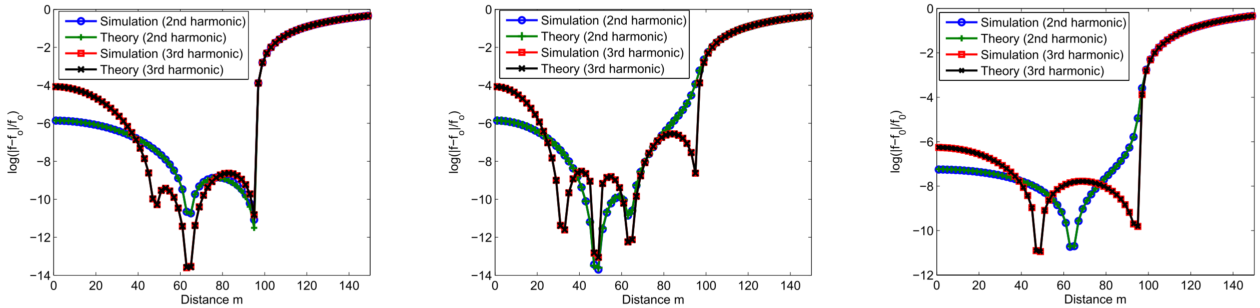
$$\text{RMSE}(f) = \sqrt{\frac{1}{L}\sum_{l=1}^{L}(\tilde{f}_l - f_o)^2} \qquad (49)$$

where L is the number of simulation trails for the Gaussian noise generation and $\tilde{f}_l$ is the estimated frequency from the lth simulation.

Fig. 2(a) and (b) shows the frequency estimation variance of all the proposed CLS algorithms, evaluated for their optimal phase intervals, versus different number of observations K and noise deviations σ, respectively. Note that in both cases, the assumption that the observation number K is large enough broke down, resulting in a performance advantage of the four-point relationship-based CLS1 and CLS2 algorithms over their three-point relationship-based counterpart as shown in Fig. 2(a). This advantage may result from the fact that within the CLS framework in (17), both four-point relationship-based CLS1 and CLS2 use the difference between voltage samples in the denominator instead of a single voltage sample in CLS3. As a comparison, the statistical performances of the convention LS and a recently developed MVDR spectrum-based frequency estimation algorithms [5], [23], are also shown in Fig. 2(a) and (b). In order to achieve unbiased frequency estimates for unbalanced power systems, the underlying observation model of both algorithms were modified by employing the WL estimation model; we refer to [44] and [47] for more detail. The consistent performance advantages of the proposed CLS approaches over two other existing WL frequency estimators can be observed in both cases.

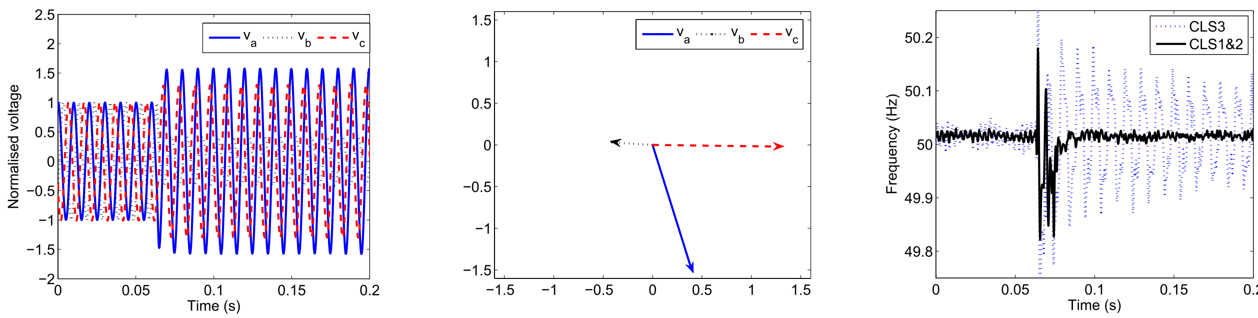
We next validated the theoretical bias analysis of the proposed CLS approaches for frequency estimation of unbalanced three-phase power systems under a harmonic pollution. The unbalanced Clarke's transformed system voltage with harmonics was generated using (40), where $|A| = 1$, $|B| = 0.2$, $|A_n| = 0.02$, and $|B_n| = 0.01$ with the associated phase set to be $\phi_A = 0.2\pi$, $\phi_B = -0.1\pi$, $\phi_{A_n} = 0.1\pi$, and $\phi_{B_n} = -0.2\pi$, respectively, and the number of observations $K = 100\,000$. As shown in Fig. 3(a) and (b), there is a good agreement between the simulated and theoretical frequency estimation bias [evaluated by using (46) and (47)] of CLS1 and CLS2 over a range of sample distances m for unbalanced power systems under the second and third harmonics, respectively. The simulation results also conform with our theoretical bias analysis in Section V-B, that with a large enough number of voltage observations K the optimal phase interval of two four-point relationship-based CLS1 and CLS2 approaches under the noise contamination gives minimum estimation bias under both even and odd harmonic pollution. Note that this desirable property cannot be obtained by their three-point relationship-based counterpart CLS3, since the optimal phase interval in both noisy environments and odd harmonic pollution, that is $m\omega_o = \pi/2$ ($m = 48$), is not the optimal solution in the even harmonic polluted cases, as shown in Fig. 3(c).

In the last set of simulations, a real-world power system was considered. The three-phase voltage was recorded at a 110/20/10 kV transformer station. The REL 531 numerical line distant protection terminal, produced by ABB Ltd., was installed in the station and was used to monitor changes



(a) (b) (c)

Fig. 3. Comparison of the estimation bias between the numerical simulation results and the theory of the proposed CLS-based frequency estimation algorithms under the second and third harmonic pollution against the sample distance m with $K = 100\,000$. (a) CLS1. (b) CLS2. (c) CLS3.



(a) (b) (c)

Fig. 4. Frequency estimation of the proposed CLS-based algorithms for a real-world unbalanced three-phase power system. (a) Waveform of the three-phase power system. (b) Phasor representation of the unbalanced three-phase voltage. (c) Frequency estimation results by all the algorithms with a sliding window of $K = 200$ voltage observations.

in the three phase-ground voltages. The measured three phase-ground voltages with a system frequency ~50 Hz were normalized with respect to the normal peak voltage values, as shown in Fig. 4(a). Initially, the three-phase power system was in a balanced condition, however, at ~0.07 s, phase v_b experienced an earth fault, causing a 58.4% voltage drop, and 57.9% and 30.5% voltage swells in phases v_a and v_c, respectively. The phasor representation of the unbalanced three-phase voltage was calculated by using the DFT technique as proposed in [4], and is shown in Fig. 4(b). Besides of the unbalanced voltage magnitudes, the phase difference between v_a and v_b and between v_b and v_c were 109.1° and 74.4°, and both deviated from the nominal 120°. The frequency tracking capabilities for the proposed CLS frequency estimators are shown in Fig. 4(c). All the approaches provided comparable and accurate responses under balanced operating conditions; however, after a short disturbance, the four-point relationship-based CLS1 and CLS2 provided more robust frequency estimates as compared with their three-point relationship-based CLS3 for the unbalanced three-phase voltage against measurement noises.

VII. Conclusion

A class of LS-based fundamental frequency estimation algorithms for unbalanced three-phase power systems have been proposed by extending three existing time-series relationships among voltage samples into the complex domain, so as to cater for the Clarke's transformed complex-valued system voltage. The LS framework has been shown to enable meaningful and accurate frequency estimation under both noise and harmonic contamination. The optimal phase interval of the proposed algorithms has been obtained via theoretical bias and variance analysis in order to achieve high immunity to both noise and harmonics, enabling performance advantages over other existing WL estimation-based unbiased frequency estimators for unbalanced power systems. We have also shown that the two four-point relationship-based CLS1 and CLS2 algorithms are more favorable in practice since a single optimal phase interval can, respectively, achieve minimum frequency estimation bias in both noise and harmonic pollution environments. However, this desirable property cannot be obtained using a three-point relationship-based counterpart CLS3, as shown in both theoretical analysis and simulations.

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