Toward bias minimization in acoustic feedback cancellation systems

Christos Boukis
Autonomic and Grid Computing Group, Athens Information Technology, Athens 19002, Greece

Danilo P. Mandic and Anthony G. Constantinides
Communications and Signal Processing Group, Department of Electrical and Electronic Engineering, Imperial College London, London, SW7 2BT United Kingdom

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A novel technique for bias suppression within acoustic feedback cancellation systems is proposed. This is achieved based on the use of all-pass filters in the forward part of the hearing aid. The poles of these filters are made time-varying, which results in a frequency response with constant magnitude and varying phase. This is a desired feature of the proposed approach, since the results from human psychoacoustics show that the human ear is not sensitive to moderate phase perturbations. The derivation of the proposed algorithms for the time variation of the location of the poles of all pass filters is based on a rigorous analysis of the phenomenon of bias in acoustic systems. Practical issues, such as the dependence of the steady-state error on the order of the all-pass filter, the number of varying poles, and their standard deviation are examined and strategies for the variation of the poles are introduced. Results obtained from a simulated hearing aid are provided to support the analysis. The quality of the processed audio signals is evaluated through subjective tests. © *2007 Acoustical Society of America.* [DOI: 10.1121/1.2431341]

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I. INTRODUCTION

A major drawback of hearing aids is the acoustic feedback from their receiver to their microphone, whereby a part of the acoustic signal emitted from the receiver propagates through the ventilation duct and is recaptured by the microphone.[1] This audio signal is then again processed and transmitted, which causes an acoustic feedback, a phenomenon detrimental to the performance which results in *echoes* and *howling*.[2] The suppression of this feedback is therefore crucial. In order to improve the quality of the emitted audio signal and to increase the maximum allowable gain (MAG).[3]

The most efficient current approach for feedback cancellation is based on the modeling of the feedback path with a finite impulse response (FIR) filter in order to produce feedback estimates which are subsequently subtracted from the microphone signal.[4,5] To cope with the variability of the acoustic feedback path, least squares (LS) stochastic gradient descent (SGD) algorithms have been used[6,7] for the training of this FIR filter. Despite their widespread use, these algorithms provide biased feedback estimates[8–10] since

(1) the data used for the optimization of the filter coefficients are collected in closed loop, which renders the standard independence assumptions ineffective,[11]
(2) the actual estimation error, that is, the difference between the feedback signal and its estimate, is not readily available.

Alternatively, we can use techniques based on the decorrelation between the input and the output of a hearing aid. These result in more accurate estimates of the feedback path. This decorrelation can be achieved either by disconnecting the forward path of the hearing aid when the coefficients of the digital filter are being adapted,[12] or by introducing delays in the forward and/or the cancellation path of a hearing aid.[13] The former technique implies noncontinuous adaptation of the coefficients. Its major drawback is the need to estimate instances when the values of the coefficients of the filter need to be updated. Moreover, there is a trade-off between the adaptation time and the achieved perceptual quality. The introduction of delays, on the other hand, implies a continuous adaptation of the coefficients of the adaptive filter. The degree of bias suppression critically depends on both, the statistics of the input signal and the properties of the feedback path.[8]

To that cause we propose a novel bias suppression technique based on the use of an all-pass filter with time-varying coefficients in the forward path of an acoustic feedback cancellation system.[14] We show, both analytically and through simulations, that this technique has the potential to achieve greater bias reduction than the existing approaches, and that it is more robust to the changes in the statistics of the input signal and the characteristics of the feedback path.[15] Practical issues, such as the effect of the parameters of the all-pass filter, like its order and the number of the varying poles, on the performance of the acoustic feedback cancellation are evaluated and several approaches for the variation of the filter poles are examined. Simulation results on both recorded and synthetic signals are provided to support the analysis.

II. HEARING AID DESCRIPTION

A hearing aid, whose block diagram is shown in Fig. 1, typically consists of the forward path $A(z)$, whose objective is to amplify appropriately the captured audio signals, the acoustic feedback path $G(z)$ which propagates the receiver's signal to the microphone and the feedback cancellation path

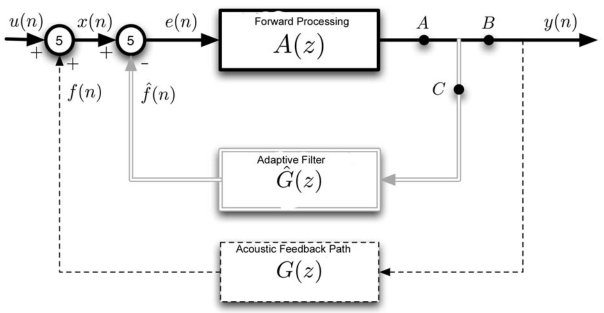

FIG. 1. Block diagram of a hearing aid with an integrated feedback cancelling adaptive filter. The dashed line indicates the inaccessible part of the system, i.e., the acoustic path.

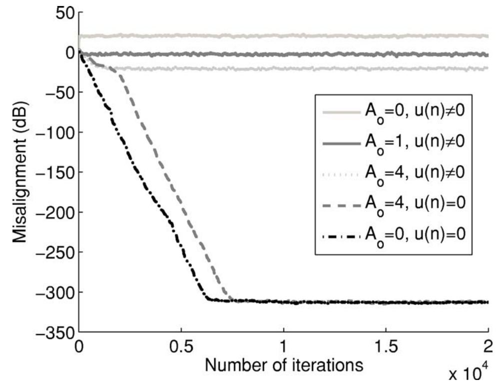

FIG. 2. The performance of the adaptive feedback cancelling filter for five distinct cases.

$\hat{G}(z)$ which estimates the parameters of the feedback path.[13] For convenience, in our analysis the forward acoustic channel[16] is neglected. The involved signals are the external input denoted by $u(n)$, the feedback and its estimate $f(n)$ and $\hat{f}(n)$, respectively, the microphone signal $x(n)$, the signal after feedback removal $e(n)$, and the output of the receiver $y(n)$.

The most significant components of the processing unit $A(z)$ of the forward path are the automatic gain control (AGC) unit, which is adjusted according to the audiogram of the user, and the amplifier. A delay z^{-d}, where $d \geq 1$, is also introduced so as to avoid having a closed loop without a delay, a mathematically ill-posed problem. Since the AGC and the transfer functions of the microphone and the receiver have known and fixed values,[17] our focus is solely on the identification of the acoustic feedback path.[16] Thus in our simulations we assume that $A(z)=A_o z^{-d}$.

III. ADAPTIVE FEEDBACK CANCELLATION

Since a part of the signal produced by the receiver leaks back to the microphone, the actual transfer function of a hearing aid,

$$T(z) \triangleq \frac{y(z)}{u(z)} = \frac{A(z)}{1 - A(z)G(z)}, \tag{1}$$

differs substantially from its intended transfer function $A(z)$. A common approach for the suppression of the undesired feedback signal is to include an FIR filter in the hearing aid, placed in parallel to the feedback path as illustrated in Fig. 1. The order of this feedback cancelling filter should be large enough to avoid the undermodeling of the feedback path, and its output is given by

$$\hat{f}(n) = \sum_{i=0}^{N-1} \hat{g}_i(n) y(n-i) = \hat{\boldsymbol{g}}^t(n)\boldsymbol{y}(n), \tag{2}$$

where $\hat{\boldsymbol{g}}(n)=[\hat{g}_0(n), \hat{g}_1(n), \ldots, \hat{g}_{N-1}(n)]^t$ are its coefficients, $\boldsymbol{y}(n)=[y(n), y(n-1), \ldots, y(n-N+1)]^t$ is the output regressor vector and $(\cdot)^t$ the matrix transpose operator. The transfer function of the hearing aid now becomes

$$T(z) = \frac{A(z)}{1 - A(z)[G(z) - \hat{G}(z)]}. \tag{3}$$

To cope with the variability of the acoustic feedback path, the coefficients of the feedback cancelling filter are adapted with a SGD algorithm[18] that aims at the minimization of the square of the error $e(n)=u(n)-\hat{f}(n)$, resulting in[12]

$$\hat{\boldsymbol{g}}(n+1) = \hat{\boldsymbol{g}}(n) + \mu e(n)\boldsymbol{y}(n), \tag{4}$$

where μ is a constant learning rate. Notice that for the derivation of (4) the dependence of the output $y(n)$ on the coefficients of the adaptive filter is deliberately neglected. This is, strictly speaking, not correct, especially for large learning rate values.[19] In order to facilitate adaptation a low-power noisy signal $w(n)$ can be added to the output $y(n)$, yielding

$$y(n) = A_o e(n-1) + w(n). \tag{5}$$

Thus, when the data are collected in a closed loop, the optimal solution in the LS sense is given by[13]

$$\hat{\boldsymbol{g}}^{\star}_{\mathrm{cl}} = E\{\boldsymbol{y}(n)\boldsymbol{y}^t(n)\}^{-1}[E\{u(n)\boldsymbol{y}(n)\} + E\{f(n)\boldsymbol{y}(n)\}] \tag{6}$$

and by

$$\hat{\boldsymbol{g}}^{\star}_{\mathrm{ol}} = E\{\boldsymbol{w}(n)\boldsymbol{w}^t(n)\}^{-1}E\{f(n)\boldsymbol{w}(n)\} \tag{7}$$

when the data are collected in open loop, that is for interrupted forward path $A(z)=0$. Comparing (6) with (7) and noting that SGD algorithms converge in the mean to unbiased solutions when they perform in open loop,[20] the bias of the feedback cancelling filter in the steady state becomes

$$\boldsymbol{q}^{\star} = \boldsymbol{g}^{\star}_{\mathrm{cl}} - \boldsymbol{g}^{\star}_{\mathrm{ol}}. \tag{8}$$

During the derivation of (6) and (7) it was assumed that the input $u(n)$ and the injected noise $w(n)$ are uncorrelated. The dependence of the output $y(n)$ on the coefficients of the feedback cancelling filter was also neglected in the derivation of (6).

The misalignment curves of the adaptive feedback cancelling filter for several values of the amplification A_o and both in the presence and absence of the input signal $u(n)$ are shown in Fig. 2. Observe that the steady-state error depends primarily on the external input $u(n)$ and to a lesser extent on the gain A_o [it is a function of A_o only when $u(n) \neq 0$]. The value of gain A_o, however, affects the settling time. The acoustic feedback path was approximated by the first 100 samples of the impulse response of ear canal measured on a KEMAR mannequin,[21] and for its identification an FIR filter of the same order was employed. The normalized LMS (NLMS) algorithm was employed for the adaptation of the filter coefficients (with learning rate $\mu=1$). The input $u(n)$ was a zero mean and unit variance random signal with Gaussian distribution. To facilitate adaptation 60 dB of white noise were added to the output $y(n)$.

IV. BIAS REDUCTION WITH TIME-VARYING ALL-PASS FILTERS

From the analysis of the previous section it is concluded that when operating in closed loop the adaptive filter produces biased estimates of the acoustic feedback path. As a consequence the acoustic feedback is not removed completely; a residual feedback signal is always present. Reducing the bias in the estimates of the acoustic channel results in reduced residual feedback.

A straightforward approach toward bias suppression, and thus more efficient feedback cancellation, is to interrupt the forward path ($A_o=0$) when adapting the filter coefficients. Moreover, if the coefficients of the filter are adapted in the absence of input then unbiased feedback estimates are produced.[5] However, this method is not preferable in acoustic feedback cancellation systems and especially in hearing aids, since the procedure of interrupting the forward path and feeding the receiver with noise might be very irritating to the user.

Bias suppression in continuously adapting feedback cancelling systems can be accomplished by introducing delays in the forward or the cancellation path of the hearing aid.[13,22] The fundamental idea of this approach is to decorrelate the signal $u(n)$ from the output $y(n)$. These decorrelating delays can be placed in the points A, B, or C of the hearing aid depicted in Fig. 1 and they can be either constant or time varying.[19] Delays in the forward path (point A or B in Fig. 1) suppress the bias only for colored inputs $u(n)$. Delays in the cancellation path (point C in Fig. 1) compensate for the inherent delay in the acoustic feedback path. Thus bias suppression is successful, irrespective of the characteristics of the input signal. If the introduced delay is larger than the actual delay within the feedback path, this results in an increase instead of reduction of the steady-state error,[23] and pre-echo or "*comb*"-filter effects can be observed.[24]

To this end we propose an approach based on all-pass filters with time-varying poles in the forward path, for which the transfer function is given by

$$H_M(z,\boldsymbol{\alpha}(n)) = \frac{z^{-M} + \sum_{i=1}^{M} r_i(n) z^{M-i}}{1 + \sum_{i=1}^{M} r_i(n) z^{-i}} = \prod_{i=1}^{M} \frac{z^{-1} - \alpha_i(n)}{1 - \alpha_i(n) z^{-1}}, \tag{9}$$

where $\boldsymbol{\alpha}(n)=[\alpha_1(n), \alpha_2(n), \ldots, \alpha_M(n)]^t$ are the values of the varying poles at time instant $n, r_i(n)$ the coefficient values ($i=1,2,\ldots,M$) and M the order of the filter as shown in Fig. 3.

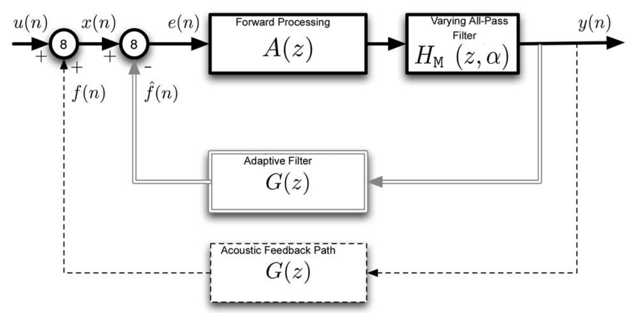

FIG. 3. Block diagram of a hearing aid with a feedback cancelling adaptive filter and a varying all-pass filter in the forward path.

The transfer function of the proposed system from Fig. 3 in the $\mathcal{Z}$ domain becomes

$$T(z,n) = \frac{A(z) H_M(z,\boldsymbol{\alpha}(n))}{1 - A(z) H_M(z,\boldsymbol{\alpha}(n))[G(z) - \hat{G}(z)]}. \tag{10}$$

Since the all-pass filter $H_M(z,\boldsymbol{\alpha}(n))$ is time varying the transfer function of the system will be varying even when the adaptive filter reaches steady state, introducing some form of "*controlled*" nonstationarity to the system.

The output of the system is no longer given by

$$y(n) = A_o e(n-1),$$

but instead by the recursive equation

$$y(n) = -\sum_{i=1}^{M} r_i(n) y(n-i) + A_o e(n-M-1) + A_o \sum_{i=1}^{M} r_i(n) y(n-M-1+i) \tag{11}$$

which results in an increase of the computational complexity by $2M$ multiplications and $2M$ additions. For example, when a first-order all-pass filter is used, we have

$$y(n) = \alpha(n) y(n-1) + A_o e(n-2) - A_o \alpha(n) e(n-1). \tag{12}$$

Equation (12) can be written in a vector-matrix form as follows:

$$\boldsymbol{y}(n) = \boldsymbol{A}(n)\boldsymbol{y}(n-1) + A_o \boldsymbol{e}(n-2) - A_o \boldsymbol{A}(n)\boldsymbol{e}(n-1), \tag{13}$$

where $\boldsymbol{A}(n)$ is an $(N \times N)$ diagonal matrix given by

$$\boldsymbol{A}(n) = \begin{bmatrix} \alpha(n) & 0 & \cdots & 0 \\ 0 & \alpha(n-1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha(n-N+1) \end{bmatrix}. \tag{14}$$

This method is in agreement with an earlier result[9] where it is stated that "*The system can be made identifiable with* $w(n)=0$ *if the signal processing unit* $A(z)$ *is nonlinear or time-variant in such a way that the autocorrelation matrix of the output* $y(n)$ *becomes nonsingular*." Namely, since the magnitude of the frequency response of this all-pass filter is flat and also remains constant, pole variations result in phase variations. This method can therefore be seen as an attempt to decorrelate the input from the output of a hearing aid by adding noise to their phase. An advantage of this approach is the preservation of the perceptual quality, since the human ear is insensitive to low-scale phase changes.[25]

A. Steady-state analysis

The introduction of the all-pass filter $H_M(z,\alpha(n))$ in the forward path of the system, modifies the output signal $y(n)$, rendering it more "random" and the corresponding correlation matrices nonsingular. Since the transfer function of the system is time varying, even when the adaptive filter has reached steady state, its output $y(n)$ will be nonstationary and the steady-state analysis provided in Sec. III only applies approximately due to the time-varying nature of the optimal coefficient. For example, when a first-order all-pass filter is introduced in the forward path the terms $E\{u(n)\boldsymbol{y}(n)\}$, $E\{f(n)\boldsymbol{y}(n)\}$, and $E\{\boldsymbol{y}(n)\boldsymbol{y}^t(n)\}$ become, respectively,

$$E\{u(n)\boldsymbol{y}(n)\} = E\{u(n)\boldsymbol{A}(n)\boldsymbol{y}(n-1)\} + A_o E\{u(n)\boldsymbol{e}(n-2)\} - A_o E\{u(n)\boldsymbol{A}(n)\boldsymbol{e}(n-1)\}, \quad (15)$$

$$E\{f(n)\boldsymbol{y}(n)\} = E\{f(n)\boldsymbol{A}(n)\boldsymbol{y}(n-1)\} + A_o E\{f(n)\boldsymbol{e}(n-2)\} - A_o E\{f(n)\boldsymbol{A}(n)\boldsymbol{e}(n-1)\}, \quad (16)$$

and

$$\begin{aligned} E\{\boldsymbol{y}(n)\boldsymbol{y}^t(n)\} &= E\{\boldsymbol{A}(n)\boldsymbol{y}(n-1)\boldsymbol{y}^t(n-1)\boldsymbol{A}(n)\} + A_o E\{\boldsymbol{e}(n-2)\boldsymbol{y}^t(n-1)\boldsymbol{A}(n)\} - A_o E\{\boldsymbol{A}(n)\boldsymbol{e}(n-1)\boldsymbol{y}^t(n-1)\boldsymbol{A}(n)\} \\ &+ A_o E\{\boldsymbol{A}(n)\boldsymbol{y}(n-1)\boldsymbol{e}(n-2)\} + A_o^2 E\{\boldsymbol{e}(n-2)\boldsymbol{e}^t(n-2)\} - A_o^2 E\{\boldsymbol{A}(n)\boldsymbol{e}(n-1)\boldsymbol{e}^t(n-2)\} - A_o E\{\boldsymbol{A}(n)\boldsymbol{y}(n-1)\boldsymbol{e}(n \\ &- 1)\boldsymbol{A}(n)\} - A_o^2 E\{\boldsymbol{e}(n-2)\boldsymbol{e}(n-1)\boldsymbol{A}(n)\} + A_o^2 E\{\boldsymbol{A}(n)\boldsymbol{e}(n-1)\boldsymbol{e}(n-1)\boldsymbol{A}(n)\}. \end{aligned} \quad (17)$$

From these equations no direct conclusion about the effect of the introduced all-pass filter, or the variation of its poles, on the steady-state error can be drawn. Intuitively, we expect a decrease in the value of the terms that contain the random pole, which is proportional to its variance.

B. Pole variation

Three approaches for the variation of the poles of the all-pass filter within the forward path are proposed. A simple approach would be to make poles $\alpha_i(n)$ vary randomly around a fixed value $\alpha_{o,i}$ according to[14]

$$\alpha_i(n) = \alpha_{o,i} + \sigma_i v_i(n), \quad (18)$$

where $v_i(n)$ is a stochastic process with zero mean, unit variance and Gaussian or uniform distribution, and σ_i is a constant that specifies the variance of the ith pole.

Alternatively, a recursive formula can be employed, based on a convex combination of the previous pole value and its random displacement, which is given by

$$\alpha_i(n+1) = \lambda\alpha_i(n) + (1-\lambda)\sigma_i v_i(n), \quad (19)$$

where σ_i and $v_i(n)$ have the same meaning as in (18) and $\lambda \in (0,1)$ is a convex parameter that controls the "*randomness*" within the update of the pole location.

Finally, in order to minimize the square of the cross correlation $\xi(n)$ between the input $u(n)$ and the output $y(n)$ of the hearing aid we may apply a stochastic gradient descent approach, that updates the poles $\alpha_i(n)$ toward the direction (Appendix A)

$$\frac{\partial\xi(n)}{\partial\alpha_i(n)} = \left(-\sum_{k=1}^{M}\frac{\partial r_k(n)}{\partial\alpha_i(n)}y(n-k) + A_o e(n-M) + A_o\sum_{k=1}^{M}\frac{\partial r_k(n)}{\partial\alpha_i(n)}e(n+M-k)\right)e^2(n)y(n) \quad (20)$$

for $i=1,2,\ldots,N$, where $r_i(n)$ are the coefficients and $\alpha_i(n)$ the poles of the introduced all-pass filter (Eq. (9)). Since the magnitude of the poles of the all-pass filter should be less than unity (for stability reasons), the following hard-bounding formula is applied:

$$\alpha_i(n+1) = \begin{cases} 0.9 & \alpha_i(n+1) > 0.9, \\ \alpha_i(n+1) & 0.9 \geq \alpha_i(n+1) \geq -0.9, \\ -0.9 & \alpha_i(n+1) < -0.9. \end{cases} \quad (21)$$

A comparison of the effect of the pole updating formula on the performance of the adaptive feedback cancelling filter is given in Fig. 4. Observe that the use of adaptive poles

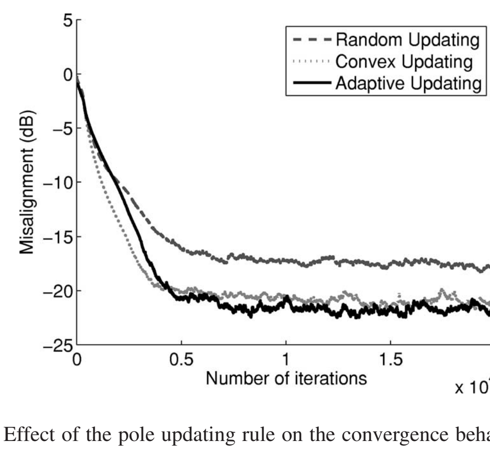

FIG. 4. Effect of the pole updating rule on the convergence behavior of the adaptive filter.

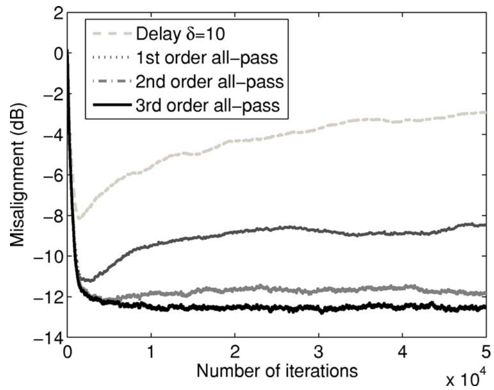

FIG. 5. Misalignment curves for speech-shaped input signals for a delay and an all-pass filter in the forward path.

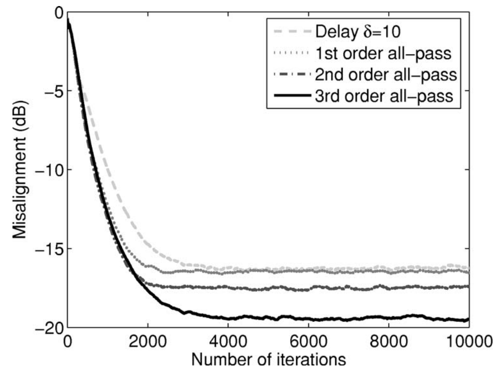

FIG. 6. Misalignment curves for white noise input for the cases of forward path delay and all-pass filter.

results in lower steady-state error. Notice that all-pass filters with adaptive poles do not require any *a priori* information since their coefficients converge provided a small step size. Moreover, they have the ability to cope with nonstationary environments more effectively. For the derivation of this graph a first-order all-pass filter with the following parameter values was used: $\alpha(0)=\alpha_o=-0.2$, $\lambda=0.2$, $\eta=10^{-8}$, $\sigma=0.2$, and $v(n)\sim\mathcal{N}(0,1)$.

V. SIMULATIONS AND DISCUSSION

Two sets of simulations were conducted with the aim to (i) illustrate the benefits of the proposed bias reduction approach over existing methods and (ii) examine the effect of the parameters of the introduced all-pass filter on the performance of the feedback cancelling filter.

A. Experimental setup

The forward path processing unit had a transfer function of the form $A(z)=A_o z^{-1}$, with $A_o=4$. The acoustic feedback path was approximated by the first 100 samples of the impulse response of an ear canal measured on a KEMAR mannequin and sampled at 22 kHz. To avoid the situation of undermodeling, the feedback cancelling filter was an FIR filter of the same order. For the adaptation of its coefficients the NLMS algorithm was employed, with $\mu=0.1$. The misalignment was used as a metric for the evaluation of the performance of the adaptive filter, defined as

$$v(n)=\sqrt{[\hat{\mathbf{g}}(n)-\mathbf{g}_o]^t[\hat{\mathbf{g}}(n)-\mathbf{g}_o]}, \tag{22}$$

where $\hat{\mathbf{g}}(n)$ is the coefficient vector of the adaptive filter and $\mathbf{g}_o$ are the samples of the impulse response of the acoustic feedback path. Notice that since it is assumed that the acoustic feedback path can be adequately modeled by this adaptive filter, both vectors had the same length.

B. Varying all-pass filter vs delay in the forward path

The convergence behavior of the adaptive filter was assessed for the cases of a delay and an all-pass filter in the forward path for colored noise (Fig. 5), white noise (Fig. 6), and speech (Fig. 7) input. The transfer function of the all-pass filter is given by (9). Unless stated otherwise, its poles were varying according to (20), with $\eta=10^{-8}$, $\alpha_i(0)=-0.2$, $\sigma_i=0.2$, and $v_i(n)$ were zero mean and unit variance random processes with Gaussian distribution.

From Fig. 5 it is observed that a reduction in the bias of 10 dB is achieved when in the forward path a third-order all-pass filter is used instead of a delay of z^{-10}. The input was colored noise derived by passing white noise of zero mean and unit variance through a stable autoregressive model of the form

$$K(Z)=\frac{1}{1-1.79z^{-1}+1.85z^{-2}-1.27z^{-3}+0.41z^{-4}}. \tag{23}$$

Time-varying all-pass filters hold another strong advantage over forward delays: they reduce the steady-state bias even for white noise input $u(n)$. This is illustrated in Fig. 6 where it is shown that a third-order all-pass filter achieves approximately 5 dB reduction of the steady-state error compared to a forward path delay.

In Fig. 7 the performance improvement achieved by the proposed use of varying all-pass filters over the standard method with delays is illustrated in terms of bias reduction for the case of speech input signal. This speech signal was recorded in a typical office room with a DAT machine at a sampling rate of 48 kHz and it was downsampled to 24 kHz. From this plot it is observed that the introduction of a third-order all-pass filter with a varying pole in the forward path

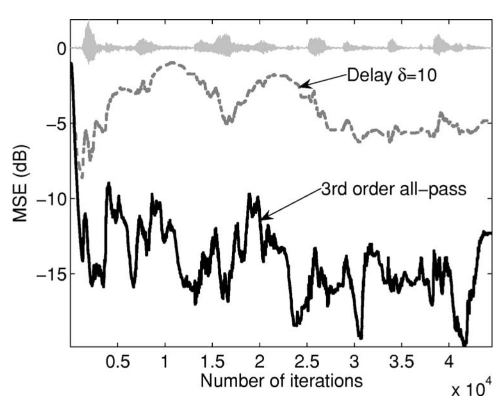

FIG. 7. Misalignment curves of a system with a delay and a system with an all-pass filter in the forward path, for speech input signal. The input signal is scaled and given within an offset in order to assess the performance of the feedback cancelling adaptive filter.

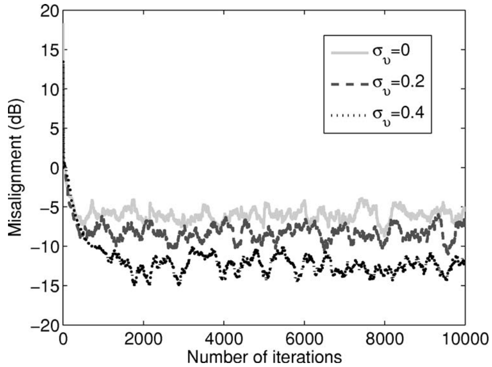

FIG. 8. Dependence of the steady-state bias on the variance of the poles of the all-pass filter.

can reduce the steady-state bias up to 10 dB. Notice that significant reduction in the bias was achieved mainly during silent intervals of the speech, indicating that the filter coefficient adaptation can be suspended or relaxed during the voiced intervals.

Notice also that the introduction of time-varying all-pass filters does not exclude the existence of delays in the forward path; on the contrary these can be combined to achieve even lower steady-state misalignment and thus more accurate acoustic feedback path estimates.

C. Effect of the parameters of the all-pass filter on the bias

The amount of bias in a hearing aid supplied with an adaptive feedback cancelling filter and an all-pass filter with varying poles is mainly a function of the variance of the poles of the all-pass filter. This can be verified from Fig. 8 where it is shown that the higher the variance of the poles the lower the bias value. Stochastic poles with uniform instead of Gaussian distribution were also tested and the results were found to be similar to those of Fig. 8.

From Fig. 5 and Fig. 6 it is concluded that the steady-state error is inversely proportional to the order of the decorrelating all-pass filter. The mean value of the poles did not have significant impact, since the steady-state misalignment was not sensitive to $\alpha_{o,i}$.

Finally in Fig. 9 it is illustrated that even a single varying pole can achieve a bias reduction that is comparable to

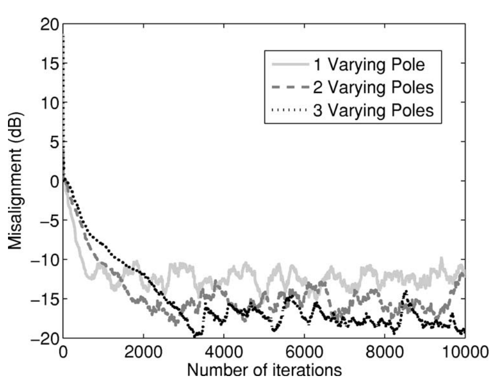

FIG. 9. Convergence behavior of the adaptive filter as a function of the number of varying poles for a third order decorrelating all-pass filter.

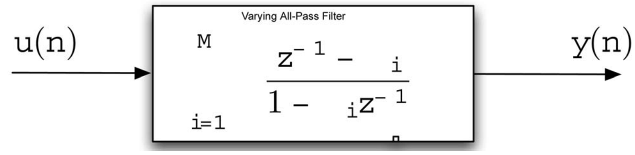

FIG. 10. The setting used for the direct effect of an all-pass filter on an audio signal $u(n)$.

the case were all the poles are varying. This can be explained from Viéta's formulas,[26] which imply that a single time-varying pole forces all the coefficients of the all-pass filter to vary, and thus it decorrelates (up to a satisfying degree) the input $u(n)$ from the output $y(n)$ of the hearing aid. For the derivation of the curves of Figs. 8 and 9, randomly updated poles according to (18) were employed with mean value $\alpha_o = -0.2$.

VI. SIGNAL QUALITY

A major task of audio processing systems is the preservation and/or enhancement of the quality of the processed signals. In other words, the elimination of the undesired echoes and ringing effects, which is accomplished by suppressing the acoustic feedback, should not be accompanied by noticeable distortion or degradation of the amplified audio signals.

To gain insight into the effect of the proposed all-pass-filters-based processing on the quality of the audio signals, their performance was further evaluated via subjective tests with 17 participants. The quality of the audio signals was evaluated using a slightly modified version of the ITU-R.BS.1116-1 impairment scale (Fig. 11), to measure the extent of the distortion.[27] The audio signals were reproduced with the use of headphones in order to simulate the hearing aid conditions more accurately.

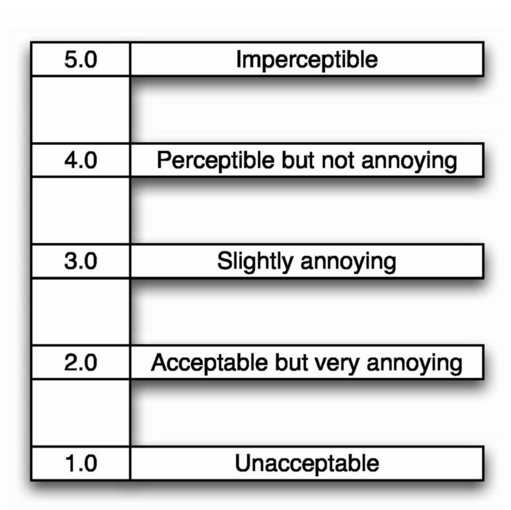

FIG. 11. The modified version of the ITU-R.BS.1116-1 impairment scale that was used in our subjective tests.

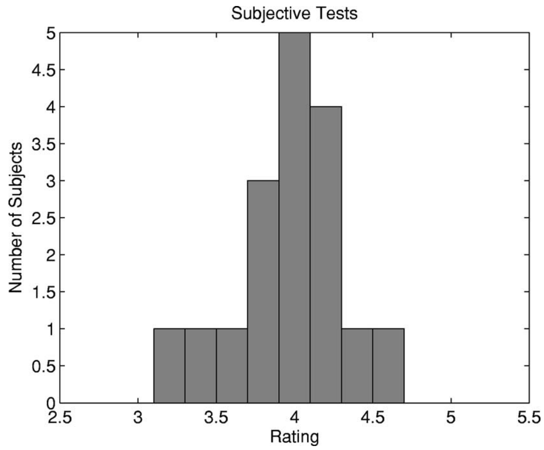

FIG. 12. Histogram of the results of subjective tests for the evaluation of the direct impact of a varying all-pass filter on an audio signal.

A. Direct impact

To assess the direct impact of a varying all-pass filter on an audio signal, a recorded speech signal sampled at 22 KHz, was filtered with a third-order all-pass filter (Fig. 10) whose coefficients were varying randomly according to (19), where $\lambda=0.2$ and $\sigma_i=0.5$ for every pole $(i=1,2,3)$. Each subject listened to the input $u(n)$ and the output $y(n)$ separately and was asked to assign a number from 1 to 5 to the latter, indicating whether the artifacts added by the processing were noticeable or not. The results are illustrated in the histogram in Fig. 12. This clearly shows that our proposed varying filters add some noticeable artifacts to the output signal, but not perceptually annoying. Listeners described the perceived distortion as a barely noticeable hiss. From the spectrograms of the input and the output signal (Fig. 13) observe the close match between the two; this is due to the introduced distortion having significantly smaller power than the original signal as it appears only during speech intervals, rendering it imperceptible during silence periods.

B. Impact on a hearing aid

For the evaluation of the effect of a varying all-pass filter on the quality of the output signal of a hearing aid, listeners were asked to compare the output of the system of Fig. 1 and the output of the system depicted in Fig. 3 to the signal $\mathcal{F}^{-1}\{A(z)\}\otimes u(n)$ which is the desired output. The input signal $u(n)$ was a recorded speech signal sampled at 22 KHz and the forward processing unit was a simple amplifier described by $A(z)=A_o z^{-1}$, $A_o=6$. The varying all-pass filter was of third-order and its poles were adapted according to the scheme presented in Appendix A.

From the results of the subjective tests, illustrated in Figs. 14 and 15, it is concluded that the participating subjects felt that the introduction of a varying all-pass filter improved the quality of the output of the hearing aid. Listeners claimed that the output of the hearing aid that included an all-pass filter was more clear than the output of the system of Fig. 1 since the metallic timbre and the echoes that are inherent in the output of the latter did not appear in the output of the system of Fig. 3. This is also supported by the spectrograms from Fig. 16, which show that the output of the hearing aid is closer to the desired output $A_o u(n)$ when a varying all-pass filter is introduced in the forward path of the system.

Therefore although all-pass filters introduce a noticeable, but not annoying hiss to the audio signals, especially during voiced intervals, they improve the overall performance of a hearing aid. More specifically they allow for

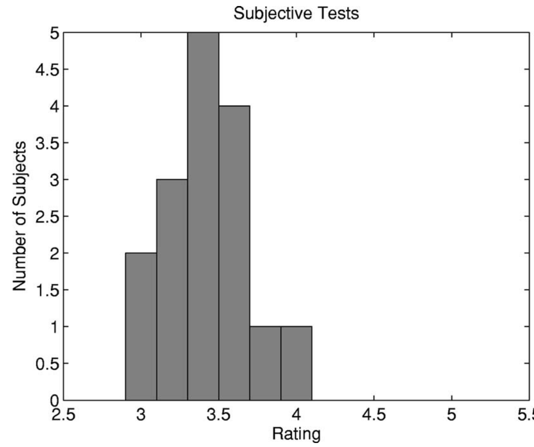

FIG. 14. Subjective tests for the evaluation of the output of the hearing aid whose block diagram is depicted in Fig. 1.

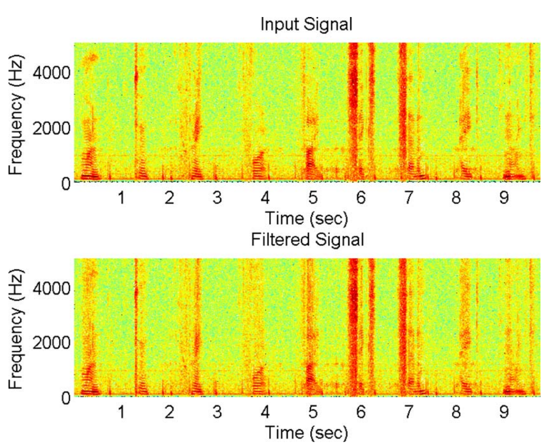

FIG. 13. (Color online) Spectrogram of the input and the output of the system of Fig. 10.

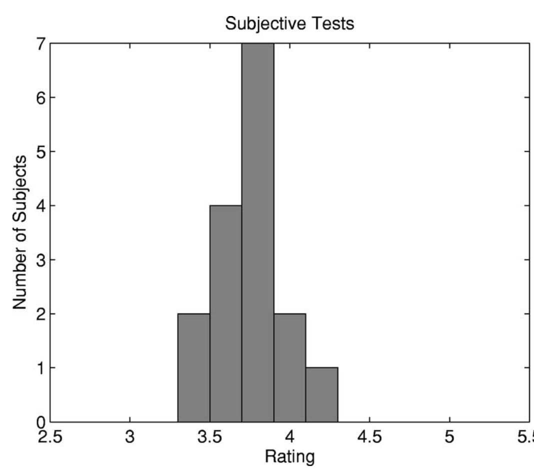

FIG. 15. Subjective tests for the evaluation of the output of the hearing aid whose block diagram is depicted in Fig. 3.

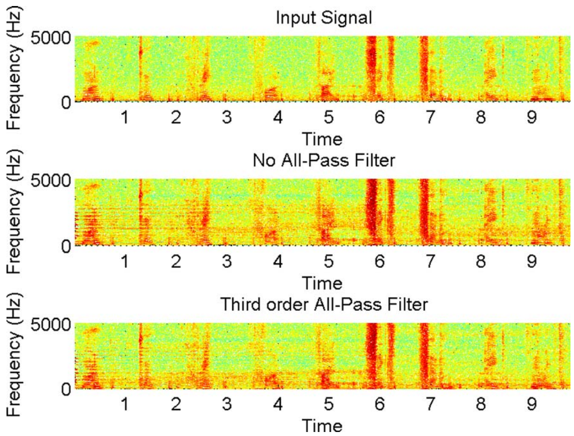

FIG. 16. (Color online). Spectrograms of the output of a hearing aid with and without an all-pass filter in its forward path.

more precise identification of the feedback path and thus for a remarkable suppression of the echoes and the metallic timbre of the output of a hearing aid.

VII. CONCLUSIONS

A novel technique for the reduction of bias in acoustic feedback cancellation systems has been proposed. This approach, which is based on the use of an all-pass filter in the forward path of a hearing aid (or any other system with acoustic feedback), has been shown to out-perform the existing ones in terms of steady-state error. Moreover, its behavior does not depend on the characteristics of the input signal. Several policies for the variation of the poles have been examined, and a method for the adaptation of the pole values has been proposed that aims at the minimization of the input-output cross correlation. The effect of the choice of the parameters of this all-pass filter on the steady-state error was studied. Experimental results from a simulated hearing aid support the analysis. Subjective tests were also conducted to assess the quality of the processed audio signals.

APPENDIX A: ADAPTING THE POLE OF THE ALL-PASS FILTER

Since the purpose of the introduced all-pass filter is to decorrelate the output of the hearing aid $y(n)$ from its input $u(n)$ an adaptive pole can be employed which aims at the minimization of the input-output cross correlation given by

$$\xi(n) = \frac{1}{2} r_{uy}^2(n) = \frac{1}{2} E\{u(n)y(n)\}^2, \qquad \text{(A1)}$$

and is updated at every time instant n according to

$$\alpha_i(n+1) = \alpha_i(n) + \eta_i \frac{\partial \xi(n)}{\partial \alpha_i(n)}. \qquad \text{(A2)}$$

Approximating the expectation of the product $u(n)y(n)$ with its instantaneous value and differentiating both sides of (A1) yields

$$\frac{\partial \xi(n)}{\partial \alpha_k(n)} = \left(\sum_{i=1}^{M} \frac{\partial r_i(n)}{\partial \alpha_k(n)} y(n-i) - A_o \sum_{i=1}^{M} \frac{\partial r_i(n)}{\partial \alpha_k(n)} e(n-M+i) \right) u^2(n) y(n). \qquad \text{(A3)}$$

Since $u(n)$ is not available its estimate $e(n)$ can be used instead resulting in

$$\frac{\partial \xi(n)}{\partial \alpha_k(n)} = \left(-\sum_{i=1}^{M} \frac{\partial r_i(n)}{\partial \alpha_k(n)} y(n-i) + A_o e(n-M) + A_o \sum_{i=1}^{M} \frac{\partial r_i(n)}{\partial \alpha_k(n)} e(n+M-i) \right) e^2(n) y(n) \qquad \text{(A4)}$$

when a varying all-pass filter of order M is introduced in the forward path with a transfer function given by (9). The coefficients $r_i(n)$ and the poles $\alpha_i(n) (i=1,2,\ldots,M)$ are related according to Viéta's formulas.[26] For $M=1$ (A4) becomes

$$\frac{\partial \xi(n)}{\partial \alpha(n)} \approx [y(n-1) - A_o e(n-1)] e^2(n) y(n) \qquad \text{(A5)}$$

and the pole adapts according to

$$\alpha(n+1) = \alpha(n) + \eta(y(n-1) - A_o e(n-1)) e^2(n) y(n) + \sigma v(n), \qquad \text{(A6)}$$

where η is the learning rate, $v(n)$ is a random variable of zero mean and unit variance, and σ is the standard deviation. This stochastic term is added to guarantee that the pole will vary even when steady state is reached. The derivation of recursive equations for the adaptation of the poles of higher-order all-pass filters is a straightforward procedure.

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