Addendum to "Augmented Second-Order Statistics of Quaternion Random Signals"

Clive Cheong Took*[,a], Danilo P. Mandic[a]

[a]*Department of Electrical and Electronic Engineering, Imperial College, London SW7 2AZ, United Kingdom*

Abstract

Further to our recently published paper [1], we would like to comment on the difference in equations (24)-(26), when considering temporal and spatial data.

In our recent paper [1], we used the equations (24)-(26) to explain the $\imath-\jmath-\kappa$ covariance matrices. Whereas these equations hold for temporal data (a common place in adaptive filtering), we would now like to clarify their interpretation for a general (possibly spatial) $\mathbb{Q}-$proper vector $\mathbf{q} = [q_1, \ldots, q_N]^T \neq [q(t), \ldots, q(t-N)]^T$, whereby $q_1 = q_{a,1} + \imath q_{b,1} + \jmath q_{c,1} + \kappa q_{d,1}$. In such a general case, equations (24)-(26) may not necessarily hold, meaning that the real-valued cross-correlation matrices of components $\mathbf{q}_a, \mathbf{q}_b, \mathbf{q}_c, \mathbf{q}_d$ are not guaranteed to vanish. This can be illustrated by inserting the block diagonal matrix (27) into (21) to demonstrate that the off-diagonal matrices (which are the cross-correlation matrices) in C_R do not vanish.

Observe, however, that the diagonal elements of these real-valued cross-correlation matrices do vanish. This implies no correlation between $q_{a,i}$ and $q_{b,i}$ and possible correlation between the off-diagonal elements $q_{a,i}$ and $q_{b,j}$ where $i \neq j$, which in practice is questionable. To further illuminate our point, consider the Wiener filter discussed in equations (29)-(33), to demonstrate that equations in the original paper (24)-(26) do hold. Here, for a $\mathbb{Q}-$proper random vector $\mathbf{q} = [q(t), \ldots, q(t-N)]^T$, since there is no correlation between $q_{a,1} = q_a(t)$ and $q_{b,1} = q_b(t)$ due to $\mathbb{Q}-$properness, then it is very unlikely in practice that $q_a(t)$ is correlated with $q_b(t-\tau)$ for $\tau = 1, \ldots, N$ due to the nature of temporal correlation between samples. On the other hand, if there is no temporal correlation between samples such as in the case of independent and identically distributed (i.i.d.) samples, then each component/channel within the quaternion signal is most probably noise, leading to vanishing cross-correlation matrices, and hence equations (24)-(26) hold.

We would also like to point a typo in Table 3 regarding the κ imaginary part of $C_{\mathbf{q}^\imath}$, it is made up of $\Im_\kappa\{\cdot\} = C_{\mathbf{q}_d\mathbf{q}_a} + C_{\mathbf{q}_a\mathbf{q}_d} + C_{\mathbf{q}_b\mathbf{q}_c} + C_{\mathbf{q}_c\mathbf{q}_b}$.

References

[1] C. Cheong Took and D. P. Mandic, "Augmented Second-Order Statistics of Quaternion Signals," *Signal Processing*, vol. 91, no. 2, pp. 214 – 224, 2011.

*Corresponding author.

Email addresses: c.cheong-took@imperial.ac.uk (Clive Cheong Took), d.mandic@ic.ac.uk (Danilo P. Mandic)