Human-Centred Multivariate Complexity Analysis

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***Abstract*—Signatures of changes in dynamical system complexity, in the response of a living system to their environment, are reflected in both the within- and cross-channel correlations in observed physiological variables. Only a joint analysis of these heterogeneous variables yields full insight into the underlying system dynamics. We illuminate the abilities of the multivariate multiscale entropy (MMSE) method to model structural richness, and illustrate its usefulness in human centred applications - complexity changes due to constraints (cognitive load, stress).**

***Index Terms*—multivariate sample entropy, complexity, coupling and causality, cognitive load, stress, Yarbus experiment.**

I. Introduction

Physiological responses of a living organism are a widely-studied form of complex system [1]. The analysis is not trivial and is conducted from heterogeneous physiological variables, from spiking neuronal activity to respiratory waveforms, with different dynamics, degrees of coupling and causality, and at multiple temporal and spatial scales [2].

The 'complexity-loss' theory states that the severity of constraints on a living system, caused by e.g. illness or ageing, is manifested by changes in the complexity of its responses - a natural measure of structural richness [3]. Standard entropy measures, such as Shannon entropy, Kolmogorov-Sinai entropy or approximate entropy, assess signal regularity (cf. randomness) but not true system complexity - represented by coupled dynamics at different scales. The multiscale entropy (MSE) method evaluates univariate sample entropy across multiple temporal scales revealing long range correlations - a key property of complex systems [4], [5] - and has been applied to estimate the complexity changes in physiological time series for numerous applications (congestive heart failure, Alzheimers disease and postural sway dynamics [6], [7]).

The above methods only cater for single-channel data and therefore fail to account for dynamical relationships that exist *between* the physiological variables. This limits their potential in e.g. medical applications - a medic would routinely examine brain and heart responses as well as eye and muscle activity. The recent multivariate multiscale entropy (MMSE) method was developed *specifically* to cater for both the within- and cross-channel dependencies for any number of data channels [6], [7], revealing coupled dynamics not observable using standard single-channel estimates (Gestalt).

We here revisit MMSE and illuminate its use in both classic and novel human-centred applications, focusing on identifying complexity signatures from physiological recordings caused by increased cognitive load and stress. The approach is validated both for homogeneous and for heterogeneous multivariate physiological variables.

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II. Multivariate Multiscale Entropy

Multivariate multiscale entropy (MMSE) estimation (Matlab code available from [8]) is performed by two steps [6], [7]:

1) The different temporal scales are defined by coarse-graining (moving average) the *p*-variate time series $\{x_{k,i}\}_{i=1}^{N}, k = 1, 2, \ldots, p$, with *N* samples in each variate. For a scale factor ϵ, the corresponding coarse coarse-grained time series: $y_{k,j}^{\epsilon} = \frac{1}{\epsilon}\sum_{i=(j-1)\epsilon+1}^{j\epsilon} x_{k,i}$, where $1 \leq j \leq \frac{N}{\epsilon}$ and $k = 1, \ldots, p$.
2) The multivariate sample entropy, $MSampEn$, is evaluated for each intrinsic scale within the multivariate $y_{k,j}^{\epsilon}$, and is plotted as a function of the scale factor ϵ.

A. MSampEn Calculation

For a *p*-variate time series, $\{x_{k,i}\}_{i=1}^{N}, k = 1, 2, \ldots, p$ of length N, the calculation of $MSampEn$ is described in Algorithm 1 [6] [7], where the multivariate embedded vectors are constructed as:

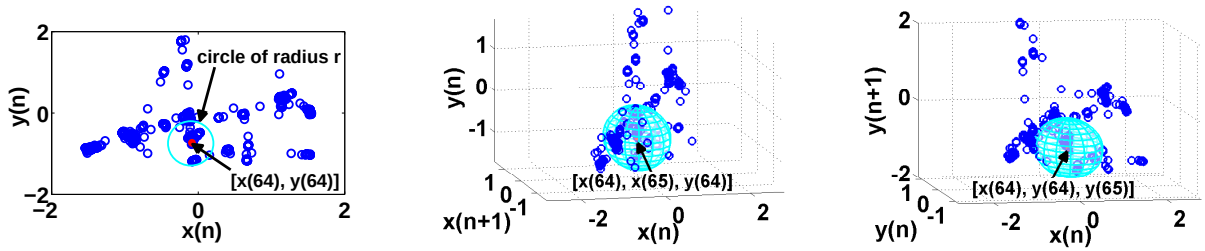
$$X_m(i) = [x_{1,i}, x_{1,i+\tau_1}, \ldots, x_{1,i+(m_1-1)\tau_1}, x_{2,i}, x_{2,i+\tau_2}, \ldots, x_{2,i+(m_2-1)\tau_2}, \ldots, x_{p,i}, x_{p,i+\tau_p}, \ldots, x_{p,i+(m_p-1)\tau_p}],$$

and $\mathbf{M} = [m_1, m_2, \ldots, m_p] \in \mathbb{R}^p$ is the embedding vector, $\boldsymbol{\tau} = [\tau_1, \tau_2, \ldots, \tau_p]$ the time lag vector, and the composite delay vector $X_m(i) \in \mathbb{R}^m$ $(m = \sum_{k=1}^{p} m_k)$.

Algorithm 1 Multivariate sample entropy ($MSampEn$)

1: Form $(N-\delta)$ composite delay vectors $X_m(i) \in \mathbb{R}^m$, where $i = 1, 2, \ldots, N-\delta$ and $\delta = \max\{\mathbf{M}\} \times \max\{\boldsymbol{\tau}\}$ and define the distance between any two vectors $X_m(i)$ and $X_m(j)$ as the maximum norm;
2: For a given composite delay vector $X_m(i)$ and a threshold r, count the number of instances P_i for which $d[X_m(i), X_m(j)] \leq r$, $j \neq i$, then calculate the frequency of occurrence, $B_i^m(r) = \frac{1}{N-\delta-1}P_i$, and define $B^m(r) = \frac{1}{N-\delta}\sum_{i=1}^{N-\delta} B_i^m(r)$;
3: Increase $m_k \to (m_k+1)$ for a specific variable k, keeping the dimension of the other variables unchanged. Thus, a total of $p\times(N-\delta)$ vectors $X_{m+1}(i)$ in $\mathbb{R}^{m+1}$ are obtained;
4: For a given $X_{m+1}(i)$, calculate the number of vectors Q_i, such that $d[X_{m+1}(i), X_{m+1}(j)] \leq r$, where $j \neq i$, then calculate the frequency of occurrence, $B_i^{m+1}(r) = \frac{1}{p(N-\delta)-1}Q_i$, and define $B^{m+1}(r) = \frac{1}{p(N-\delta)}\sum_{i=1}^{p(N-\delta)} B_i^{m+1}(r)$;
5: Finally, for a tolerance level *r*, estimate $MSampEn$ as

$$MSampEn(\mathbf{M}, \boldsymbol{\tau}, r, N) = -ln\left[\frac{B^{m+1}(r)}{B^m(r)}\right]. \quad (1)$$



(a) Scatter plot of [x(n), y(n)] (b) Scatter plot of [x(n), x(n+1), y(n)] (c) Scatter plot of [x(n), y(n), y(n+1)]

Fig. 1. Geometry behind $MSampEn$. Scatter plots for a 2-variate gaze signal (see Fig. 4(c)) for ($m = 2$) and the two 3-variate subspaces for ($m = 3$).

B. Geometric Interpretation of MSampEn

Underpinning the multivariate sample entropy method is the estimation of the conditional probability that two similar sequences will remain similar when the next data point is included. This is achieved by calculating the average number of neighbouring delay vectors for a given tolerance level (r) and repeating the process after increasing the embedding dimension, from (m) to ($m + 1$), a geometric interpretation of which is shown in Fig. 1. Fig. 1(a) shows the set of delay vectors for a 2-variate gaze signal ([x(n), y(n)] with $\boldsymbol{\tau} = [1, 1]$ & $\mathbf{M} = [1, 1]$) and illustrates the neighbours[1] for the point [x(64), y(64)]. Upon increasing the embedding dimension from ($m = 2$) to ($m = 3$), we have two different subspaces spanning: (i) the vectors [x(n), x(n+1), y(n)] (Fig. 1(b)) and (ii) the vectors [x(n), y(n), y(n+1)] (Fig. 1(c)). The $MSampEn$ algorithm accounts fully for both within- and cross-channel correlations by examining the composite of all such subspaces.

C. Interpretation of the MMSE curves

The complexity of multi-channel data is assessed from the MMSE plots ($MSampEn$ as a function of the scale factor). A multivariate time series is more complex than another one if for the majority of the time scales the multivariate entropy values are higher than those of the other time series. A monotonic decrease of the multivariate entropy values with the scale factor indicates that the signal in hand only contains useful information at the smallest scale and has no structure, and is therefore not dynamically complex (white noise).

III. Simulation Results

The potential of the MMSE method can be conveniently illustrated on a 6-variate time series, where originally all the data channels were realizations of mutually independent white noise. We then gradually decreased the number of variates that represent white noise (from 6 to 5, 3, 1 and 0) and simultaneously increased the number of data channels of independent 1/f noise (from 0 to 1, 3, 5 and 6). The 1/f noise exhibits long range correlations and is therefore complex [4], [5]. Fig. 2 shows that as the number of variates representing 1/f noise increased, the $MSampEn$ (complexity) at higher scales also increased, and when all the six data channels contained 1/f noise, the complexity at larger scales was the highest. The more variables/channels had long range correlations the higher the overall complexity of the underlying system - a key feature for use in human centred scenarios.

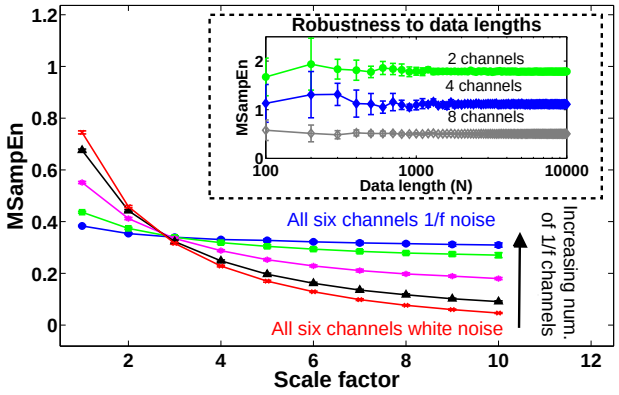


Fig. 2. MMSE analysis for 6-channel data containing white and 1/f noise, each with 10,000 data points. The curves represent an average of 20 independent realizations and error bars standard deviation (SD). Figure insert in top right shows the robustness of $MSampEn$ estimates for white noise and illustrates that, the greater the number of channels, the more robust the $MSampEn$ estimate (errorbars become smaller).

Fig. 3 illustrates that, unlike standard MSE, multivariate MSE caters for cross-channel correlations - a crucial advantage of the algorithm. Indeed, the complexity of the correlated bivariate 1/f noise with maximum correlation coefficient ($cc = 1$) was the highest at large scales; the complexity decreased as the degree of correlation between the channels decreased and was lowest for the uncorrelated white noise.

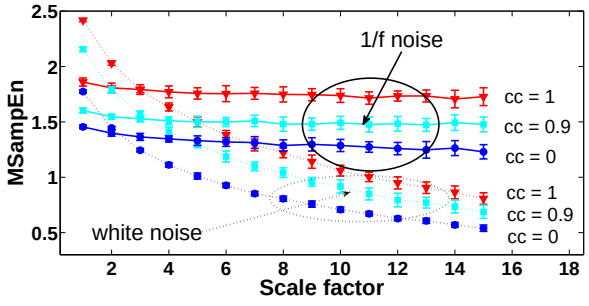


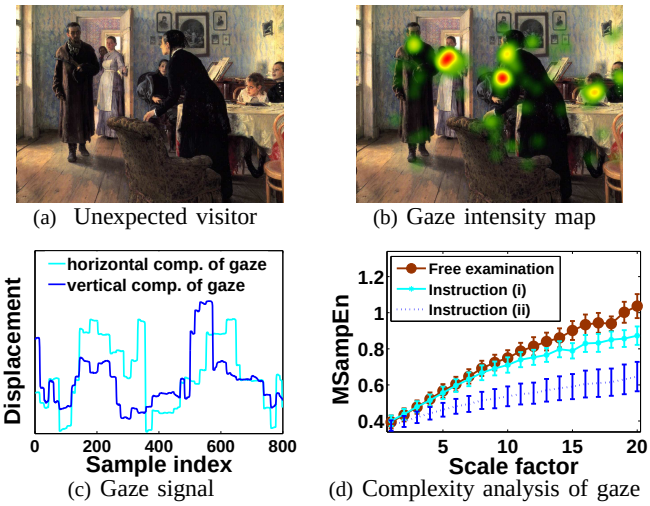
Fig. 3. Multivariate multiscale entropy (MMSE) analysis for bivariate white and 1/f noise, each with 10,000 data points. The curves represent an average of 20 independent realizations and error bars the standard deviation (SD).

[1]Neighbouring vectors at a point in an m-dimensional space can be represented by the points enclosed by an m-sphere or an m-cube, for the Euclidean and maximum norm respectively.

A. Analysis of Eye Gaze Dynamics

Psychologist Alfred L. Yarbus famously illustrated the impact of cognitive load on scanning eye patterns by presenting subjects with an image (see Fig. 4(a)) and recording gaze trajectories in response to different instructions [9]. To re-investigate this classic study from a completely novel perspective, we set out to examine whether cognitive load is reflected in the complexity of the gaze dynamics. Seven healthy, naive subjects were asked to both examine the image in Fig. 4(a) freely and to complete six different instructions over 100 s trials (see [9] for more details), while bivariate (vertical and horizontal) eye gaze was recorded (a segment of which is shown in Fig. 4(c)).

Fig. 4(d) shows the average gaze complexity over all subjects, for both constrained and free examination, and illustrates that the cognitive instructions can be uniquely identified in the gaze complexity space. Compared to all instruction trials, the gaze complexity of free examination was the highest over high scale factors (>10), supporting the general 'complexity-loss' theory, that is, the less constrained the cognitive task the higher the complexity.



(a) Unexpected visitor (b) Gaze intensity map (c) Gaze signal (d) Complexity analysis of gaze

Fig. 4. Average MMSE analysis of the classic 'Yarbus experiment' illustrating that induced cognitive load reduces the gaze complexity. (a) The presented image. (b) Gaze intensity map for the instruction relating to the ages of the people. (c) Segment of raw gaze data, both horizontal and vertical components. (d) Average complexity results for 'free examination' and instructions (i) *estimate the material circumstances of the family* and (ii) *remember positions of people and objects in the room*, where the errorbars denote the standard error - std/sqrt(num. of trials).

B. Heart and Respiratory Function During Stress

Stress-induced illnesses are a major concern in modern mankind, the American Institute of Stress estimates that 75-90% of all visits to primary care physicians are for stress related problems. Stress is manifested by changes in several psycho-physiological modalities - a perfect match for the multivariate nature of the MMSE method.

Three naive, healthy subjects participated in a study (two parts each lasting 20 mins), in which respiration waveforms and electrocardiography (ECG) were recorded while the subject was seated comfortably and instructed not to talk or move unnecessarily. The baseline physiological response was established ('normal state') by engaging the subject in a relaxing task - watching a movie. Next, the subject was presented with a series of demanding mathematical logic questions and was instructed to respond via a keypad as quickly and accurately as possible ('stressed state'). An increased level of background noise and verbal interference from the experiment coordinator were used to increase the level of subject engagement.

Fig. 5 shows the average complexity results obtained for the bivariate data[2] [ECG, respiration] for both the 'normal' and 'stressed' states. The MMSE approach was clearly able to separate the two states in the complexity-space.

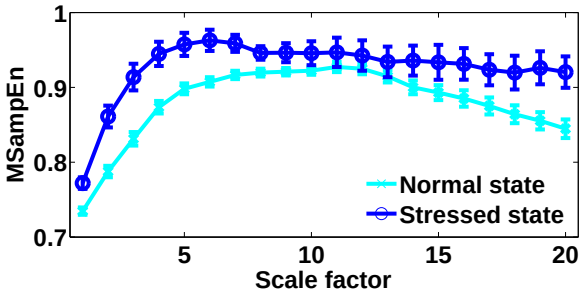


Fig. 5. Average MMSE analysis for 'normal' and 'stressed' states, based on heart and respiratory functions. Error bars denote the standard error.

IV. Conclusion

The recently introduced multivariate multiscale entropy (MMSE) method has been illuminated as an enabling tool for the complexity analysis of real-world multivariate data. It has been shown to model the dynamical couplings between physiological variables, giving an insight into the underlying system complexity, a feature not achievable using standard univariate measures. The advantages of MMSE have been exemplified for detecting signatures caused by increased cognitive load and stress, highlighting its appeal in human-centred applications.

References

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[2]For each subject and sub-experiment, the data was divided into 100 s segments and at least 5 artifact-free segments were extracted and analysed. The ECG was bandpass filtered to occupy the frequency range (0.5 - 20) Hz and the respiration data was bandpass filtered to occupy the frequency range (0.05 - 3) Hz. All data was recorded at 1200 Hz and downsampled to 120 Hz.