Signal Processing for Vector Sensors: Crossroads of Tools and Ideas

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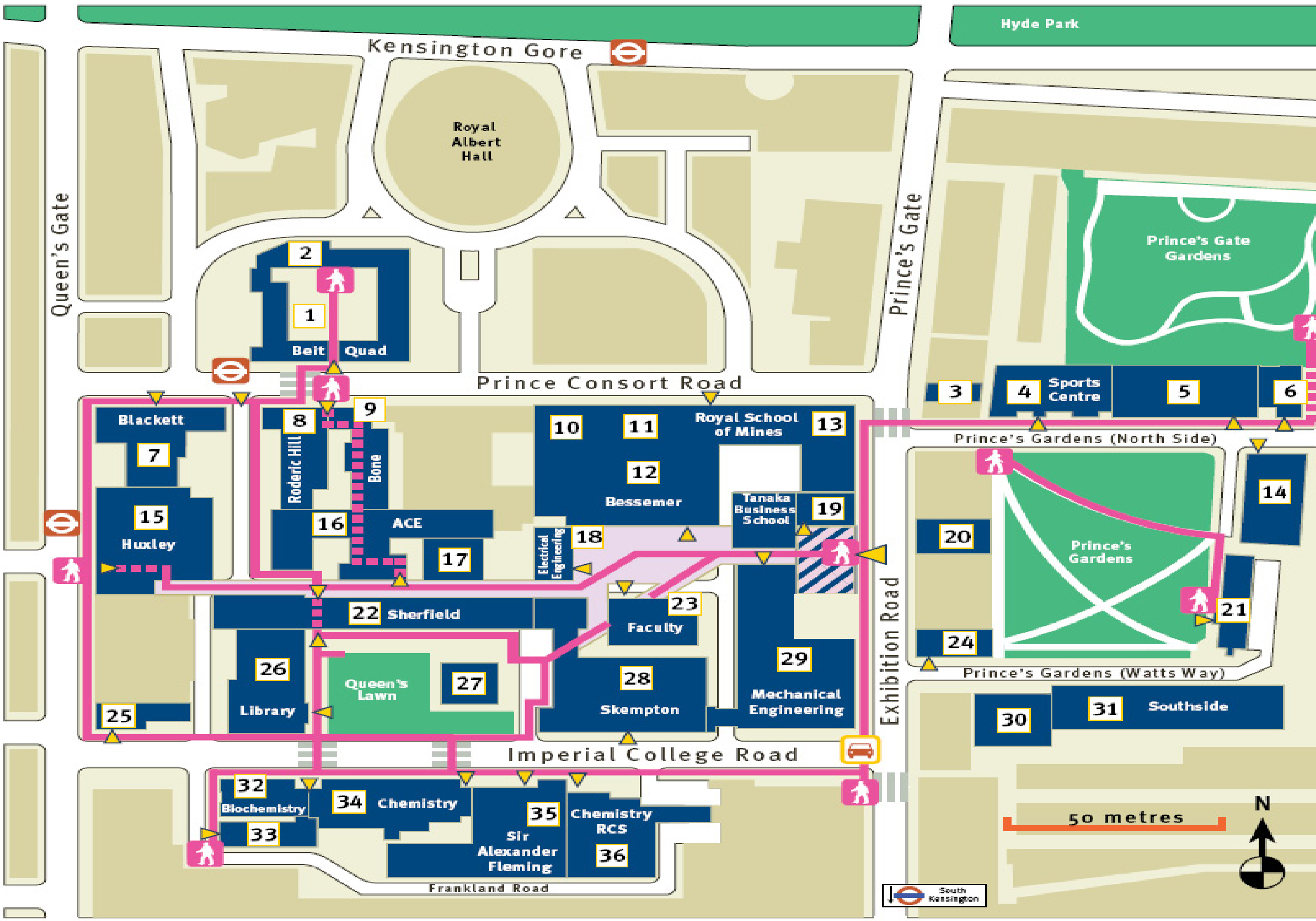
About Imperial College



Outline:-

Why vector sensors

Vectors in $\mathbb{R}^N$ versus complex $\mathbb{C}$ and quaternion $\mathbb{H}$ representations

Complex and quaternion valued processing of real valued problems

Circularity, properness, nonstationarity

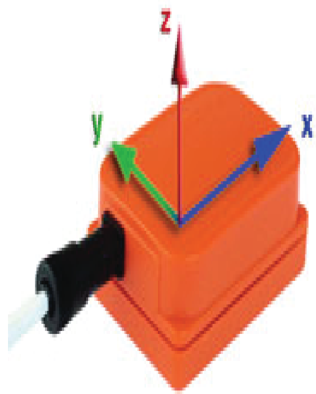
Augmented statistics and widely linear models

Data fusion via vector spaces

Applications:-

– Body sensors and wearable technologies
– Radar and sonar
– Renewable energy applications
– Biomedical applications

Vector sensors



Renewable Energy

2D and 3D anemometers

control of wind turbine

Body motion sensor

3D - position, gyroscope, speed

gait, biometrics

Wearable techologies

Biomechanics

virtual reality

Why Modelling in $\mathbb{C}$?

- Complex signals by design (communications, analytic signals, equivalent baseband represenation to eliminate spectral redundancy)

- By convenience of representation (radar, sonar, wind field)

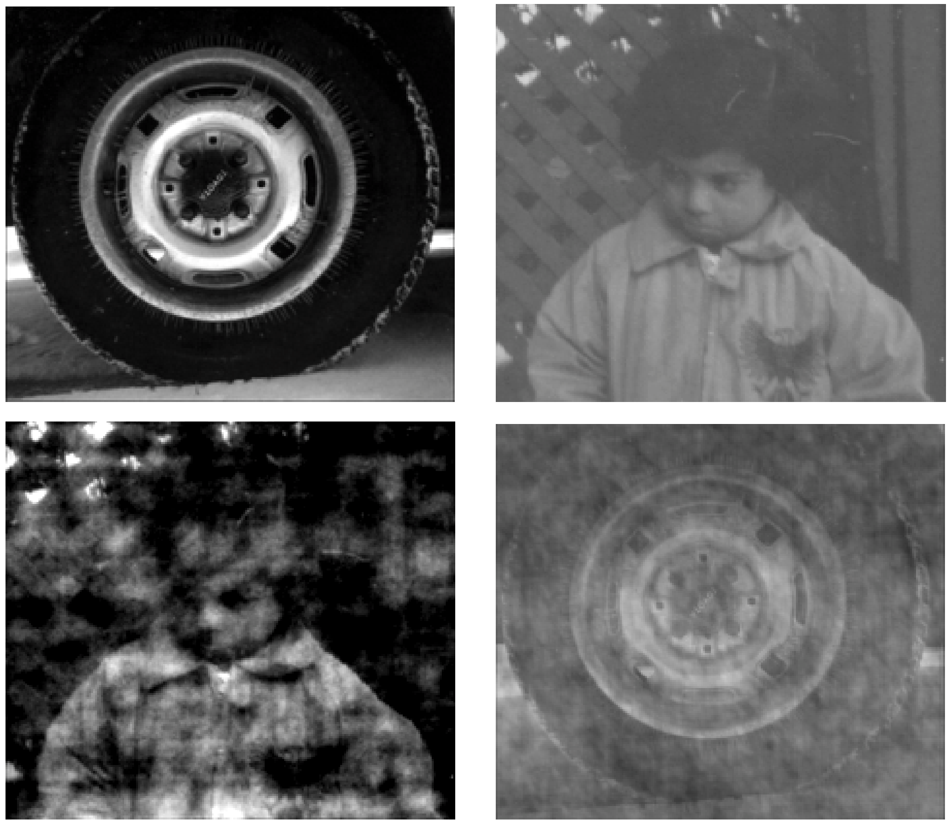
- Problem: Different algebra (no ordering - operator "$\leq$" makes no sense!), and the notion of pdf has to be induced

- Problem: Special form of nonlinearity (the only continuously differentiable function in $\mathbb{C}$ is a constant (Liouville theorem)

- Solution: Special statistics – augmented complex statistics (started in mathematics in 1992)

- We can differentiate between several kinds of noises (doubly white circular with various distributions $n_r \perp n_i$ & $\sigma^2_{n_r} = \sigma^2_{n_i}$, doubly white noncircular $n_r \perp n_i$ & $\sigma^2_{n_r} > \sigma^2_{n_i}$, noncircular noise)

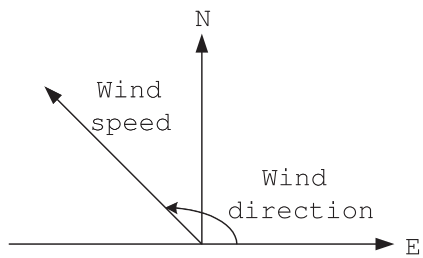
Human Visual System – Importance of Phase Information



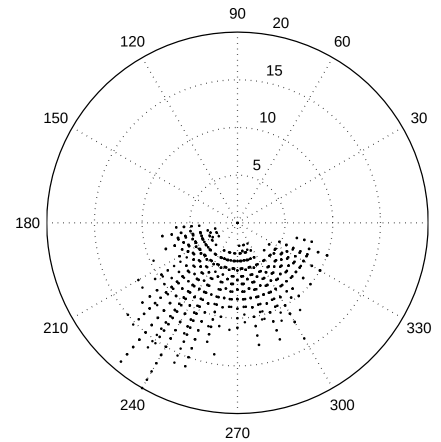
Surrogate images. *Top:* Original images I_1 and I_2; *Bottom:* Images $\hat{I}_1$ and $\hat{I}_2$ generated by exchanging the amplitude and phase spectra of the original images.

Noncircularity of Distributions - Wind Modelling $(v(k) = |v(k)|e^{\jmath\Phi(k)})$

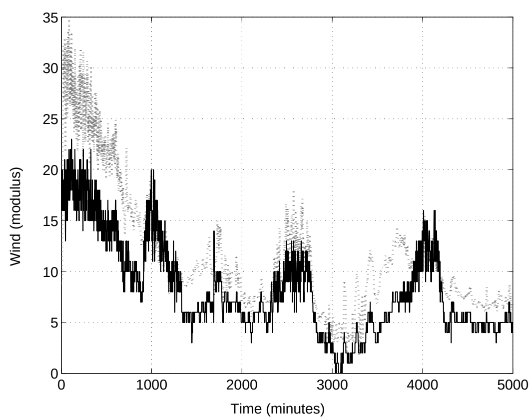
(e) Gill Inst. 2D ultrasonic anemometer



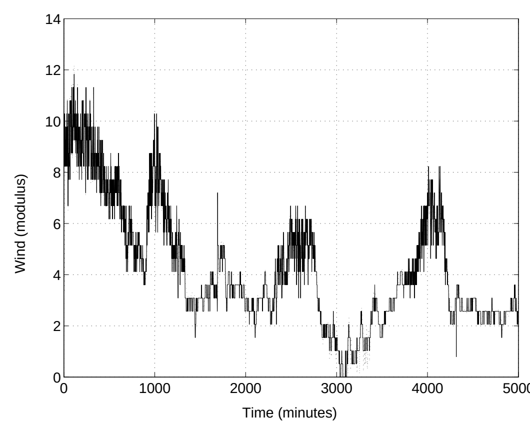
(f) Wind as a complex vector.



(g) Wind lattice (rose) – distribution of wind speeds over directions.



(h) Dual univariate model



(i) Complex model

Isomorphism Between $\mathbb{C}$ and $\mathbb{R}^2$

$$z \rightarrow z^a \quad \leftrightarrow \quad \begin{bmatrix} z \\ z^* \end{bmatrix} = \begin{bmatrix} 1 & \jmath \\ 1 & -\jmath \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

whereas in the case of complex–valued signals, we have

$$\mathbf{z} \rightarrow \mathbf{z}^a \quad \leftrightarrow \quad \begin{bmatrix} \mathbf{z} \\ \mathbf{z}^* \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \jmath\mathbf{I} \\ \mathbf{I} & -\jmath\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix}$$

For convenience, the "augmented" complex vector $\mathbf{v} \in \mathbb{C}^{2N \times 1}$ can be introduced as

$$\begin{aligned} \mathbf{v} &= [z_1, z_1^*, \ldots, z_N, z_N^*]^T \\ \mathbf{v} &= \mathbf{A}\mathbf{w}, \qquad \mathbf{w} = [x_1, y_1, \ldots, x_N, y_N]^T \end{aligned}$$

where matrix $\mathbf{A} = diag(\mathbf{J}, \ldots, \mathbf{J}) \in \mathbb{C}^{2N \times 2N}$ is block diagonal and transforms the composite real vector $\mathbf{w}$ into the augmented complex vector $\mathbf{v}$.

The Multivariate Complex Normal Distribution

Recall, the relationships like "$<$" or "$\geq$" make no sense in $\mathbb{C}$.

$$\mathbf{V} = cov(\mathbf{v}) = E[\mathbf{v}\mathbf{v}^H] = \mathbf{A}\mathbf{W}\mathbf{A}^H$$

Using the result by Vanden Bos 1995

$$\begin{aligned} \mathbf{w} &= \mathbf{A}^{-1}\mathbf{v} = \frac{1}{2}\mathbf{A}^H\mathbf{v} \\ det(\mathbf{W}) &= \left(\frac{1}{2}\right)^{2N} det(\mathbf{V}) \\ \mathbf{w}^T\mathbf{W}^{-1}\mathbf{w} &= \mathbf{v}^H\mathbf{V}^{-1}\mathbf{v} \end{aligned}$$

The multivariate *generalised complex normal distribution* (GCND) can now be expressed as

$$f(\mathbf{v}) = \frac{1}{\pi^N\sqrt{det(\mathbf{V})}} e^{-\frac{1}{2}\mathbf{v}^H\mathbf{V}^{-1}\mathbf{v}}$$

and has been derived without any restriction. (Van Den Bos, 1995)

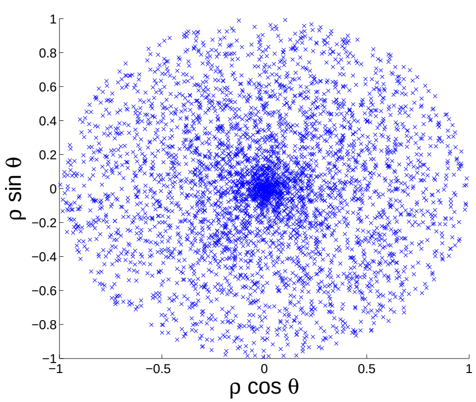
Circular Complex Random Variables

Circularity = Rotation invariant distribution $p(\rho,\theta) = p(\rho,\theta-\phi)$

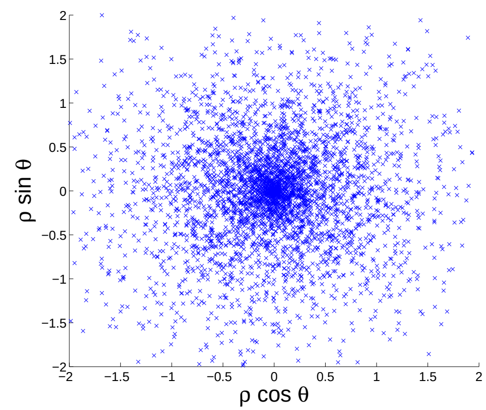
1. Take a real–valued random variable ρ with a pdf $p(\rho)$;

2. Take another real–valued random variable θ, which must be uniformly distributed on $[0, 2\pi]$ and independent of ρ;

3. Construct $Z = X + jY$ as $X = \rho\cos(\theta), \qquad Y = \rho\sin(\theta)$



(j) Uniform circular



(k) Gaussian circular

Other Definitions of Circularity

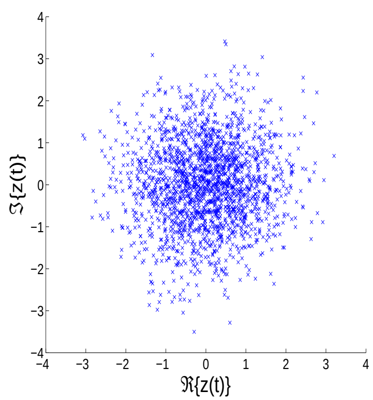
Via Probability dens. func., Characteristic Function, Cumulants (Amblard at al., 1996)

- *Probability density function.* A complex random variable Z is circular if its pdf is a function of only the product zz^*, that is[1]

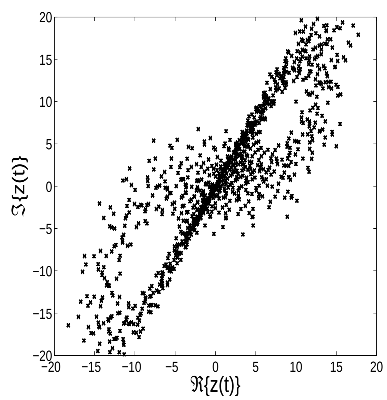
$$p_{Z,Z^*}(z, z^*) = p_{Z_\phi, Z_\phi^*}(z_\phi, z_\phi^*)$$

and for for Gaussian CCRVs we have

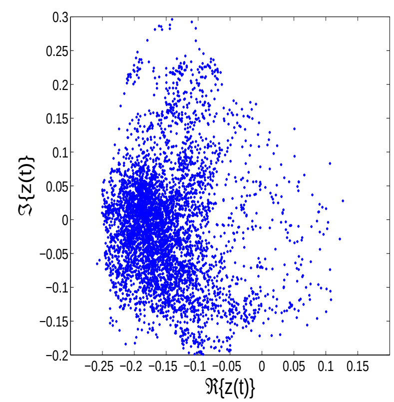
$$p_{Z,Z^*}(z, z^*) = \frac{1}{\pi\sigma^2} e^{-zz^*/\sigma^2}$$



(l) Complex AR(4)



(m) Complex Lorenz



(n) Complex wind

[1]The pdf of a circular complex random variable is function of only the modulus of Z, **and not of z^*.**

What are we Doing Wrong - Widely Linear Model

Consider the MSE estimator of a signal y in terms of another observation x

$$\hat{y} = E[y|x]$$

For zero mean, jointly normal y and x, the solution is

$$\hat{y} = \mathbf{h}^T \mathbf{x}$$

In standard MSE in the complex domain $\hat{y} = \mathbf{h}^H \mathbf{x}$, however

$$\hat{y}_r = E[y_r | x_r, x_i] \quad \& \quad \hat{y}_i = E[y_i | x_r, x_i]$$

$$thus \qquad \hat{y} = E[y_r | x_r, x_i] + \jmath E[y_i | x_r, x_i]$$

Upon employing the identities $x_r = (x + x^*)/2$ and $x_i = (x - x^*)/2\jmath$

$$\hat{y} = E[y_r | x, x^*] + \jmath E[y_i | x, x^*]$$

and thus arrive at the **widely linear** estimator for general complex signals

$$y = \mathbf{h}^T \mathbf{x} + \mathbf{g}^T \mathbf{x}^*$$

We can now process general (noncircular) complex signals!

Dealing with Complex Statistics

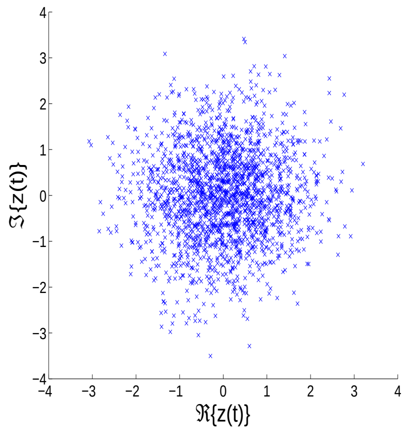
- In general, the covariance matrix
$$\mathcal{C} = cov(\mathbf{z}) = E\left[\mathbf{z}\mathbf{z}^H\right]$$
does not
completely describe the second order statistics of $\mathbf{z}$, and another quantity
$$\mathcal{P} = pcov(\mathbf{z}) = E\left[\mathbf{z}\mathbf{z}^T\right]$$
called the **pseudocovariance** or **complementary covariance**, needs to be taken into account;

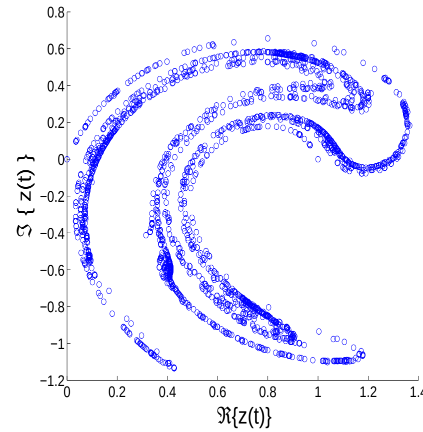
- The probability density function of Gaussian complex random variables has a form similar to that for real Gaussian variables only for *proper*, or *second order circular*, random processes $\mathbf{z}$ for which the pseudocovariance
$$\mathcal{P} = E\left[\mathbf{z}\mathbf{z}^T\right] = \mathbf{0}$$
vanishes (hint: $E[z \times z^T] = E[x^2] - Ey^2] = \sigma_x^2 - \sigma_y^2$);

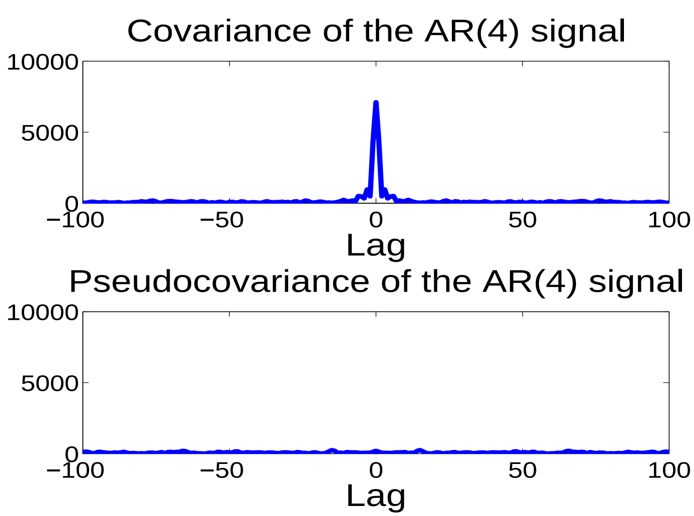
- However, general complex random processes are *improper*.

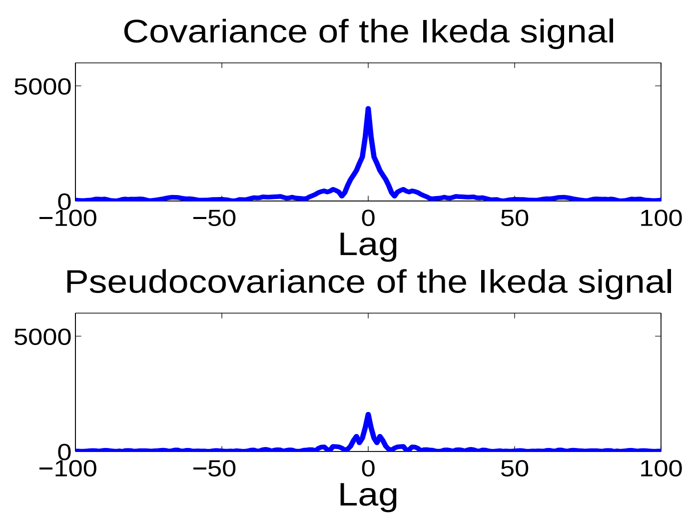
Practical Example

Complex AR(4) process (circular) Complex Ikeda map (noncircular)





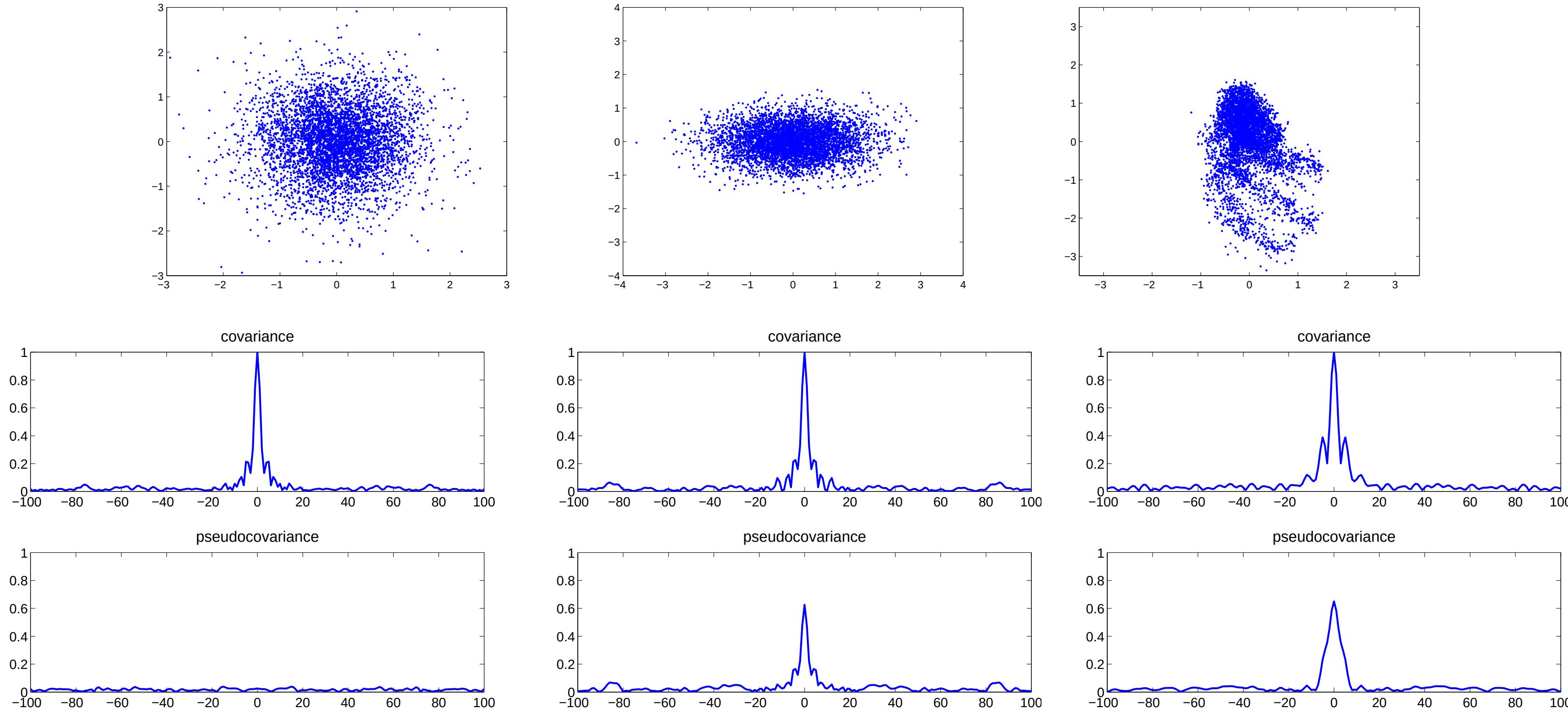




Complex AR(4) process (proper) Complex Ikeda map (improper)

The Role of Noise – Double Whiteness

PDFs: DW circular noise DW noncircular noise noncircular noise



Covariances: DW circular noise DW noncircular noise noncircular noise

- Doubly white circular noise (proper) $\Rightarrow n_r \perp n_i \;\&\; \sigma^2_{n_r} = \sigma^2_{n_i}$
- Doubly white noncircular noise (improper) $\Rightarrow n_r \perp n_i \;\&\; \sigma^2_{n_r} > \sigma^2_{n_i}$

Measuring (Non)-Circularity

Obviously, since $\sigma_x^2 \geq \sigma_y^2$, any ratio of the powers of the real and imaginary part of a general complex signal is a candidate for a measure of the degree of circularity. **Remember:** $|S_{\mathcal{P}}(\omega)|^2 \leq S_{\mathcal{C}}(\omega)S_{\mathcal{C}}(-\omega)$

An unbounded measure

$$\xi = \sqrt{\sigma_x^2/\sigma_y^2} \qquad \xi = 1 \rightarrow \text{proper}, \quad \xi > 1 \rightarrow \text{improper}$$

Another measure

$$\kappa = 1 - \det(\mathcal{C}_a)\det{}^{-2}\mathcal{C}_{zz} \qquad 0 \leq \kappa \leq 1,\ \kappa = 0 \rightarrow \text{proper signal}$$

Or, circularity coefficient

$$r = \frac{|E\{z^2\}|}{E\{|z|^2\}}, \qquad 0 < r < 1,\ r = 0 \rightarrow \ \text{proper signal}$$

r – square of the eccentricity ϵ of an ellipse centred in $\mathbb{C}$; For $\epsilon = 0$ the shape is a circle $\leftrightarrow$ proper (2nd order circular) signal with $r = 0$.

Comparison of degrees of noncircularity κ for the various classes of signals

	Circular AR(4)	Noncircular ARMA	Ikeda map	Wind (*low*)	Wind (*medium*)	Wind (*high*)
κ	0.0016	0.9429	0.1229	0.2703	0.4305	0.8117
r	0.0093	0.9198	0.3549	0.5199	0.6484	0.8398
ξ	1.05	4.8901	1.4173	1.1908	1.2876	1.3736

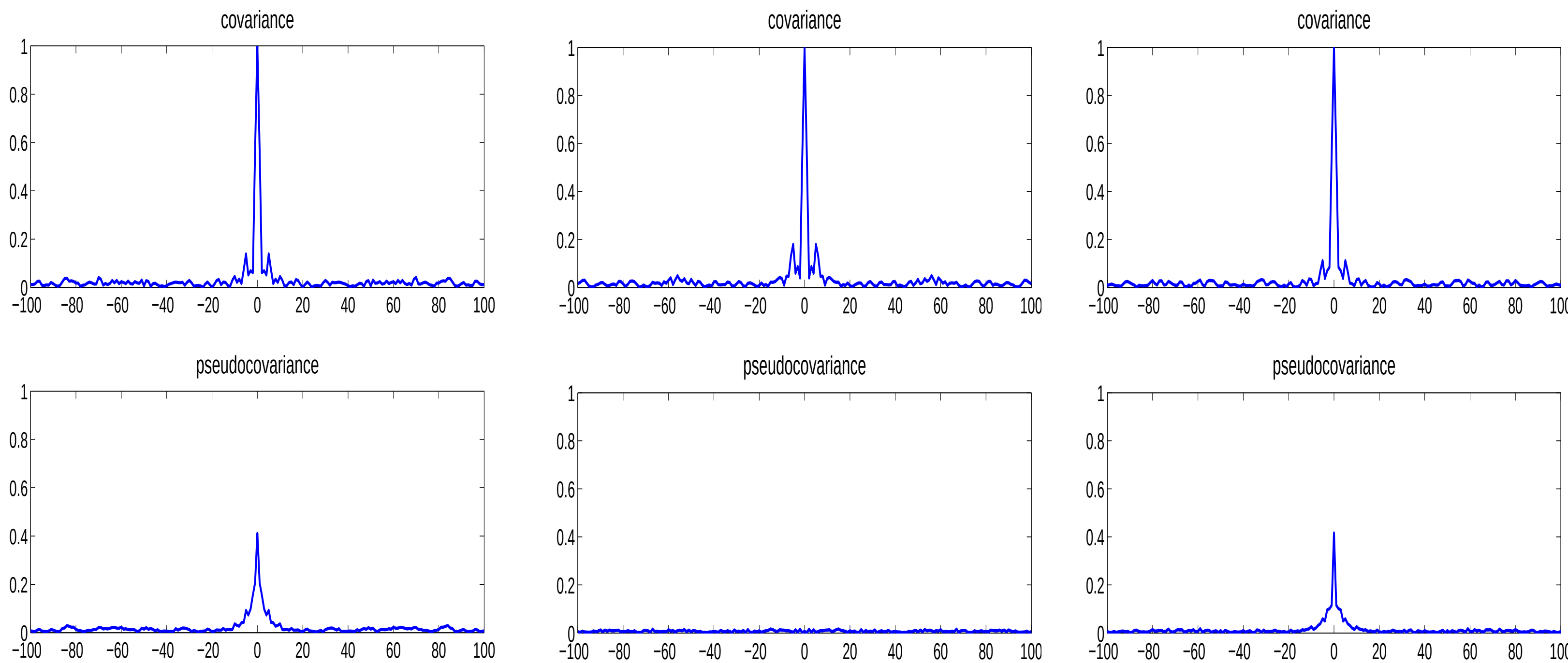
Solution: Widely Linear Stochastic Modelling

Widely linear model

$$y(k) = \mathbf{h}(k)\mathbf{x}(k) + \mathbf{g}(k)\mathbf{x}^*(k) + n(k)$$

Widely linear normal equations

$$\begin{bmatrix} \mathbf{h}^* \\ \mathbf{g}^* \end{bmatrix} = \begin{bmatrix} \mathcal{C} & \mathcal{P} \\ \mathcal{P}^* & \mathcal{C}^* \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{c} \\ \mathbf{p}^* \end{bmatrix}$$



Covariances: Original Ikeda Standard AR model of Ikeda Widely linear AR of Ikeda

Can we Quantify the Benefits of WL Modelling?

$$\hat{z}_l = \mathbf{a}^T\mathbf{z}(k) \quad \rightarrow \quad \varepsilon_l^2 = E[|z(k)|^2] - E[|\hat{z}_l(k)|^2]$$
$$\hat{z}_{wl} = \mathbf{h}^T\mathbf{z}(k) + \mathbf{g}^T\mathbf{z}^*(k) \quad \rightarrow \quad \varepsilon_{wl}^2 = E[|z(k)|^2] - E[|\hat{z}_{wl}(k)|^2]$$

Let us examine (Picinbono, Chevalier)

$$\delta\varepsilon^2 = \varepsilon_{wl}^2 - \varepsilon_l^2 = \mathbf{c}_a^T \mathcal{C}_a^{*-1} \mathbf{c}_a^* - \mathbf{c}^T \mathcal{C}^{*-1} \mathbf{c}^*$$

This expression is a bit awkward, as the pseudocovariance information is embedded into the augmented covariance matrix $\mathcal{C}_a$.

After some tedious matrix manipulation, we arrive at

$$\delta\varepsilon^2 = \left[\mathbf{p} - \mathcal{P}^*\mathcal{C}^{*-1}\mathbf{c}^*\right]^H \left[\mathcal{C}^* - \mathcal{P}^*\mathcal{C}^{-1}\mathcal{P}\right]^{-1} \left[\mathbf{p}^* - \mathcal{P}^*\mathcal{C}^{-1}\mathbf{c}\right]$$

where $\mathbf{c}$ and $\mathbf{p}$ are respectively the first column of $\mathcal{C}$ and $\mathcal{P}$.

Observe that for proper signals $\mathcal{P} = \mathbf{0}$ implies $\delta\varepsilon^2 = 0$, that is, **both the standard and widely linear model perform in the same way**.

For improper signals, $\mathcal{P}$ is nonzero and $\delta\varepsilon^2 > 0$, the WL model is superior.

Learning: Cauchy–Riemann Equations and Drawbacks

$$\frac{\partial u(x,y)}{\partial x} = \frac{\partial v(x,y)}{\partial y}, \qquad \frac{\partial v(x,y)}{\partial x} = -\frac{\partial u(x,y)}{\partial y}$$

The Jacobian matrix of a complex function $f(z) = u + \jmath v$, is given by

$$\mathbf{J} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix} \qquad \Leftrightarrow \qquad \begin{bmatrix} \text{“}1\text{''} & \text{“}1\text{''} \\ \text{“}-1\text{''} & \text{“}1\text{''} \end{bmatrix}$$

Thus, $f(z) = z^*$ is not analytic as its Jacobian $\mathbf{J} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

Functions which depend on both $z = x + \jmath y$ and $z^* = x - \jmath y$ are not analytic

$$J(z, z^*) = zz^* = x^2 + y^2 \quad \Rightarrow \quad \mathbf{J} = \begin{bmatrix} 2x & 2y \\ 0 & 0 \end{bmatrix} \quad \Leftrightarrow \quad \frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y} \quad \frac{\partial v}{\partial x} \neq -\frac{\partial u}{\partial y}$$

One typical example is the cost function $J = \frac{1}{2}e(k)e^*(k) = \frac{1}{2}|e(k)|^2$.

The $\mathbb{CR}$ calculus

Based on our earlier examples of nonanalytic functions $f(z) = z^*$ and $f(z) = |z|^2 = zz^*$, observe that:-

- A function $f(z)$ can be non–holomorphic in the complex variable $z = x + \jmath y$, but still be analytic in real variables x and y, as for instance, $f(z) = z^*$ and $f(z) = zz^* = x^2 + y^2$;

- Both $f(z) = z^*$ and $f(z) = zz^*$ are holomorphic in z for $z^* = const$, and are also holomorphic in z^* when $z = const$.

The main idea behind both Wirtinger calculus and Brandwood's result, is to introduce so called *conjugate coordinates*

$$f(z) = f(z, z^*) = g(x, y) = \Re\{f\} + \jmath\Im\{f\} = u(x, y) + \jmath v(x, y)$$

For an excellent overview see the web material by Kenneth Kreutz-Delgado

The Derivative of a Cost Function $\frac{1}{2}e(k)e^*(k)$ and CLMS

As $\mathbb{C}$-derivatives are not defined for real functions of complex variable

$$\mathbb{R}-\text{der:} \quad \frac{\partial}{\partial \mathbf{z}} = \frac{1}{2}\left[\frac{\partial}{\partial \mathbf{x}} - \jmath\frac{\partial}{\partial \mathbf{y}}\right] \qquad \mathbb{R}^*-\text{der:} \quad \frac{\partial}{\partial \mathbf{z}^*} = \frac{1}{2}\left[\frac{\partial}{\partial \mathbf{x}} + \jmath\frac{\partial}{\partial \mathbf{y}}\right]$$

and the gradient

$$\nabla_{\mathbf{w}} J = \frac{\partial J(e,e^*)}{\partial \mathbf{w}} = \left[\frac{\partial J(e,e^*)}{\partial w_1}, \ldots, \frac{\partial J(e,e^*)}{\partial w_N}\right]^T = 2\frac{\partial J}{\partial \mathbf{w}^*} = \underbrace{\frac{\partial J}{\partial \mathbf{w}^r} + \jmath\frac{\partial J}{\partial \mathbf{w}^i}}_{pseudogradient}$$

The standard Complex Least Mean Square (CLMS) (Widrow *et al.* 1975)

$$\begin{aligned} y(k) &= \mathbf{x}^T(k)\mathbf{w}(k) \\ e(k) = d(k) - y(k) & \qquad e^*(k) = d^*(k) - \mathbf{x}^*(k)\mathbf{w}^*(k) \\ \text{and} & \quad \nabla_{\mathbf{w}} J = \nabla_{\mathbf{w}^*} J \\ \mathbf{w}(k+1) &= \mathbf{w}(k) - \mu\frac{\partial \frac{1}{2}e(k)e^*(k)}{\partial \mathbf{w}^*(k)} = \mathbf{w}(k) + \mu e(k)\mathbf{x}^*(k) \end{aligned}$$

Thus, no need for tedious computations – The CLMS is derived in one line.

The Augmented (widely linear) CLMS (ACLMS)

Widely linear model $y(k) = \mathbf{h}^{\mathsf{T}}(k)\mathbf{z}(k) + \mathbf{g}^{\mathsf{T}}(k)\mathbf{z}^*(k)$

$$\mathbf{h}(k+1) = \mathbf{h}(k) - \mu\nabla_{\mathbf{h}^*}J \qquad \Rightarrow \qquad \nabla_{\mathbf{h}^*}J = -e(k)\mathbf{x}^*(k)$$

$$\mathbf{g}(k+1) = \mathbf{g}(k) - \mu\nabla_{\mathbf{g}^*}J \qquad \Rightarrow \qquad \nabla_{\mathbf{g}^*}J = -e(k)\mathbf{x}(k)$$

Therefore, the ACLMS update

$$\begin{aligned} \mathbf{h}(\mathbf{k+1}) &= \mathbf{h}(\mathbf{k}) + \mu\mathbf{e}(\mathbf{k})\mathbf{x}^*(\mathbf{k}) \\ \mathbf{g}(\mathbf{k+1}) &= \mathbf{g}(\mathbf{k}) + \mu\mathbf{e}(\mathbf{k})\mathbf{x}(\mathbf{k}) \end{aligned}$$

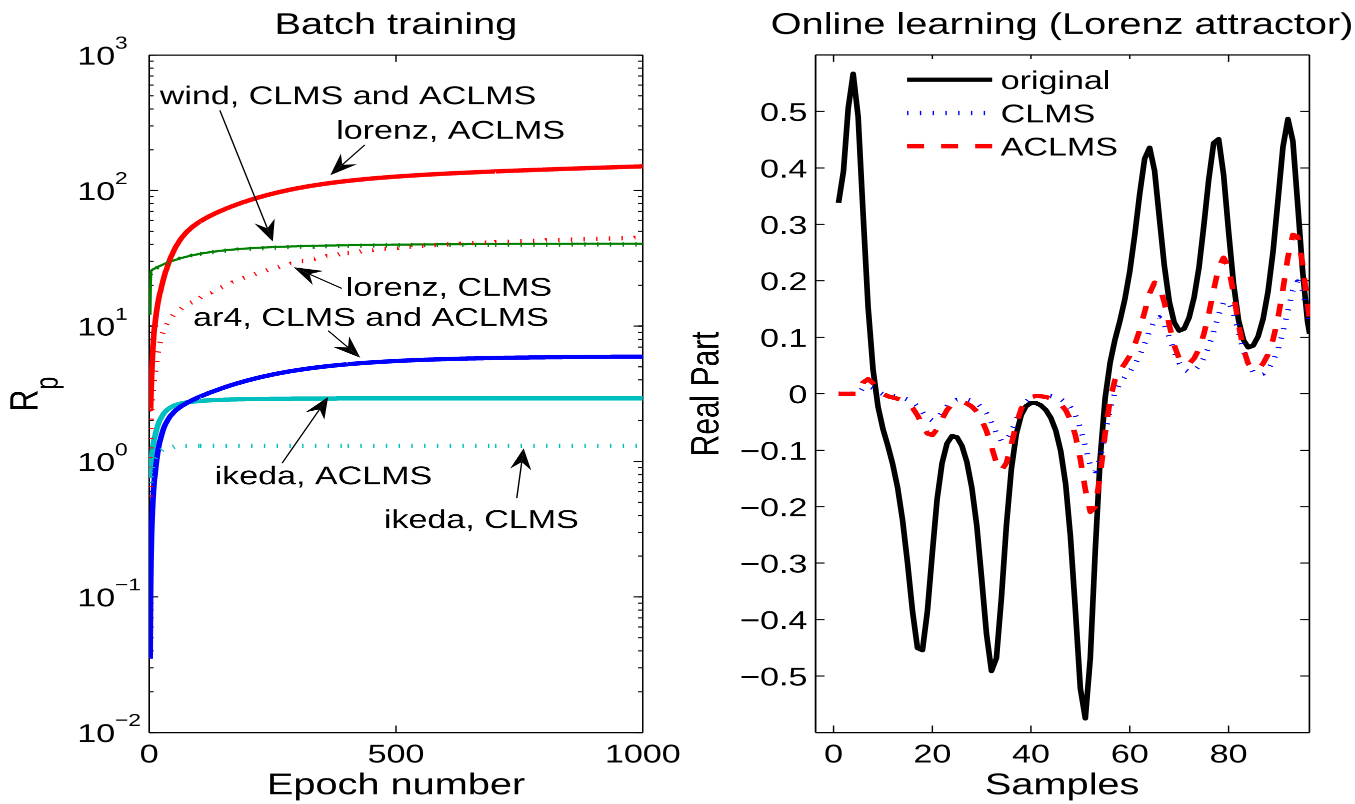
or in a more compact form (using augmented input and weight vectors)

$$\mathbf{w}^{\mathbf{a}}(\mathbf{k+1}) = \mathbf{w}^{\mathbf{a}}(\mathbf{k}) + \eta\mathbf{e}^{\mathbf{a}}(\mathbf{k})\mathbf{x}^{\mathbf{a}^*}(\mathbf{k})$$

where $\eta = \mu_h = \mu_g$, $\mathbf{w}^{\mathbf{a}}(\mathbf{k}) = [\mathbf{h}^{\mathbf{T}}(\mathbf{k}),\ \mathbf{g}^{\mathbf{T}}(\mathbf{k})]^{\mathbf{T}}$, $\mathbf{x}^{\mathbf{a}}(\mathbf{k}) = [\mathbf{x}^{\mathbf{T}}(\mathbf{k}),\ \mathbf{x}^{\mathbf{H}}(\mathbf{k})]^{\mathbf{T}}$, $e^a(k) = d(k) - \mathbf{x}^{\mathbf{a}^{\mathbf{T}}}(\mathbf{k})\mathbf{w}^{\mathbf{a}}(\mathbf{k})$ (Mandic et al. 2008).

Performance of ACLMS

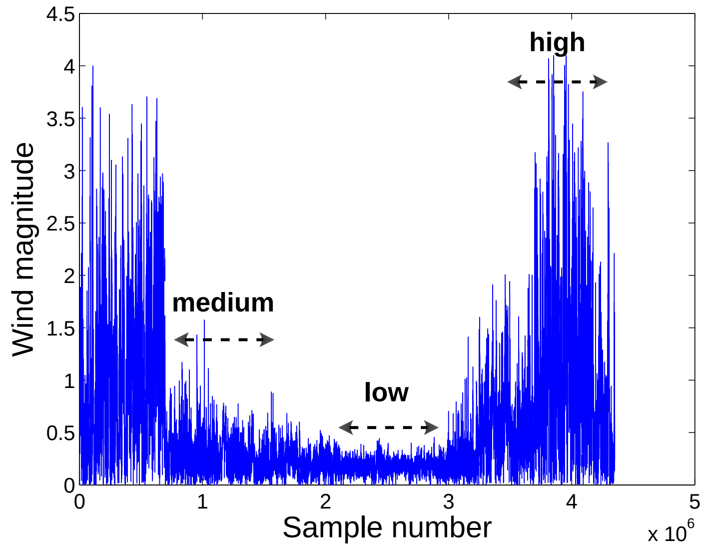
Evaluated for both second order circular (proper) and improper signals.



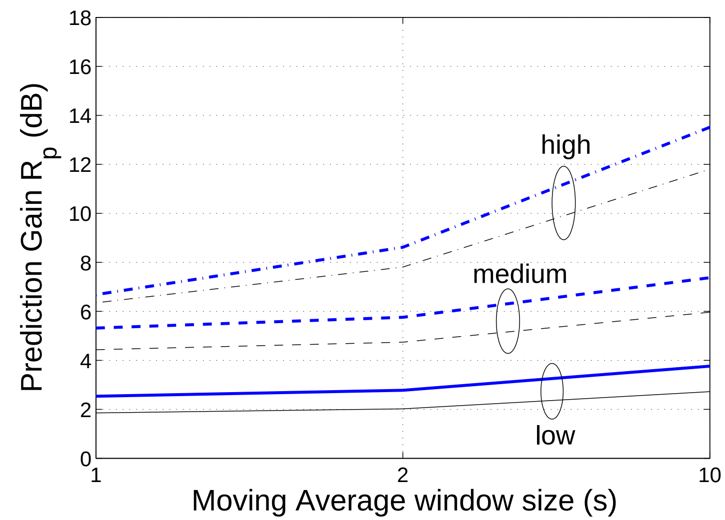
The ACLMS outperforms CLMS for second order noncircular signals.

Wind Modelling - Dynamics vs Circularity

Data recorded in an urban environment over one day



(o) Modulus of complex windover one day



(p) CLMS vs ACLMS for different wind regimes. CLMS - black, ACLMS - blue

Different wind regimes $\rightsquigarrow$ different dynamics,

$$v(k) = |v(k)|e^{\jmath\Phi(k)},\ |v| \text{ - speed, } \Phi \text{ - direction}$$

Different dynamics $\rightsquigarrow$ different circularity properties $\rightsquigarrow$ impact of ACLMS

The CRTRL vs ACRTRL

- **Complex Real Time Recurrent Learning (CRTRL)**

$$\pi_n^\star(k) = \Phi'^*(net(k))\left(u_n^*(k) + \sum_{l=1}^{N} w_{l+M+1}^*(k)\pi_n^\star(k-l)\right)$$

- **Augmented Complex Real Time Recurrent Learning (ACRTRL)**

$$\pi_{w_q}^\circ(k) = \Phi'(net(k))\left(\sum_{l=1}^{N} a_l(k)\pi_{w_q}^\circ(k-l) + \sum_{l=1}^{N} \alpha_l(k)\pi_{w_q}^\star(k-l)\right)$$

$$\pi_{w_q}^\star(k) = \Phi'^*(net(k))\left(u^*(k-q) + \sum_{l=1}^{N} a_l^*(k)\pi_{w_q}^\star(k-l) + \sum_{l=1}^{N} \alpha_l^*(k)\pi_{w_q}^\circ(k-l)\right)$$

The weight update becomes

$$\mathbf{w}^a(k+1) = \mathbf{w}^a(k) + \mu\big(e^*(k)\boldsymbol{\pi}^\circ(k) + e(k)\boldsymbol{\pi}^\star(k)\big)$$

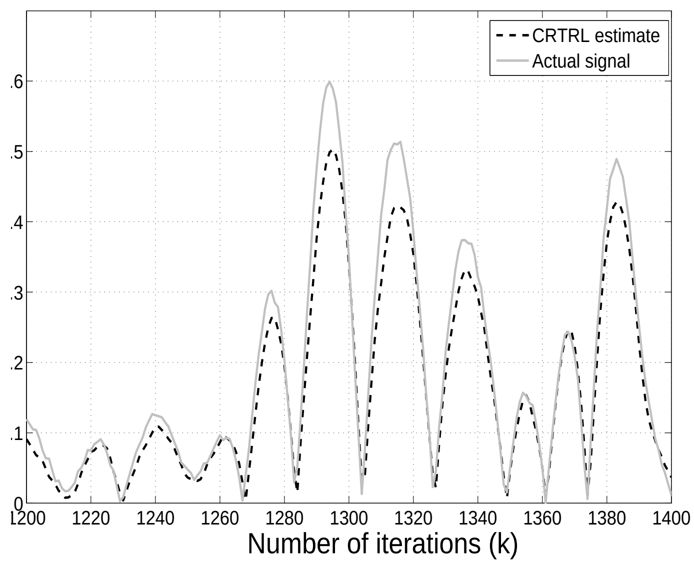
The extension to full RNNs if straightforward

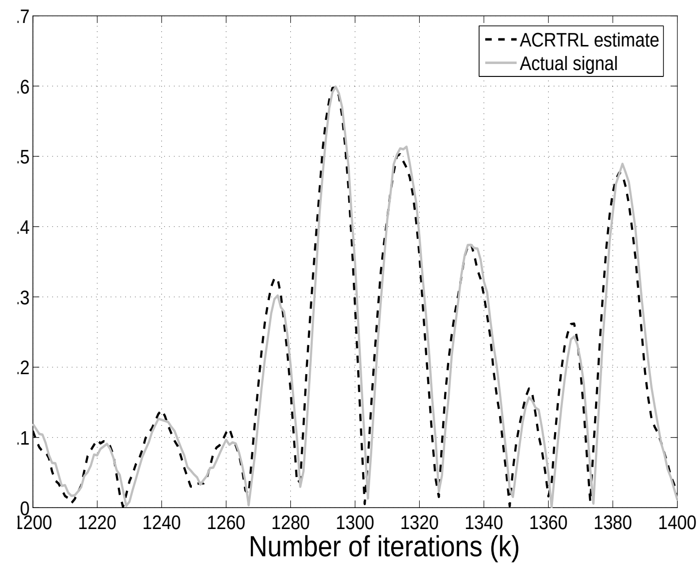
Simulation Results

Prediction gains $R_p = 10 log \sigma_x^2 / \sigma_e^2$ for circular and noncircular signals

Signal	Nonlinear	AR4 (noncirc)	AR4 (circ)	Wind	Radar
ACRTRL	3.91	4.10	3.6	9.80	9.45
CRTRL	3.76	3.54	3.6	6.32	7.22

One step ahead prediction of a complex radar signal [ICEX - S. Haykin website]

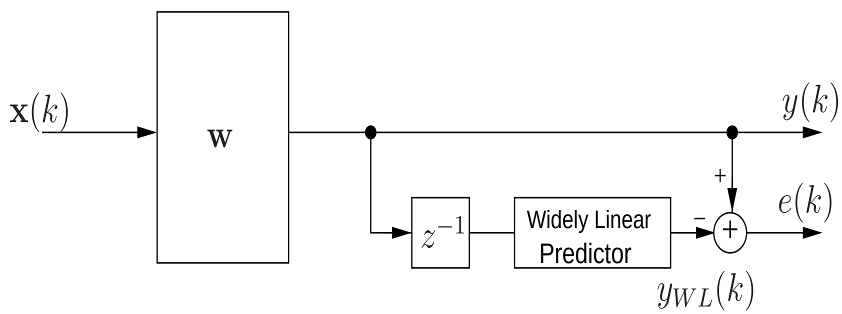




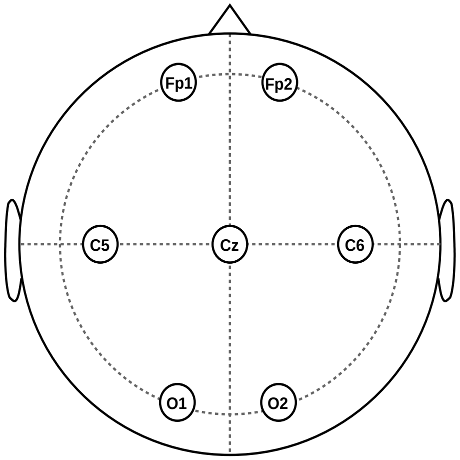
Standard CRNN (CRTRL) Widely linear RNN (ACRTRL)

Real World Example: BSE for EEG data

IEEE Transactions on CAS I, 2010 (Javidi, Cichocki, Mandic)



The Blind Source Extraction (BSE) scheme



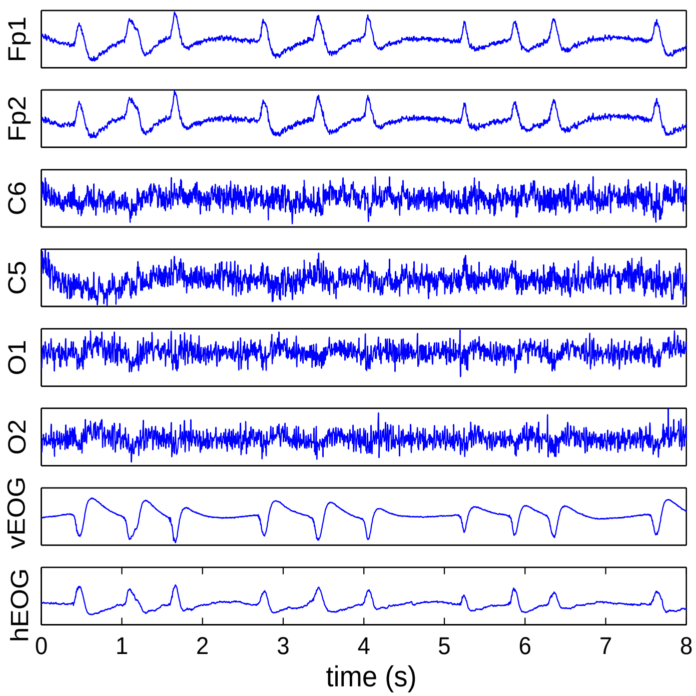
EEG electrode placement

$$\mathcal{J}_1(\mathbf{w}, \mathbf{h}, \mathbf{g}) = \frac{\mathbf{E}\{|\mathbf{e}(\mathbf{k})|^2\}}{\mathbf{E}\{|\mathbf{y}(\mathbf{k})|^2\}}$$

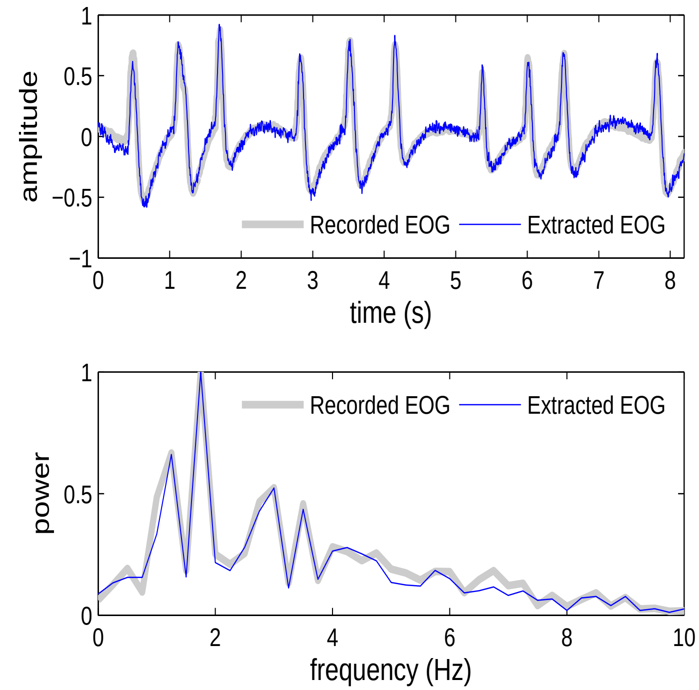
$$\mathbf{w}_{\mathbf{opt}} = \arg \max_{||\mathbf{w}||_2=1} \mathcal{J}_1(\mathbf{w}, \mathbf{h}, \mathbf{g})$$

Sources extracted based on the degree of WL predictability, and then removed from the mixtures.

Separation of EOG Artifacts from EEG



Recorded data



Extracted EOG artifact

◦ Excellent matching of the power spectra of the original and extracted signal (for visualisation - scaled to match the original)

◦ The algorithm operates in real time

The Existing Algorithms

What is currently out there?

- Augmented Statitics and Widely Linear Modelling: Neeser and Massey, Picinbono and Bondon, Amblard et al.
- Statistics being further developed by Scharf and Schreier, Picinbono and Chevalier, Walden
- Algorithms for communications by Schoeber et al., Koivunen, Erikkson, Olila
- Algorithms for Blind Source Separation: Douglas, Eriksson et al., Novey and Adali
- Algorithms for Beamforming: Delmas, Chevalier,
- Performance bounds: Delmas, Picinbono, Schreier
- Much work is needed to provide rigorous performance bounds and practical tests in various applications

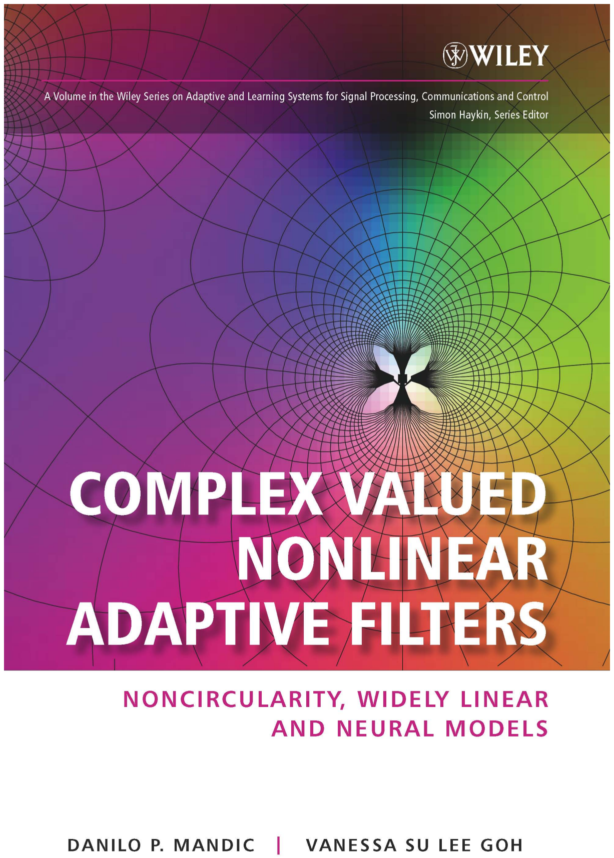
In Our Team We Have Developed

- Augmented LMS [Proc CIP 2008, Renewable Energy 2009]
- Augmented Kalman filter [Neural Computation 2007]
- Recursive algorithms for widely linear IIR filters [IEEE TSP 2009]
- Augmented Complex CRTRL for RNNs [Neural Networks 2007]
- Augmented affine projection algorithm [SP 2009]
- Augmented Echo State Networks [2008, 2010]
- Quaternion least mean square (QLMS), quaternion IIR filters, quaternion NNs [2009-]
- Widely linear quaternion model, QLMS, WL-QLMS, Augmented Q–Statistics [2008 - 2010]

Conclusions - Gains to be achieved

- Signal processing for vector sensors benefits from casting the problem into the complex (and quaternion) domain, and their division algebras;
- The mean square error of widely linear estimators is reduced for noncircular signals, whereas for circular signals the performance will be the same as that for standard models;
- Signal processing algorithms benefit from exploiting special matrix structures arising in augmented complex statistics, such as symmetries, diagonality, and subspace structure;
- Catering for complex noncircularity provides an additional degree of freedom, aiding the detection and separation algorithms;
- The uncertainty in estimation problems is reduced, as e.g. circular and noncircular noises can be separated, and the number of signals that we may resolve is incresed.

A Comprehensive Account of Widely Linear Modeling



- Unified approach to the design of complex valued adaptive filters and neural networks
- Augmented learning algorithms based on widely linear models
- Suitable for processing both second order circular (proper) and noncircular (improper) complex signals
- ACLMS, augmented Kalman filters, augmented CRTRL, linear and nonlinar IIR filters
- Adaptive stepsizes, dynamical range reduction, collaborative adaptive filters, statitical tests for the validity of complex representations

Thank you

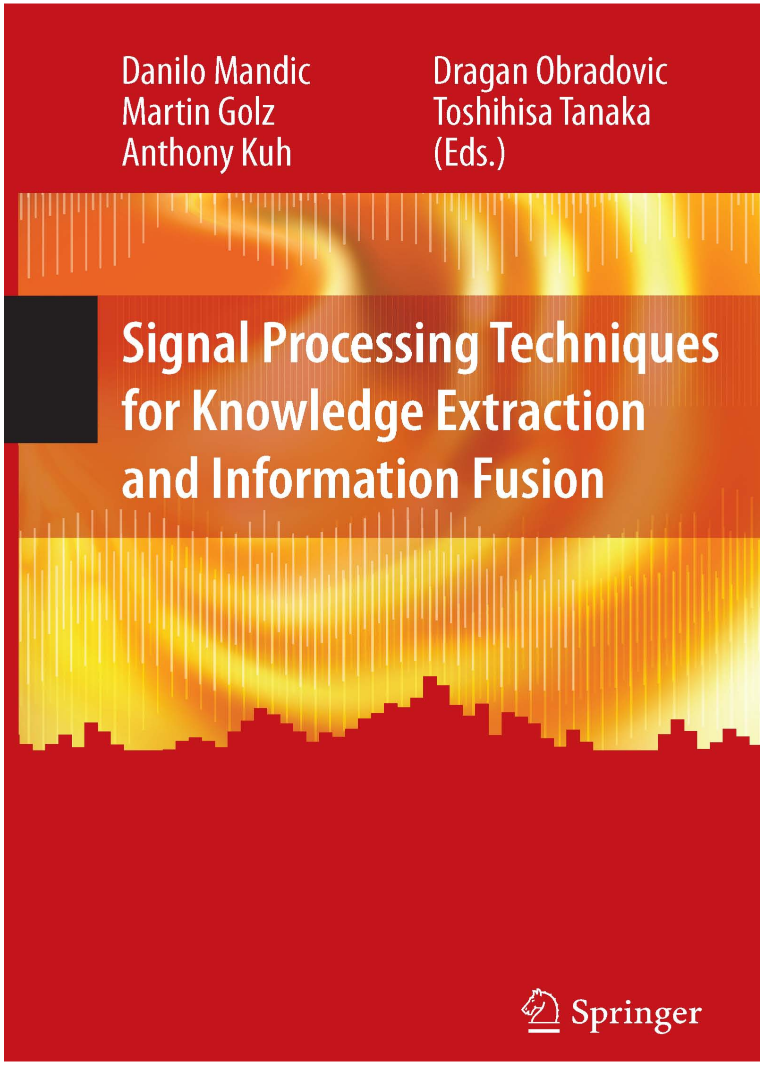
十分感谢！

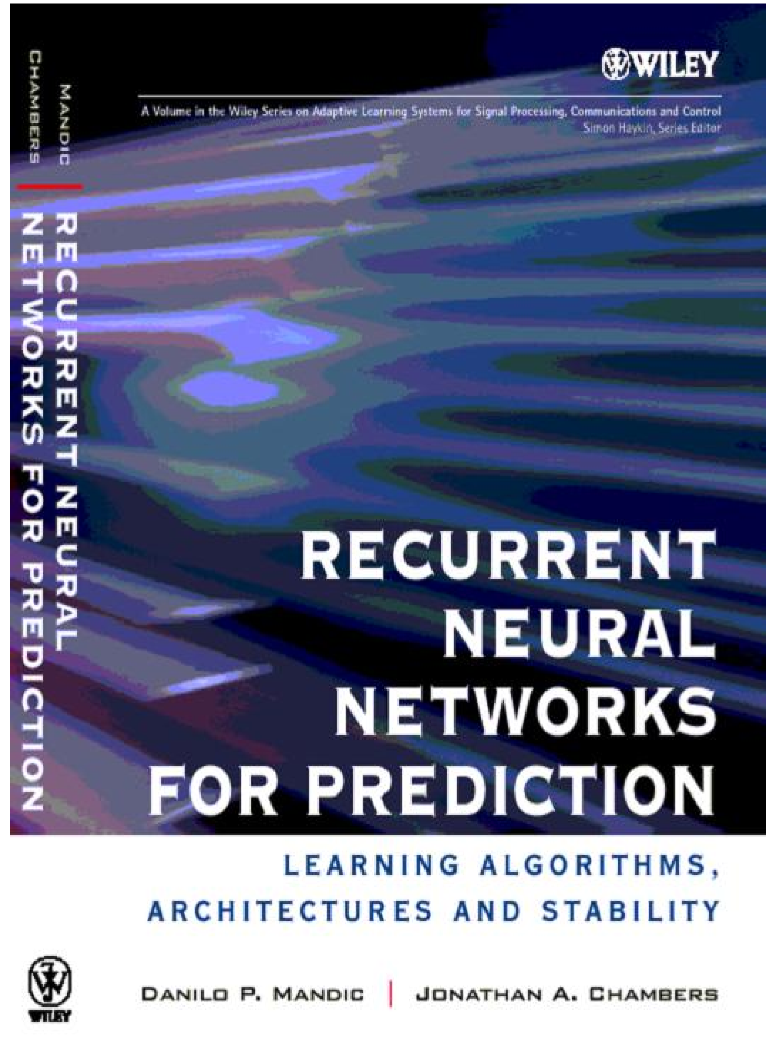
Conclusions

It is fitting to end this talk with the quote from Richard Penrose's *The Road to Reality: A Complete Guide to the Laws of the Universe*.

> *"We shall find that complex numbers, as much as reals, and perhaps even more, find a unity with nature that is truly remarkable. It is as though Nature herself is as impressed by the scope and consistency of the complex–number system as we are ourselves, and has entrusted to these numbers the precise operations of her world at its minutest scales."*

Some of Our Related Work





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