A DFT Enhanced Complex LMS for Digital Adaptive Spur Cancellation

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***Abstract*—A discrete Fourier transform (DFT) enhanced complex least mean square (CLMS) algorithm, which utilizes the underlying time series relationship among the consecutive fundamental DFT components, is proposed to adaptively mitigate the spur pollution in multi-standard transceivers. The transient and steady-state performances of the proposed algorithm are investigated, demonstrating faster convergence speed and higher signal-plus-noise-to-interference ratio (SNIR) performance at the steady state, compared to the conventional LMS algorithm. Computer simulations in the single-spur cancellation setting support the analysis.**

***Index Terms*—discrete Fourier transform (DFT), complex least mean square (CLMS), spur cancellation**

I. Introduction

A difficult challenge in modern design of consumer RF integrated circuits is to support a growing number of wireless standards (e.g. GSM, 3G, LTE, WiFi) in one single radio device, in order to keep the RF/analog circuitry in a small size for flexibility and energy-saving concerns. Due to a multiplicity of the operating frequencies and the physical proximity between the components, one major problem encountered by the implementations of such mobile architectures are spurious tones (spurs), which are clock harmonics which result from the leakage of other operating frequencies in a multi-standard transceiver. These spurs may subsequently leak into the phase-locked-loop to create a false demodulation [1], degrading the signal-plus-noise-to-interference ratio (SNIR) of the receiver as much as 8-10 dB [2], or directly coupling back within the noisy signals.

Methods to mitigate spurs have been recently studied in the literature. Among them, a group of active cancellation techniques which cancel the spurs digitally have become particularly attractive, as it is more cost-effective to deal with spurs in the digital domain at a reasonable complexity instead of restricting specifications of the analog part of front-ends. These methods can be classified into notch filter based techniques [2] and adaptive filtering algorithms [3], [4], and in [4] the two classes are unified as equivalent schemes.

Recently, a novel complex-valued least squares enhanced DFT algorithm [5], [6] was proposed, which utilizes the underlying time series relationship among the consecutive fundamental DFT components, instead of time series itself, and exhibits higher accuracy of frequency estimation of a single-tone exponential. By considering a similar underlining signal processing modelling setup, we employ its essential features and propose a DFT enhanced CLMS (DFT-CLMS) algorithm to deal with the adaptive spur cancellation, for which the closed-form expression of the post-cancellation SNIR is provided to illustrate its performance advantages over the conventional CLMS algorithm [4]. The analysis is supported by illustrative simulations in practical scenarios.

II. One-spur Model and CLMS based Adaptive Spur Cancellation

The observed signal $d(n)$ in a multi-standard transceiver in discrete-time sense is composed of three parts, given by [4]

$$d(n) = x(n) + b(n) + s(n) \tag{1}$$

where $x(n)$, $b(n)$ and $s(n)$ are respectively the data signal, additive white noise, and additive spur, all assumed to be independent with each other. The received data $x(n)$ and the noise $b(n)$ are further assumed to be zero-mean complex-valued Gaussian processes, with respective variances σ_x^2 and σ_b^2. The spur is defined as

$$s(n) = Ae^{j(\omega_0-\omega)n+j\varphi(n)+j\varphi_0} \tag{2}$$

where ω_0 is the nominal frequency of the spur, which is normalized and known *a priori*. The symbol A denotes the amplitude, φ_0 is the initial phase, ω is a small frequency shift between the actual frequency and the nominal frequency, so that $\omega \ll \omega_0$, and these are all unknown parameters. The phase noise (PN) $\varphi(n)$ follows a Brownian process, the evolution of which is given by [7]

$$\varphi(n+1) = \varphi(n) + \xi(n) \tag{3}$$

with $\varphi(0) = 0$ and $\xi(n)$ real-valued additive white noise with variance $\sigma_\xi^2 \ll 1$. Based on (1), the SNIR of the observed signal before spur cancellation can be expressed as

$$\mathrm{SNIR}_0 = -10\log_{10}\left(\frac{A^2}{\sigma_x^2+\sigma_b^2}\right) \tag{4}$$

$$Q_1 = \frac{A^2(\omega^2+\sigma_\xi^2)}{B^2} + 2\omega^2(1-\mu B^2)\frac{A^2[\mu B^2-(\omega^2+\sigma_\xi^2)(1-\mu B^2)]}{B^2[\mu^2 B^4+\omega^2(1-\mu B^2)^2]} \tag{11}$$

As the nominal frequency ω_0 of the spur is known *a priori*, a complex exponential signal $u(n)$, given by

$$u(n) = Be^{j\omega_0 n + j\varphi_R} \tag{5}$$

can be synthesized and used as a reference for spur cancellation, where B is the amplitude of the reference and φ_R is the initial phase. In [4], the conventional CLMS algorithm was employed to cancel the spurs in the presence of the frequency drift ω [8], for which the functional relation is

$$e(n) = d(n) - w(n)u(n) \tag{6}$$

$$w(n+1) = w(n) + \mu u^*(n)e(n) \tag{7}$$

where $e(n)$ is the error or the compensated signal, $w(n)$ the weight coefficient updated by the step-size μ, and $(\cdot)^*$ the complex conjugation operator.

With the above described assumptions on ω and σ_ξ^2, $e(n)$ can be considered as approximately wide-sense stationary, and hence, we are able to derive the asymptotic performance of CLMS by analyzing the misalignment or the weight error coefficient $v(n)$, defined as

$$v(n) = w(n) - w_{\rm opt}(n) \tag{8}$$

in which $w_{\rm opt}(n) = s(n)/u(n)$ is the ideal coefficient, for which the output error of the CLMS algorithm becomes

$$e(n) = x(n) + b(n) - v(n)u(n) \tag{9}$$

By investigating the standard mean square convergence of the CLMS algorithm [9]–[13], and after a few algebraic manipulations, the steady-state variance of the misalignment $v(n)$ can be derived as [4]

$$\sigma_v^2 = E[|v(\infty)|^2] = \frac{\mu(\sigma_x^2+\sigma_b^2)}{2-\mu B^2} + \frac{Q_1}{\mu B^2(2-\mu B^2)} \tag{10}$$

where Q_1 is given in (11). Based on (9) and (10), the SNIR performance of CLMS for the spur cancellation is given by [4]

$$\mathrm{SNIR}_{\rm CLMS} = -10\log_{10}\left(\frac{B^2\sigma_v^2}{\sigma_x^2+\sigma_b^2}\right) \tag{12}$$

III. Proposed Adaptive Spur Cancellation based on DFT-LMS

Since the nominal frequency ω_0 of the spur is known *a propri*, we can effectively design the sampling frequency f_s to be Nf_0, where $f_0 = \omega_0/2\pi$, and then perform the N-point DFT transformation on the observed signal $d(n)$ to obtain its DFT fundamental component, given by [5], [6]

$$\dot{d}(k) = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} d(n+k)e^{-j\frac{2\pi n}{N}} \tag{13}$$

where k denotes the time index of the fundamental DFT component, and the scaling factor $1/\sqrt{N}$ is used for power invariance. Based on (1), we can express $\dot{d}(k)$ as

$$\dot{d}(k) = \dot{x}(k) + \dot{b}(k) + \dot{s}(k) \tag{14}$$

where $\dot{x}(k)$, $\dot{b}(k)$ and $\dot{s}(k)$ are respectively the fundamental DFT component of $x(n)$, $b(n)$, and $s(n)$. After some algebraic manipulations, it is not difficult to find out that the mean and variance of $\dot{x}(k)$ and $\dot{b}(k)$ are the same as those of $x(n)$ and $b(n)$, but have the following iterative characteristics,

$$E[\dot{x}(k)\dot{x}^*(k+1)] = \frac{N-1}{N}\sigma_x^2 \tag{15}$$

where

$$\dot{x}(k+1) = e^{j\frac{2\pi}{N}}\dot{x}(k) + \frac{1}{\sqrt{N}}e^{j\frac{2\pi}{N}}[x(k+1) - x(k+1-N)] \tag{16}$$

and $\dot{s}(k)$ can be represented as

$$\dot{s}(k) = \frac{A}{\sqrt{N}}e^{j\phi_0}e^{j2\pi k/N}e^{-j\omega k}\sum_{n=0}^{N-1} e^{-j\omega n}e^{j\varphi(n+k)} \tag{17}$$

By exploiting the ergodicity of $\xi(n)$, the form $\varphi(n+k)$ can be derived from (3) as

$$\varphi(n+k) = \sum_{i=0}^{n+k-1}\xi(i) = \sum_{i=0}^{n-1}\xi(i) + \sum_{j=0}^{k-1}\xi(j) \tag{18}$$

Substituting (18) into (17) and after some rearranging of forms, we arrive at

$$\dot{s}(k) = A\sqrt{N}e^{j\phi_0}e^{j2\pi k/N}e^{-j\omega k}e^{j\varphi(k)} \tag{19}$$

where for small phase variance σ_ξ^2 and frequency drift ω,

$$\sum_{n=0}^{N-1} e^{-j\omega n}e^{j\varphi(n)} \approx N \tag{20}$$

and $\varphi(k)$ is a zero-mean colored Gaussian noise with variance $k\sigma_\xi^2$. Based on (14) and (19), the SNIR of $\dot{d}(k)$ now becomes $A^2N/(\sigma_x^2+\sigma_b^2)$, which is N times larger than its time domain counterpart given in (4). Correspondingly, the fundamental DFT component of the reference signal $\dot{u}(k)$, denoted by $\dot{u}(k)$, is given by

$$\dot{u}(k) = B\sqrt{N}e^{j\varphi_R}e^{j2\pi k/N} \tag{21}$$

Finally, the proposed DFT enhanced CLMS (DFT-CLMS) algorithm for adaptive spur cancellation can be described as

$$\dot{e}(k) = \dot{d}(k) - \dot{w}(k)\dot{u}(k) \tag{22}$$

$$\dot{w}(k+1) = \dot{w}(k) + \mu\dot{u}^*(k)\dot{e}(k) \tag{23}$$

$$Q_2 = \frac{A^2(\omega^2+\sigma_\xi^2)}{B^2} + 2\omega^2(1-\mu NB^2)\frac{A^2[\mu NB^2-(\omega^2+\sigma_\xi^2)(1-\mu NB^2)]}{B^2[\mu^2N^2B^4+\omega^2(1-\mu NB^2)^2]} \tag{32}$$

$$Q_3 = E\left[\Re\{\dot{v}(k)\dot{u}(k)[\dot{x}^*(k)+\dot{b}^*(k)]\}\right] = \frac{N-1}{\mu N^2B^2} + \frac{1-\mu NB^2}{\mu N^{3/2}B^2} + E\left[\Re\{\dot{v}(k-1)\dot{u}(k-1)[x(k)+b(k)-x(k-N)-b(k-N)]\}\right] \tag{33}$$

Similar to (8), the corresponding misalignment $\dot{v}(k)$ of DFT-CLMS can be defined as

$$\dot{v}(k) = \dot{w}(k) - \dot{w}_{\mathrm{opt}}(k) \tag{24}$$

where $\dot{w}_{\mathrm{opt}}(k)$ is the ideal coefficient, used to provide an output strictly equal to the polluting spur $\dot{s}(k)$, which is defined as

$$\dot{w}_{\mathrm{opt}}(k) = \frac{\dot{s}(k)}{\dot{u}(k)} = \frac{A}{B}e^{j(\varphi_0-\varphi_R)}e^{-j\omega k}e^{j\varphi(k)} \tag{25}$$

and can be iteratively obtained by

$$\dot{w}_{\mathrm{opt}}(k+1) = \dot{w}_{\mathrm{opt}}(k)e^{-j\omega}e^{j\xi(k)} \tag{26}$$

Using (24) and (25), equation (22) becomes

$$\dot{e}(k) = \dot{x}(k) + \dot{b}(k) - \dot{v}(k)\dot{u}(k) \tag{27}$$

Upon applying the first-order approximation of the McLaurin series on the right hand side (RHS) of (26), the evolution of $\dot{v}(k)$ in (24) becomes

$$\begin{aligned}\dot{v}(k+1) = (1-\mu NB^2)\dot{v}(k) + \mu\dot{u}^*(k)[\dot{x}(k)+\dot{b}(k)] \\ + \dot{w}_{\mathrm{opt}}(k)[j\omega - j\dot{\xi}(k)]\end{aligned} \tag{28}$$

Taking the statistical expectation $E[\cdot]$ on both sides of (28), the step-size μ which guarantees the mean convergence of the proposed DFT-CLMS algorithm should satisfy

$$0 < \mu < \frac{2}{NB^2} \tag{29}$$

and the time constant τ, used to measure the rate of its convergence [14], is then given by

$$\tau \approx \frac{1}{2\mu NB^2} \tag{30}$$

Upon multiplying both sides of (28) by $\dot{v}^*(k+1)$, taking the statistical expectation $E[\cdot]$, and using the independence assumptions made in Section II, we arrive at the mean square evolution of the misalignment $\dot{v}(k)$, in the form

$$\begin{aligned}E[|\dot{v}(k+1)|^2] = (1-\mu NB^2)^2E[|\dot{v}(k)|^2] + \mu^2NB^2[\sigma_x^2+\sigma_b^2] \\ + Q_2 + 2\mu(1-\mu NB^2)Q_3\end{aligned} \tag{31}$$

The detailed expression of Q_2 is given in (32), and by utilizing (15) and (16), Q_3 is iteratively described in (33), where $\Re\{\cdot\}$ is the real part operator. The steady-state variance of the misalignment $\dot{v}(k)$ from (31) is then obtained by considering $k \to \infty$, to give

$$\begin{aligned}\sigma_{\dot{v}}^2 = E[|\dot{v}(\infty)|^2] = \frac{\mu(\sigma_x^2+\sigma_b^2)}{2-\mu NB^2} + \frac{Q_2}{\mu NB^2(2-\mu NB^2)} \\ + \frac{2(1-\mu NB^2)Q_3}{NB^2(2-\mu NB^2)}\end{aligned} \tag{34}$$

In order to implement effective spur cancellation and investigate the performance of the proposed DFT-CLMS algorithm, we need to transform $\dot{e}(k)$ back into the time domain to obtain the equivalent spur-compensated signal $e(n)$. This can be achieved through the following inverse DFT operation

$$e(n) = \sqrt{N}\sum_{m=0}^{N-1} E_m\left(\inf\left(\frac{n-1}{N}\right)N+1\right)e^{j\frac{2\pi nm}{N}} \tag{35}$$

where $\inf(\cdot)$ is the operator which rounds down the argument to the nearest integer and

$$E_m(l) = \begin{cases} D_m(l) & m \neq 1 \\ \dot{e}(l) & m = 1 \end{cases} \tag{36}$$

in which $D_m(l)$ is the mth DFT component of $d(n)$. After some calculus, the SNIR of the proposed DFT-CLMS algorithm for spur cancellation can be evaluated as

$$\mathrm{SNIR}_{\mathrm{Proposed}} = -10\log_{10}\left(\frac{\sigma_{\dot{v}}^2NB^2}{N(\sigma_x^2+\sigma_b^2)}\right) \tag{37}$$

where $\sigma_{\dot{v}}^2$ is given in (34). Note that the conventional CLMS algorithm can be regarded as a special case of the proposed one when $N = 1$.

IV. SIMULATIONS

We investigated the performances of the conventional CLMS and the proposed DFT-CLMS algorithms for single spur cancellation in an OFDM transmission system. The number of subcarriers of the OFDM waveform was 64, the cyclic prefix in each OFDM symbol was 16, and the data on each subcarrier was QPSK-modulated. The waveform was transmitted through a Rayleigh fading channel. On the receiver side, the signal was polluted by a single spur, whose nominal center frequency f_0 was at $f_0 = 6$ MHz in the baseband, and additive complex white Gaussian noise, with the SNR $= \sigma_x^2/\sigma_b^2$ was set to 10 dB. The amplitudes of the spur $s(n)$ and the reference signal $u(n)$, that is, A and B, were set to $A = 1$ and $B = 1.3$ respectively, and the initial SNIR was 0 dB. All the simulations results were obtained by averaging 500 independent trials.

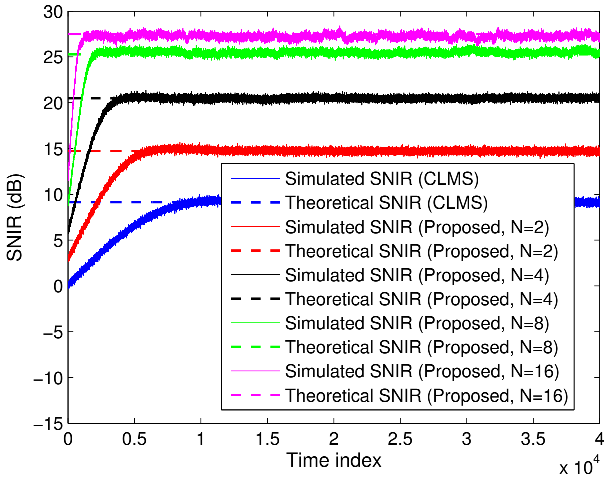
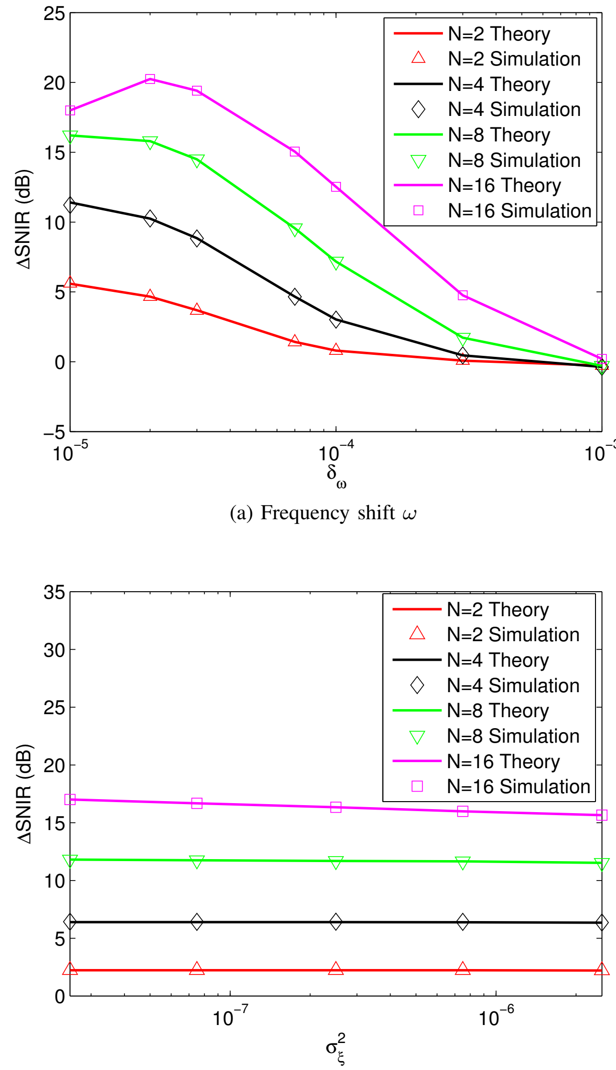


Fig. 1. Comparison of the mean transient SNIR performances of both the considered algorithms.



(a) Frequency shift ω

(b) Phase noise variance σ_ξ^2

Fig. 2. Improvement in SNIR of the proposed algorithm over the conventional CLMS for different values of (a) frequency shift ω, and (b) phase noise variance σ_ξ^2.

In the first set of simulations, the equivalent variance of the Brownian modelled phase noise was $\sigma_\xi^2 = 2.5 \times 10^{-8}$, and the small frequency shift was $\omega = 10^{-5}\omega_0$. Fig. 1 illustrates the transient SNIR of both the conventional CLMS algorithm [4] and the proposed one, for different DFT sizes N. Using a step-size $\mu = 1 \times 10^{-5}$, the theoretical steady state SNIRs of both algorithms, evaluated by using (12) and (37), are also provided as a reference. The proposed DFT-CLMS algorithm enabled both faster convergence speed and higher SNIR performance in the steady state; these performance advantages were more pronounced when the proposed spur cancellation algorithm employed a larger DFT size N. The faster convergence was also expected since the time constant τ of the proposed DFT-CLMS algorithm in (30) is inversely proportional to the DFT size N, and the excellent agreement between the simulated steady state SNIR and the theoretical one of both algorithms can be observed.

We next considered the steady-state SNIR improvement of the proposed spur cancellation algorithm over the conventional CLMS, defined as $\Delta\text{SNIR} = \text{SNIR}_{\text{Proposed}} - \text{SNIR}_{\text{CLMS}}$, for different values of the frequency shift ω and phase noise variance σ_ξ^2, as shown in Fig. 2 (a) and (b), respectively. It is obvious that the proposed algorithm provided higher spur cancellation accuracy than the CLMS algorithm for a smaller frequency drift ω, and again, the close match between the theoretical SNIR at the steady state and the simulated one can be observed.

V. Conclusions

We have introduced a discrete Fourier transform enhanced least mean square (DFT-LMS) algorithm to mitigate the spur pollution in multi-standard transceivers. This has been achieved by utilizing the underlying time series relationship among the consecutive fundamental DFT components. Both the transient and the steady-state performances of the proposed algorithm have been studied, illustrating its faster convergence speed and enhanced steady state SNIR performance over the conventional CLMS algorithm. Simulations on spur polluted OFDM signals support the analysis.

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