A Balancing Voltage Transformation for Robust Frequency Estimation in Unbalanced Power Systems

Yili Xia*, Kai Wang*, Wenjiang Pei* and Danilo P. Mandic†
*School of Information Science and Engineering, Southeast University, 2 Sipailou, Nanjing 210096, P. R. China.
E-mail: yili.xia06@gmail.com, kaiwang@seu.edu.cn, wjpei@seu.edu.cn
†Department of Electrical and Electronic Engineering, Imperial College London, SW7 2AZ London, UK.
E-mail: d.mandic@imperial.ac.uk

***Abstract*—This paper addresses the detection of the fundamental frequency of power systems under unbalanced and distorted conditions. By using the second order information, both the autocorrelation and pseudo-autocorrelation, within the Clarke's transformed voltage, a novel balancing voltage transformation (BVT) is proposed to accurately detect the underlying phase angle evolution of the positive sequence component. This removes the biggest obstacle in current power systems and makes possible to use any frequency estimator for single-tone exponential on unbalanced power systems. The robustness of the proposed phase angle detection technique is illustrated for two well-known and efficient frequency estimators, that is, a discrete Fourier transform (DFT) coefficient interpolation method [1] and the weighted linear predictor (WLP) [2]. A window technique is used to cater for the fast and computationally affordable frequency estimation purposes. Simulations over a range of unbalanced conditions, including voltage dips and swells, frequency deviations and the presence of higher order harmonics support the analysis.**

I. Introduction

Frequency is an important power quality parameter which is only allowed to vary within a small predefined range, a consequence of the dynamic unbalance between the generation and the load [3]. Its accurate estimation is essential, as maintaining the nominal frequency value is a prerequisite for both the stability of the grid and for normal operation of electrical devices.

A variety of architectures and algorithms have been proposed for this purpose, including phase-locked loops (PLL) [4], least squares techniques [2], [5-7], Kalman filters [8], adaptive notch filters [9] and recursive DFT methods [1], [10], [11]. In three-phase systems, standard frequency estimators employ Clarke's transformation [8], a framework that simultaneously considers all the available information among the three-phase reference channels, to produce the complex-valued system voltage, from which the instantaneous phase information can be directly extracted. Most frequency estimation algorithms based on Clarke's transformation work well under normal, balanced, power system conditions. However, most suffer from performance degradation under unbalanced voltage conditions. These occur when e.g. different amplitudes within the three-phase voltage or phase deviations from their nominal position are taking place, resulting in the so-called 'negative sequence' and an unavoidable phase angle estimation error, represented by oscillations at double the system frequency.

Solutions to cancel out the effect of the negative sequence are typically based on signal preprocessing, using such as symmetrical component analysis and advanced weighted least-square estimation to extract the positive sequence, so that standard phase angle detection designed for balanced power systems can be directly applied to eliminate this [4], [6]. A recent preprocessing scheme exploits the "augmented" statistical nature of the complex-valued system voltage under unbalanced power systems to extend frequency estimation algorithms to more general forms in order to cater for the system imbalance [12-14]. It has been illustrated that under unbalanced conditions, the Clarke's transformed voltage is second order noncircular (improper), for which the probability density function is not rotation invariant [15]. For a noncircular complex-valued zero mean random variable $v = v_r + \jmath v_i$, the powers in the real and imaginary parts are different, and its autocorrelation $r = E[vv^*] = \sigma_{v_r}^2 + \sigma_{v_i}^2$ does not have sufficient number of degrees of freedom for full statistical description, so that the pseudo-autocorrelation $p = E[vv] = \sigma_{v_r}^2 - \sigma_{v_i}^2 + 2\jmath E[v_r v_i]$ should also be taken into account for complete second-order modelling [16-18].

Based on such 'augmented' second-order statistics, in this work, a unified framework to extract the instantaneous phase angle information of the positive sequence component within the noncircular system voltage is proposed. This is achieved by using a novel balancing voltage transformation (BVT) which considers all the available second order information. The proposed transformation maps the noncircular system voltage onto a circular positive sequence component without loss of information, where the phase angle information is detected directly, hence facilitating the implementation of the existing frequency estimators for a single tone exponential. To illustrate the robustness of the proposed BVT technique, two well-known and efficient frequency estimators, that is, the weighted linear predictor (WLP) [2] and a coefficient interpolated discrete Fourier transform (DFT) method [1], are been implemented using a sliding window, for on-line frequency estimation based on the transformed voltage. Simulations over a range of unbalanced conditions support the analysis.

This work was partially supported by the National Natural Science Foundation of China (Grant No. 61201173, 61271058, 61401094), the Specialized Research Fund for the Doctoral Program of Higher Education of China (Grant No. 20110092110008), the Natural Science Foundation of Jiangsu Province (Grant No. SBK201140040, BK2011060, BK20140645).

II. Unbalanced Three-Phase Power Systems

The three-phase voltages of a power system in a noise-free environment can be represented in a discrete time form as

$$\begin{aligned} v_a(k) &= V_a\cos(2\pi f k\triangle\mathrm{T}+\phi)\\ v_b(k) &= V_b\cos(2\pi f k\triangle\mathrm{T}+\phi-\frac{2\pi}{3})\\ v_c(k) &= V_c\cos(2\pi f k\triangle\mathrm{T}+\phi+\frac{2\pi}{3}) \end{aligned} \tag{1}$$

where V_a, V_b, V_c are the peak values of each fundamental voltage component at time instant k, $\triangle\mathrm{T} = \frac{1}{f_s}$ is the sampling interval, f_s is the sampling frequency, ϕ is the initial phase, and f is the system frequency. In the analysis of power systems, three-phase voltages are typically transformed into a zero-sequence v_0 and the direct and quadrature-axis components, v_α and v_β, by means of the orthogonal ($\alpha\beta0$ or Clarke's) transformation matrix, given by [8]

$$\begin{bmatrix} v_0(k)\\ v_\alpha(k)\\ v_\beta(k)\end{bmatrix} = \sqrt{\frac{2}{3}}\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\\ 1 & -\frac{1}{2} & -\frac{1}{2}\\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}\end{bmatrix}\begin{bmatrix} v_a(k)\\ v_b(k)\\ v_c(k)\end{bmatrix} \tag{2}$$

The factor $\sqrt{2/3}$ is used to ensure that the system power is invariant under this transformation. In balanced system conditions, V_a, V_b, V_c are identical, giving $v_0(k) = 0$, $v_\alpha(k) = A\cos(2\pi f k\Delta\mathrm{T}+\phi)$ and $v_\beta(k) = A\sin(2\pi f k\Delta\mathrm{T}+\phi)$. Under the transform in (2), it can be shown that the amplitude $A = \frac{\sqrt{6}(V_a+V_b+V_c)}{6}$ is constant, while $v_\alpha(k)$ and $v_\beta(k)$ represent the orthogonal coordinates of a point whose position is time variant at a rate proportional to the system frequency. Practical applications typically employ only v_α and v_β parts in the form of the complex voltage $v(k)$, given by

$$v(k) = v_\alpha(k) + \jmath v_\beta(k) = Ae^{j(\omega k+\phi)} \tag{3}$$

where $\omega = 2\pi f\Delta\mathrm{T}$ is the normalised angular frequency. Notice that the amplitude-phase distribution of $v(k)$ on the complex plane is rotation invariant, since for any real θ both v and $ve^{j\theta}$ rotate around a circle with radius A. In terms of augmented complex statistics, this means that $v(k)$ is second order circular (proper) [15], for which the powers in the v_α and v_β are equal, and standard autocorrelation matrix C has enough degrees of freedom to fully describe the second order statistics, while the pseudo-autocorrelation matrix $\mathrm{P} = E[\mathbf{v}\mathbf{v}^T] = \mathbf{0}$ vanishes [16-18]. When the three-phase power system operates in abnormal conditions, for example, under different levels of dips or transients (typically $V_a \neq V_b \neq V_c$), then the complex Clarke's voltage becomes [12-14]

$$v(k) = Ae^{j\omega k} + Be^{-j\omega k} \tag{4}$$

where

$$\begin{aligned} A &= \frac{\sqrt{6}(V_a+V_b+V_c)}{6}e^{j\phi}\\ B &= \left(\frac{\sqrt{6}(2V_a-V_b-V_c)}{12} - \jmath\frac{\sqrt{2}(V_b-V_c)}{4}\right)e^{-j\phi} \end{aligned} \tag{5}$$

Note that this expression is theoretically accurate for both the balanced and unbalanced system conditions. For balanced system condition, $V_a = V_b = V_c$ and $B = 0$, whereas for unbalanced conditions, $B \neq 0$, and the so introduced negative sequence $Be^{-j\omega k}$ causes the samples of $V(k)$ generated from (5) to deviate from the circle with constant radius, making the distribution of $v(k)$ rotation dependent (noncircular) [12-14]. This, in turn, results in the failure of standard phase angle calculation algorithms designed for balanced power systems, represented by the unavoidable estimation bias and oscillations at double the system frequency [6], [7], [12-14].

III. The Proposed Balancing Voltage Transform (BVT) for Unbalanced Power Systems

To cancel out the negative sequence component within (4) which biases the frequency estimation of standard strictly linear algorithms, for unbalanced systems, our aim is to find a suitable data transformation for $v(k)$. To this end, we consider the following balancing voltage transform (BVT)

$$\begin{aligned} \tilde{v}(k) &= v(k) - av^*(k)\\ &= v(k) - \frac{B}{A^*}v^*(k)\\ &= Ae^{j\omega k} + Be^{-j\omega k} - \frac{B}{A^*}\left(A^*e^{-j\omega k} + B^*e^{j\omega k}\right)\\ &= \left(A - \frac{|B|^2}{A^*}\right)e^{j\omega k}\\ &= (1-|a|^2)Ae^{j\omega k} \end{aligned} \tag{6}$$

where the ratio $a = \frac{B}{A^*}$ indicates the degree of system imbalance. In this way, for both the balanced and unbalanced system conditions, the transformed voltage $\tilde{v}(k)$ contains only the positive sequence component $e^{j\omega k}$ with a modified voltage magnitude; the transformed system voltage has therefore a rotation invariant amplitude-phase distribution, and is hence second order circular (proper) [15].

Indeed, for balanced conditions, the term $|a| = 0$, and we have a standard model, whereas for unbalanced conditions, a comparison of (4) and (6) shows the circular nature of (6). The problem then boils down to the estimation on the system imbalance ratio a. This is not feasible based solely on $v(k)$ or its complex conjugate $v^*(k)$, and we need to resort to the full autocorrelation structure of $v(k)$, where e.g. based on (4) the autocorrelation coefficient with lag m, is given by

$$\begin{aligned} r(m) &= E[v(k)v^*(k-m)]\\ &= E[|A|^2e^{j\omega m}] + E[|B|^2e^{-j\omega m}]\\ &\quad + E[AB^*e^{j\omega(2k-m)}] + E[A^*Be^{-j\omega(2k-m)}]\\ &\approx |A|^2e^{j\omega m} + |B|^2e^{-j\omega m} \end{aligned} \tag{7}$$

In a similar way, the pseudo-autocorrelation coefficient $p(m)$ of $v(k)$ at lag m can be expanded as

$$\begin{aligned} p(m) &= E[v(k)v(k-m)]\\ &= E[A^2e^{j\omega(2k-m)}] + E[B^2e^{-j\omega(2k-m)}]\\ &\quad + E[ABe^{j\omega m}] + E[ABe^{-j\omega m}]\\ &\approx ABe^{j\omega m} + ABe^{-j\omega m} \end{aligned} \tag{8}$$

Combining (7) and (8), we arrive at

$$\begin{aligned} r(m) - a^* p(m) &= |A|^2 e^{j\omega m} + |B|^2 e^{-j\omega m} \\ &\quad - \frac{B^*}{A}\left(ABe^{j\omega m} + ABe^{-j\omega m}\right) \\ &= (|A|^2 - |B|^2) e^{j\omega m} \end{aligned} \tag{9}$$

and similarly,

$$r(m) - \frac{p(m)}{a} = (|B|^2 - |A|^2) e^{-j\omega m} \tag{10}$$

Taking the complex conjugate of both sides of (10), we obtain

$$r^*(m) - \frac{p^*(m)}{a^*} = (|B|^2 - |A|^2) e^{j\omega m} \tag{11}$$

Suppose that for a specific time lag m, $r(m)$ and $p(m)$ can be estimated from sufficiently long observations of $v(k)$. In (9) and (11), there are two unknown terms, $e^{j\omega m}$ and a, therefore, it is possible to construct a quadratic polynomial of a, in the form

$$p^*(m)a^2 - \big(r(m) + r^*(m)\big)a + p(m) = 0 \tag{12}$$

for which the two roots are given by

$$a = \frac{r(m) + r^*(m) \pm \sqrt{\big(r(m) + r^*(m)\big)^2 - 4|p(m)|^2}}{2p^*(m)} \tag{13}$$

Since, from (5), $|A| > |B|$, and thus $|a| < 1$, we finally arrive at the solution for the degree of system imbalance in the form

$$a = \frac{r(m) + r^*(m) - \sqrt{\big(r(m) + r^*(m)\big)^2 - 4|p(m)|^2}}{2p^*(m)} \tag{14}$$

Substituting (14) into (6), we obtain the transformed system voltage $\tilde{v}(k)$, which admits application of any frequency estimator designed for single tone exponential.

IV. Fast and Accurate Frequency Estimators for Power Systems

It is well known that the maximum likelihood (ML) estimator of the frequency under the model in (6) is given by the argument of the periodogram maximiser, that is

$$\hat{f}_{\text{ML}} = \arg\max_f \left(\Big| \sum_{k=0}^{N-1} \tilde{v}(k) e^{-j2\pi f k} \Big|^2 \right) \tag{15}$$

However, the numerical maximisation of (15) is not a computationally simple task and may suffer from convergence and resolution problems. A variety of architectures and algorithms have been proposed to cater for fast and accurate unbiased frequency estimation, required by current power grids. However, this means that statistical accuracy has to be traded for computational efficiency. A review of recent advances in the design of rapid and computationally affordable frequency estimators can be found in [19]. We here verify the validity of the proposed voltage transformation on two classic block based frequency estimators: the discrete Fourier transformation (DFT) coefficient interpolation method [1] and the weighted linear predictor (WLP) [2].

True ML estimation of the frequency of a single complex sinusoid requires locating the peak in a periodogram. In [20], Palmer suggested using the DFT for locating this maximum, resulting in a coarse frequency estimator. Subsequently, Rife and Boorstyn [21] refined Palmer's approach with an iterative search algorithm to perform a fine search. Motivated by their two-step structure, different reduced-complexity periodogram-based estimators have been proposed. Typically, these estimators first use N-point DFT to perform a coarse search for the periodogram peak, that is, to find the sample index p_N with the largest magnitude within the N bins. With the assumption that SNR is relatively high, it is very likely that the true frequency lies in the interval $[p_N - 0.5, p_N + 0.5]$. Then, a fine search for the true frequency is performed within this interval by using different coefficient interpolation procedures [1], [22]. We here use the approach proposed by Aboutanios and Mulgrew [1], which employs a simple but efficient iterative search strategy, and can be summarised as

$$\begin{aligned} \tilde{V}(n) &= \sum_{k=0}^{N-1} \tilde{v}(k) e^{-\frac{j2\pi kn}{N}} \\ p_N &= \arg\max_n (\tilde{V}(n)) \\ \tilde{V}_{p_N \pm 0.5} &= \sum_{k=0}^{N-1} \tilde{v}(k) e^{-j2\pi k \frac{p_N \pm 0.5 + \delta}{N}} \\ \delta &= \delta + \frac{1}{2} \frac{|\tilde{V}_{p_N+0.5}| - |\tilde{V}_{p_N-0.5}|}{|\tilde{V}_{p_N+0.5}| + |\tilde{V}_{p_N-0.5}|} \\ \hat{f} &= \frac{p_N + \delta}{N} f_s \end{aligned} \tag{16}$$

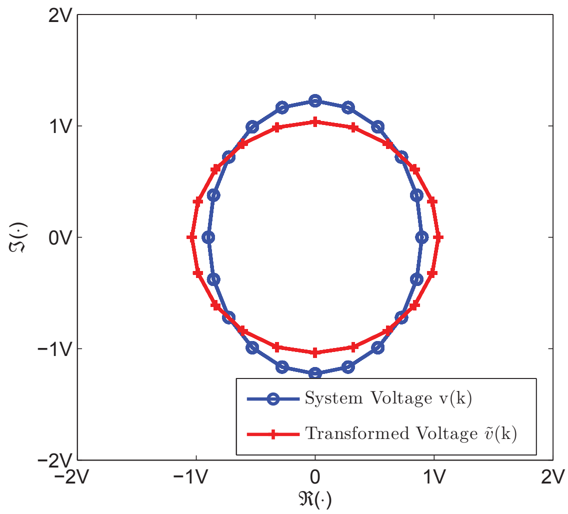
In [2], Kay developed a weighted linear predictor (WLP) from a different statistical model, where the additive noise is approximated by coloured Gaussian phase noise. This approximation is valid at a high SNR, where the phase measurements are samples in a weighted linear regression model. For a linear model with coloured noise, weighted least squares estimation is equivalent to ML estimation, and closed-form expression for the estimated frequency can be summarised as

$$\begin{aligned} w(k) &= \frac{3}{2}\frac{N}{N^2-1}\left(1 - 4\left(\frac{k - (\frac{N}{2} - 1)}{N}\right)^2\right) \\ \hat{\omega} &= \sum_{k=0}^{N-2} w(k) \angle\big(\tilde{v}^*(k)\tilde{v}(k+1)\big) \\ \hat{f} &= \frac{f_s}{2\pi}\hat{\omega} \end{aligned} \tag{17}$$

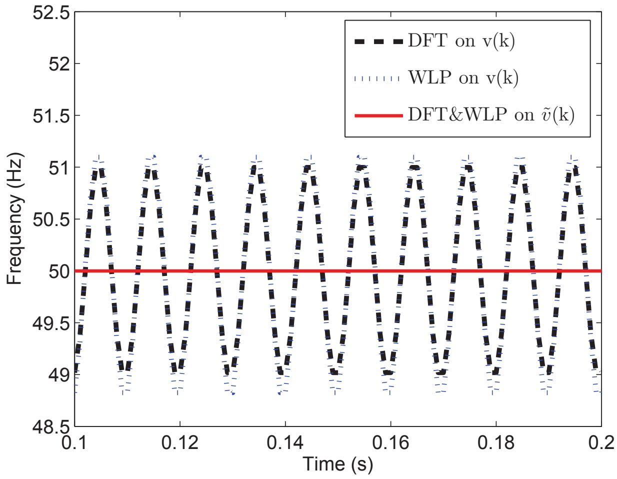
Therefore, as long as the proposed voltage transformation can detect precisely the phase angle of the positive sequence component, the high estimation accuracy of the frequency estimators considered here can be maintained.

V. Simulations

To illustrate the suitability of the proposed BVT technique for the generality of system conditions, performance analysis of the considered WLP and DFT based frequency estimators



(a)



(b)

Fig. 1. Frequency estimation under the unbalanced power system condition with a single-phase fault on phase v_a. (a) Geometric view of the distribution of the system voltage $v(k)$ and its transformation $\tilde{v}(k)$ via 'real-imaginary' scatter plots. The horizontal axis represents the real part of the system voltages, whereas the vertical axis represents their imaginary part. (b) Frequency estimation results of WLP and DFT methods on $v(k)$ and $\tilde{v}(k)$, with sliding window length of $N = 20$.

was investigated for various power system unbalanced conditions. Simulations were conducted in the Matlab programming environment at a sampling frequency of 1000 Hz, while the nominal power system frequency was 50 Hz. To cater for on-line frequency estimation, a sliding window was used for both the frequency estimation methods and for the calculation of the autocorrelation and pseudo-autocorrelation coefficients $r(m)$ and $p(m)$ in (7) and (8); the window was shifted by one step forward as the time evolved. Ten samples were used to update the estimates of both the second order statistics, $r(m)$ and $p(m)$, and also to update the system imbalance coefficient a within the proposed voltage transformation. In principle, the time lag m can be any integer, however, for practical purpose, the lag $m = 0$ should be ruled out, as

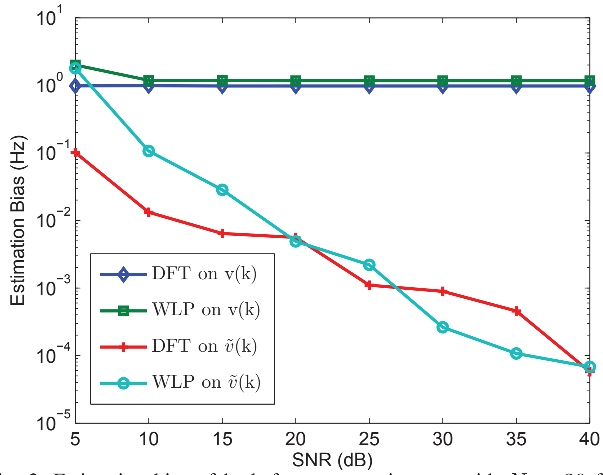
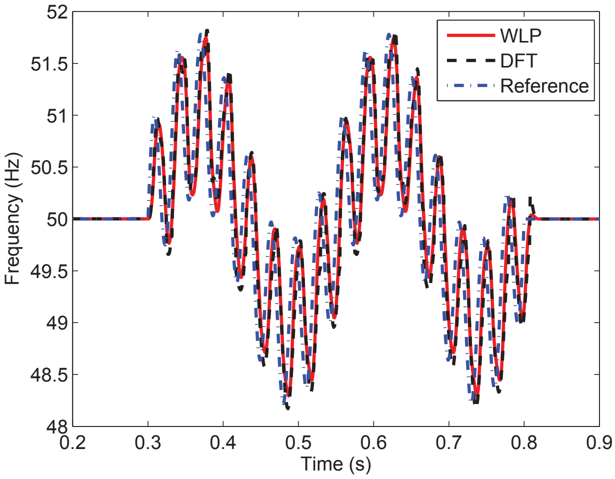


Fig. 2. Estimation bias of both frequency estimators with $N = 20$ for the original system voltage $v(k)$ and its transformation $\tilde{v}(k)$ at different SNRs, obtained by averaging 10000 independent trials.

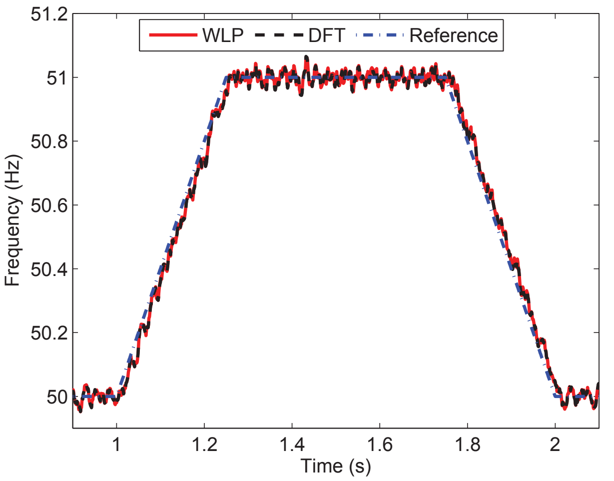
the estimate $r(0) = \sigma_v^2 + \sigma_n^2$ can be heavily deteriorated by the noise. Without loss of generality, we set $m = 1$ in the simulations, while the window size N varied in different scenarios. In principle, a large N improves the performance of the considered frequency estimators, reducing the estimation bias and variance, however, this also delays the response to the system frequency variation. To quantify the degree of system unbalance in different conditions, the noncircularity index $\eta = |\tau_v^2|/\sigma_v^2$ was used, where $\tau_v^2 = E[v(k)v(k)]$ is the pseudo-correlation of $v(k)$ [17]. The values of η lie in the interval [0, 1), with the value of $\eta = 0$ indicating that $v(k)$ is perfectly circular, otherwise indicating a second order noncircular $v(k)$, containing a negative sequence component.

In the first set of simulations, the simulated power system experienced a single-phase fault, where a 40% voltage dip occurred in phase v_a. The noncircularity index of the complex-valued system voltage $v(k)$ obtained by the $\alpha\beta$ transformation of the three-phase voltage was $\eta = 0.11$. A geometric view of its noncircular distribution via the 'real-imaginary' scatter plot is shown in Fig. 1(a). This, in turn, indicates that the negative sequence component $B(k)$ is not negligible, and therefore, the proposed voltage transformation should be performed on $v(k)$ before applying the standard frequency estimators. The distribution of the transformed system voltage $\tilde{v}(k)$ is shown in Fig. 1(a), where its circular nature (single-tone exponential) can be clearly observed, thus facilitating unbiased and minimum variance estimation even for the standard frequency estimators, as shown in Fig. 1(b), otherwise, a direct implementation of standard frequency estimators would result in unavoidable estimation oscillations.

In the next stage, the robustness of the proposed voltage transformation in noisy environments was investigated based on the analysis of estimation bias of the considered frequency estimators. As shown in Fig. 2, in the high SNR region, the unbiased property of both frequency estimators for unbalanced power systems was maintained when using the proposed



(a)



(b)

Fig. 3. Frequency tracking ability of both frequency estimators under frequency variation, based on the transformed system voltage $\tilde{v}(k)$ with $N = 20$. The power system was unbalanced and experienced a two-phase fault. (a) The system frequency experienced a frequency modulation from 0.3 s to 0.8 s. (b) The system frequency experienced frequency rise and decay, the measurements were obtained for SNR = 50 dB.

voltage transformation, whereas a direct implementation of both frequency estimators to the noncircular system voltage $v(k)$ resulted in unavoidable large bias. It is worth mentioning that although the computational complexity of the DFT based method (16) is higher as compared to the WLP method (17), the DFT based interpolation method in (16) is more robust in the presence of heavy noise.

In the next experiment, frequency tracking ability of both methods was studied. The previously considered unbalanced power system experienced a combined sinusoidal variations from 0.3s to 0.8 s with $f(t) = 50 + \sin(8\pi(t - 0.3)) + 0.8\sin(64\pi(t - 0.3))$. As shown in Fig. 3(a), based on the transformed system voltage $\tilde{v}(k)$, both methods were able to track the system frequency dynamics rapidly, and the response time required by both methods was approximately 0.02 s,

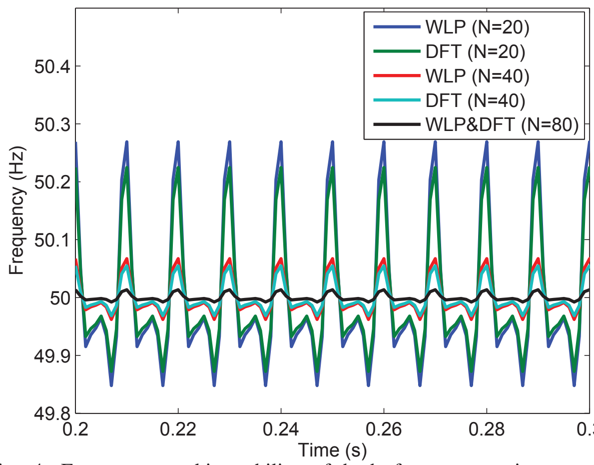
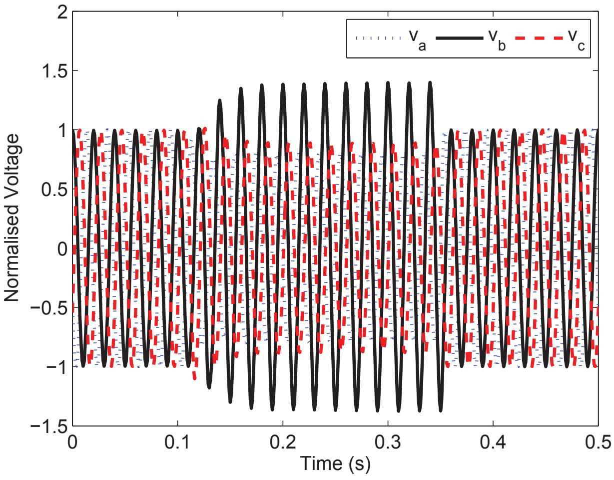


Fig. 4. Frequency tracking ability of both frequency estimators for the transformed system voltage $\tilde{v}(k)$ and for different sliding window lengths. The unbalanced power system experienced a two-phase fault and was contaminated with higher order harmonics at 10% for the 3rd, 5% for the 5th and 1% for the 7th harmonics.

that is, the sliding window length. The DFT based method tracked frequency dynamics more accurately. In Fig. 3(b), at t = 1 s, the unbalanced power system was contaminated by noise to give SNR = 50 dB, meanwhile, the system frequency underwent a rise at a rate of 4 Hz/s, staying at 51 Hz for 0.5 s, followed by a 4 Hz/s decay from 1.8 s to 2 s. In all cases, based on the transformed system voltage, both estimators tracked the frequency variations fast and accurately.

In the next set of simulations, to access the impact of higher order harmonics on the performance of the proposed voltage transformation, an unbalanced system was further distorted by 10% of 3rd, 5% of 5th and 1% of 7th harmonic of the fundamental frequency, to give a noncircularity index $\eta = 0.1318$. The estimation results of both frequency estimators with different sliding window lengths N are shown in Fig. 4. Due to the breakdown of the single tone voltage modelling, both estimators exhibited unavoidable estimation bias and oscillations for this highly distorted system voltage. A relatively long sliding window of $N = 80$, lasting for 0.08 s, can effectively attenuate the impact of higher order harmonics with the maximum estimation error of < 0.02 Hz.

In the last set of simulations, the robustness of the proposed voltage balancing transformation was investigated in the context of a real-world power system. The three-phase voltage was recorded at a 110/20/10 kV transformer station. The REL 531 numerical line distant protection terminal, produced by ABB Ltd., was installed in the station and was used to monitor changes in the three phase ground voltages. The measured three phase-ground voltages with a system frequency around 50 Hz were sampled at 1 kHz and were normalised with respect to their normal peak voltage values. As shown in Fig. 5(a), at around t = 0.12 s, the system experienced a voltage unbalance problem, where the voltage of phase v_b swelled to 139.8% of its normal value and meanwhile, both phases v_a and v_c experienced 19.7% and 9.7% voltage drops respectively.



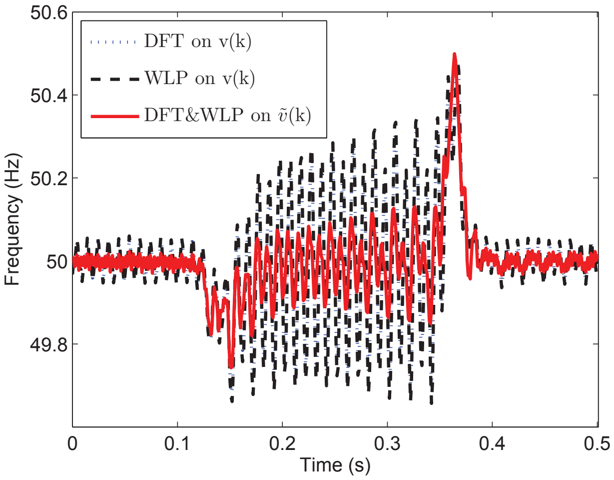


Fig. 5. Frequency estimation performance of both frequency estimators on the original system voltage $v(k)$ and its transformation $\tilde{v}(k)$ for the real-world unbalanced three-phase power system. (a) The waveforms of the three-phase voltage. (b) Frequency estimation results.

As shown in Fig. 5(b), the proposed voltage balancing transformation enabled both DFT and WLP frequency estimators provided accurate frequency estimates in both balanced and unbalanced voltage conditions.

VI. Conclusion

We have introduced a novel balancing voltage transformation (BVT) for unified online estimation of the fundamental system frequency of both balanced and unbalanced power systems. This has been achieved by accounting for the full second order statistics of the system voltage, yielding accurate phase angle evolution of the positive sequence component within the system voltage. This has made it possible to apply any frequency estimator designed for single tone exponential (balanced power system) directly on the transformed system voltage. Simulations over a range of unbalanced conditions, including voltage dips and swells, phase deviation, the presence of higher order harmonics, and for real-world unbalanced power systems support the analysis.

References

[1] E. Aboutanios and B. Mulgrew, “Iterative frequency estimation by interpolation on Fourier coefficients,” *IEEE Trans. Signal Process.,* vol. 53, no. 4, pp. 1237-1242, 2005.

[2] S. Kay, “A fast and accurate single frequency estimator,” *IEEE Trans. Acoust., Speech, Signal Process.,* vol. 37, no. 12, pp. 1987-1990, 1989.

[3] M. S. Sachdev and M. M. Giray, “A least error squares technique for determining power system frequency,” *IEEE Trans. Power App. Syst.,* vol. PAS-104, no. 5, pp. 1025-1038, 1983.

[4] P. Rogriguez, J. Pou, J. Bergas, J. Ignacio Candela, R. P. Burgos, and D. Boroyevich, “Decoupled double synchronous reference frame PLL for power converters control,” *IEEE Trans. Power Electron.,* vol. 22, no. 2, pp. 584-592, 2007.

[5] A. K. Pradhan, A. Routray, and A. Basak, “Power system frequency estimation using least mean square technique,” *IEEE Trans. Power Del.,* vol. 20, no. 3, pp. 1812-1816, 2005.

[6] H.-S. Song and K. Nam, “Instantaneous phase-angle estimation algorithm under unbalanced voltage-sag conditions,” *Proc. Inst. Electr. Eng.,* vol. 147, no. 6, pp. 409-415, 2000.

[7] M. D. Kušljević, J. J. Tomić, and L. D. Jovanović, “Frequency estimation of three-phase power system using weighted-least-square algorithm and adaptive FIR filtering,” *IEEE Trans. Instrum. Meas.,* vol. 57, no. 10, pp. 322-329, 2010.

[8] P. K. Dash, A. K. Pradhan, and G. Panda, “Frequency estimation of distorted power system signals using extennded complex Kalman filter,” *IEEE Trans. Power Del.,* vol. 14, no. 3, pp. 761-766, 1999.

[9] M. Mojiri, D. Yazdani, and A. Bakhshai, “Robust adaptive frequency estimation of three-phase power system,” *IEEE Trans. Instrum. Meas.,* vol. 59, no. 7, pp. 1793-1802, 2010.

[10] B. G. Quinn, “Estimating frequency by interpolation using Fourier coefficients,” *IEEE Trans. Signal Process.,* vol. 42, no. 5, pp. 1264-1268, 1994.

[11] T. Lobos and J. Rezmer, “Real-time determination of power system frequency,” *IEEE Trans. Instrum. Meas.,* vol. 46, no. 4, pp. 877-881, 1997.

[12] Y. Xia and D. P. Mandic, “Widely linear adaptive frequency estimation of unbalanced three-phase power systems,” *IEEE Trans. Instrum. Meas.,* vol. 61, no. 1, pp. 74-83, 2012.

[13] Y. Xia, S. C. Douglas, and D. P. Mandic, “Adaptive frequency estimation in smart grid applications: Exploiting noncircularity and widely linear adaptive estimators,” *IEEE Signal Process. Mag.,* vol. 29, no. 5, pp. 44-54, 2012.

[14] Y. Xia and D. P. Mandic, “Augmented MVDR spectrum-based frequency estimation for unbalanced power systems,” *IEEE Trans. Instrum Meas,* vol. 62, no. 7, pp. 1917-1926, 2013.

[15] B. Picinbono, “On circularity,” *IEEE Trans. Signal Process,* vol. 42, no. 12, pp. 34733482, 1993.

[16] B. Picinbono and P. Chevalier, “Widely linear estimation with complex data,” *IEEE Trans. Signal Process,* vol. 43, no. 8, pp. 2030-2033, 1995.

[17] D. P. Mandic and S. L. Goh, *Complex Valued Nonlinear Adaptive Filters: Noncircularity, Widely Linear and Neural Models.* John Wiley & Sons, 2009.8

[18] Y. Xia, B. Jelfs, M. M. Van Hulle, D. P. Mandic and J. C. Príncipe, “An augmented echo state network for nonlinear adaptive filtering of complex noncircular signals,” *IEEE Trans. Neural Netw.,* vol. 22, no. 1, pp. 74-83, 2011.

[19] B. G. Quinn, “Estimating frequency by interpolation using Fourier coefficients,” *Digit. Signal Process.,* vol. 19, no. 6, pp. 942-948, 2009.

[20] L. C. Palmer, “Coarse frequency estimation using the discrete Fourier transform,” *IEEE Trans. Inform. Theory,* vol. 20, no. 1, pp. 104-109, 1974.

[21] D. C. Rife and R. R. Boorstyn, “Single-tone parameter estimation from discrete-time observations,” *IEEE Trans. Inform. Theory,* vol. 20, no. 5, pp. 591-598, 1974.

[22] M. D. Macleod, “Fast nearly ML estimation of parameters of real or complex signle tones or resolved multiple tones,” *IEEE Trans. Signal Process.,* vol. 46, no. 1, pp. 141148, 1998.