The Journal of Physiological Sciences Advance Publication by J-STAGE
doi:10.2170/physiolsci.RP004708
This version is to be replaced by the final version after page-setting and proofing.

REGULAR PAPER

Comparison Analysis of Embedding Dimension between Normal and Epileptic EEG Time Series

Ye YUAN[1], Yue LI[1], and Danilo P. MANDIC[2]

[1]Department of Information and Engineering, Jilin University, Changchun, China; and [2]Department of Electrical and Electronic Engineering, Imperial College, London, UK

Received on Mar 24, 2008; accepted on Jun 25, 2008; released online on Jun 28, 2008; doi:10.2170/physiolsci.RP004708

Correspondence should be addressed to: Yue Li, Department of Information and Engineering, Jilin University, 71#, Nanhu Campus, Jilin University, Changchun, 130012, China. Tel: +86-0431-8515-2020, E-mail: liyue84@gmail.com

Abstract: The embedding dimension of normal and epileptic electroencephalogram (EEG) time series is analyzed by two different methods, Cao's method and differential entropy method respectively. The results of the two methods indicate consistently that the embedding dimension of EEG signals during seizure changes and becomes different from that of normal EEG signals, and the embedding dimension varies intensively during seizure, whereas the embedding dimension of normal EEG signals keeps stable basically. The embedding dimension results also reflect the variation of freedom degree of human brain nonlinear dynamic system (NDS) during seizure. In addition, it is found based on the results of Cao's method that normal EEG signals are of some degree of randomness, whereas epileptic EEG signals has determinism. [The Journal of Physiological Sciences 58(4), 2008, in press]

Key words: electroencephalogram (EEG), embedding dimension, Cao's method, differential entropy, epilepsy.

The electroencephalogram (EEG) is the most often used signal for the analysis of the epileptic seizure activity of the brain. Epilepsy, from which approximate 1% of the people in the world suffer, is a group of brain disorders characterized by the recurrent paroxysmal electrical discharges of the cerebral cortex, that result in irregular disturbances of the brain functions, which are associated with the significant changes of the EEG signal [1, 2]. The neuronal network composing the brain is spatially extended and thus EEG signals capture spatial local electrical activity in the brain [3], it is known that biological neurons can be modeled by a set of nonlinear differential equations. The minimal embedding dimension gives the upper number of nonlinear dynamic system (NDS) freedom degrees and the minimal number of differential equations demanded for mathematical modeling of NDS [4]. Therefore, the change of the structure of brain NDS during seizure can be shown by the change of embedding dimension of EEG signals if the human brain is considered as a nonlinear dynamic system. Study on EEG signals has drawn more and more attention [5–8]. In the past, many works have been already done in analyzing epileptic EEG based on nonlinear dynamic methods [9–14], for example, Xiaoli Li *et al.* (2006) proposed epileptic seizure prediction based on mutual information and Cao's method [9], and L. Diambra *et al.* (1999) presented a technique for automatic detection of epileptic spikes based on an optimal embedding dimension [10]. All these methods can clearly show the difference between normal and epileptic EEG signals based on nonlinearity or statistical information; nevertheless they do not show the change of the structure of brain NDS during seizure.

As the EEG signal is time-varying, dynamical measures should be computed within certain time scale, for which the local stationarity assumption is valid. If too long statistics to quantify the EEG are used, then the local information will be washed away, and most of the temporal characteristics will be deprived from their diagnostic values.

In this paper, we will investigate the difference of embedding dimension between normal and epileptic EEG signals through the use of two methods, Cao's method and differential entropy method [15, 16]. The reason why two kinds of methods are applied to analyze the embedding dimension of normal and epileptic EEG signals is that there has not been a criterion for determining the exact value of embedding dimension of a real-world time series up to now, and we hope to compare the results from two different methods for more accuracy to avoid the influence of the method itself on the results.

EEG TIME SERIES

The raw EEG time series used in this paper were recorded by a Nihon Kohden EEG recorder (model 7310; Nihon Kohden Inc., Tokyo, Japan). The electrodes were placed on scalp to record 16 channels of EEG data according to the international standard 10–20 system. Data were digitized with a 12-bit analog-to-digital converter at a sampling rate of 200 Hz and stored on a computer hard disk. We recorded 80-second-long EEG data, thus 16,000 sample points for each channel. Two groups of EEG time series were obtained, both of which were collected from the same 39-year-old male patient. Volunteer was relaxed in an awake state with eyes closed. One group was recorded when the patient was normal, and the other group was recorded when the patient was during induced epileptic seizure. Some of the EEG signals used in this paper are shown in Fig. 1, (a) and (b). From Fig. 1 (b) it can be observed that epileptic EEG time series show epileptic characteristic such as spike discharge and slow waves.

METHODS

Cao's method. To determine the embedding dimension of the time series $x_1, x_2, \cdots, x_N$ by Cao's method, firstly compute

$$E_1(d) = E(d+1)/E(d) \qquad (1)$$

and

$$E_2(d) = E^*(d+1)/E^*(d) \qquad (2)$$

secondly, plot $E_1(d)$ and $E_2(d)$ versus dimension d, and find the dimension d_0 where $E_1(d)$ stops changing from the plot, then d_0 is the minimum embedding dimension we look for; here, $E_2(d)$ is used to determine whether a time series is random or not. The steps and explanation in detail of Cao's method are shown in Appendix 1. The delay time τ which is needed for the computation of embedding dimension by Cao's method is determined by mutual information method [17].

Differential entropy method. A method based on differential entropy for determining the optimal embedding parameters is introduced in Ref. [16], which employs a single criterion-the "entropy ratio" between the phase space representation of a signal and ensemble of its surrogates. The advantage of the differential entropy method is that it determines the embedding dimension m and delay time τ together, whereas other methods, for example, another method used in this paper—Cao's method—need to determine the embedding dimension after the delay time is determined, if the delay time were too small to cover the minimal time span needed to capture the dynamics of a signal, the tap input length-embedding dimension would become rather large, resulting in an increased complexity of training; in turn, if delay time is greater than optimal, the nature of the resulting model becomes too discrete, resulting in a failure of the filter to capture the underlying signal dynamics, hence the need for an optimization method to jointly determining the embedding dimension and delay time [16].

The entropy ratio (ER) introduced in Ref. [16] is:

$$R_{ent}(m,\tau) = I(m,\tau)(1+\frac{m\ln N}{N}) \qquad (3)$$

Plot $R_{ent}(m,\tau)$ versus m and τ, then the minimum of the plot of the entropy ratio yields the optimal set of embedding parameters, namely optimal embedding dimension m and delay time τ. The steps and explanation in detail of differential entropy method are shown in Appendix 2.

The Journal of Physiological Sciences Advance Publication by J-STAGE
doi:10.2170/physiolsci.RP004708
This version is to be replaced by the final version after page-setting and proofing.

SIMULATIONS AND RESULTS

How the simulations are organized. If the dynamic system is invariant, then the determination of embedding dimension can be made on any segment of the measured signal. This is not true in a time-varying environment. As the EEG time series are time-varying, they have to be divided into several small segments for computing embedding dimension as time evolves. The data we used here are 16-channel normal and epileptic EEG signals, the length of the signal is 80s for each channel, the sampling frequency is 200Hz, and therefore there are 16,000 points for each channel signal. Each channel signal is divided into 16 segments, namely $S_1, S_2, \ldots, S_{16}$, each segment $S_i (i = 1,2,\ldots,16)$ is of 1,000 points. Thus we can obtain 16 embedding dimension values for each channel signal. As for the 16 channels of normal or epileptic EEG time series, we can get a 16 × 16 matrix built by the embedding dimension values respectively.

Results of Cao's method. For this paper, as one plot will be produced corresponding to every EEG segment by Cao's method to determine the embedding dimension, we cannot list all the plots here, we take the channel corresponding to scalp electrode F3 from 16-channel normal and epileptic EEG data respectively for example, and the embedding dimension plots produced by Cao's method are shown in Fig. 2, in which the plots in the left side corresponds to normal EEG segments and the plots in the right side correspond to epileptic EEG segments. It can be observed from the first plot in the left side in Fig. 2 that E_1 curve is in a rising process before $d = 10$ and stops changing after $d = 10$ (i.e., after $d = 10$, E_1 curve becomes into a straight line), thus the corresponding embedding dimension of this segment is 10, and in a similar way the embedding dimension of the EEG segment corresponding to the first plot in the right side in Fig. 2 is 16.

Cao's method is applied to compute the embedding dimension of $S_i (i = 1,2,\ldots,16)$. The results of embedding dimensions for normal and epileptic EEG time series obtained by Cao's method are shown in Table 1, in which for each cell the upper number corresponds to the embedding dimension value of the epileptic EEG segment and the lower number corresponds to the embedding dimension value of the normal EEG segment. Let E_{ij} ($i, j = 1,2,\ldots,16$) stand for the value of the point of the matrix built by the embedding dimension values, whose coordinates are (*i*, *j*). The average values are computed for each row and line of the matrix. Let AV_i stand for the average value of the *i*th row of the matrix, namely $AV_i = \frac{\sum_{j=1}^{16} E_{ij}}{16}$; let AV_j stand for the average value of the *j*th line of the matrix, namely $AV_j = \frac{\sum_{i=1}^{16} E_{ij}}{16}$, the results of AV_i and AV_j are shown in Table 1. Apparently, the value of AV_i stands for the embedding dimension of each channel of 16-channel EEG signals, and the value of AV_j stands for the embedding dimension of the 16-channel EEG signals at some corresponding time if the 16-channel EEG signals are regarded as a whole. The variances are computed for each row and line of the matrix as well, let Var_i and Var_j stand for the variance of the *i*th row and *j*th line of the matrix, respectively. As shown in Table 1, variances of embedding dimensions of epileptic EEG time series are much larger than those of normal EEG time series, which means that embedding dimension of EEG time series varies more fiercely during seizure than that of normal EEG time series. The average value of embedding dimension (ED) of EEG signals can be written as

$$ED = \frac{\sum_{i=1}^{16}\sum_{j=1}^{16} E_{ij}}{16 \times 16},$$

therefore it can be obtained that the average value of embedding dimension of the normal EEG signals is $ED_{normal} = 8$ and the average value of embedding dimension of the epileptic EEG signals $ED_{epileptic} = 17$, namely the average value of embedding dimension of epileptic EEG signals is over 2 times larger than that of normal EEG signals. The RMSE (Root mean square error) is computed for the results of the embedding dimensions of normal and epileptic EEG signals, the results are $RMSE_{normal} = 0.1753$ and $RMSE_{epileptic} = 0.3091$, which also reflects that the embedding dimension of epileptic EEG signals varies more intensely during seizure than that of normal EEG signals does.

Figure 3 is plotted based on the results of embedding dimension shown in Table 1. As shown in Fig.3, the curves constructed by the embedding dimension values of normal and epileptic EEG time series form two fluctuation belts, but the fluctuation belt corresponding to embedding dimensions of epileptic EEG time series is much wider than that corresponding to embedding dimensions of normal EEG time series, which implies that the embedding dimension of epileptic EEG signals has a bigger variation interval than normal EEG signals. The AV_j curves (the curve marked by black points) corresponding to normal and epileptic both fluctuate around the ED lines, but the AV_j curve corresponding to epileptic EEG signals fluctuates more violently than that corresponding to normal EEG signals, which also means that the embedding dimension of epileptic EEG signals varies more intensely during seizure than that of normal EEG signals does.

Results of differential entropy method. An example about how to determine the optimal embedding parameters of an EEG segment is shown in Fig. 4. As the lowest point of the plot, whose X axis is embedding dimension m and Y axis is delay time τ, appears at (2, 7), the optimal embedding parameters of this EEG segment can be determined as $m_{opt} = 2$ and $\tau_{opt} = 7$.

In a similar way with Cao's method, differential entropy method is applied to compute the embedding dimension of EEG segments $S_i (i = 1,2,\ldots,16)$. The results of the embedding dimension determined by differential entropy method are shown in Table 2, and Fig. 5 is plotted based on the results. Figure 5 shows the same rule with Fig. 3, which is plotted based on the results of Cao's method, namely that the embedding dimension of epileptic EEG signals has a wider variation belt than that of normal EEG signals, the embedding dimension of epileptic EEG signals varies between 2 and 5 considerably, whereas the embedding dimension of normal EEG signals keeps constant basically. In Table 2, AV_i and AV_j are constant for the normal EEG signals, whose definitions are the same as those defined in "Results of Cao's method" Section of this paper. AV_i and AV_j vary between 2 and 3 for epileptic EEG signals. Var_i and Var_j are also computed for the results of embedding dimension shown in Table 2. The RMSE obtained based on the results obtained by differential entropy method are $RMSE_{normal} = 0.0132$ and $RMSE_{epileptic} = 0.0562$. The results of variances and RMSE both indicate that the embedding dimension of EEG signals from an epileptic varies with time more intensely during seizure than that from a normal person does. The average value of embedding dimension of normal and epileptic EEG signals are $ED_{normal} = 2$ and $ED_{epileptic} = 3$ respectively.

DISCUSSIONS

The results of the methods both show that the embedding dimension values of epileptic EEG signals vary intensively during seizure, whereas the embedding dimension values of normal EEG signals keep stable basically; the embedding dimension of EEG signals becomes much larger during seizure than that of normal EEG signals, the average value of embedding dimension of epileptic EEG signals is over 2 times larger than that of normal EEG signals based on the results of Cao's method, the results of differential entropy method also show that the embedding dimension of epileptic EEG signals is larger than that of normal EEG signals. Although there is great difference between the results of the two methods, namely that the embedding dimension results obtained by differential entropy method are much lesser than those obtained by Cao's method, the phenomena that the results of the two methods show are consistent, namely that the embedding dimension of EEG signals during seizure becomes larger than that of normal EEG signals, and the embedding dimension of EEG signals varies with time fiercely during seizure and has a bigger variation interval than that of normal EEG signals, whereas the embedding dimension of normal EEG signals keeps stable basically.

An interesting phenomenon can be found in Fig. 2 that normal EEG signals are of randomness, whereas epileptic EEG signals are of determinism, since based on Cao's method, for random data, the future values are independent of the past values, thus $E_2(d)$ will be equal to 1 for any d in this case, however, for deterministic data, $E_2(d)$ is related to d, as a result, it cannot be a constant for all d, in other words, there must exist some $d's$ such that $E_2(d) \neq 1$[15]. For the left-hand plots corresponding to normal EEG signals in Fig. 2, except 2 or 3 plots in which $E_2(d)$ varies slightly around 1, others all show that after some slight variation at the beginning, $E_2(d)$ converges to 1 quickly, but we still cannot say that normal EEG signals are totally random, since $E_2(d)$ is not equal to 1 for all ds; for the right-hand plots corresponding to epileptic EEG signals in Fig. 2, we can say that epileptic EEG signals are of strong determinism, since $E_2(d)$ in the right-hand plots keeps unequal to 1 and varies fiercely. The determinism of epileptic EEG time series implies that there exists some connection between future values and past values of epileptic EEG time series and the EEG time series during seizure can be predicted. At present, there still exist arguments on whether the EEG is random or deterministic, but the mainstream holds that the EEG is generated by a deterministic chaotic process [18]. The results of this paper verify that epileptic EEG time series are of determinism. Although the results of this paper show that normal EEG time series are of randomness, yet we cannot say that normal EEG time series are completely random, since different methods have different fitness for different conditions such as low or high-dimensional determinism [18] and $E_2(d)$ corresponding to normal EEG time series is not always equal to 1 (see Fig. 2). Based on the results of this paper, we can only determine that epileptic EEG time series are of stronger determinism than normal EEG time series. Moreover, based on the results of this paper, a potential means for detecting epileptic seizure can also be found, that is, the difference of determinism between normal and epileptic EEG time series could be helpful for detection of epileptic seizure.

If the human brain is considered as a NDS, the minimal embedding dimension gives the upper number of NDS freedom degrees and the minimal number of differential equations demanded for mathematical modeling of NDS. The results of embedding dimension show that epileptic EEG time series are of higher degree of freedom than normal EEG time series. Although we cannot obtain the accurate mathematical model of human brain NDS, there must be relationship between the NDS and its output, and the property of a NDS can be showed by its output. Normal and epileptic EEG time series have different embedding dimensions, which also indicates that normal and epileptic may correspond to different NDSs, i.e., the NDS of human brain changes during seizure, which is meaningful for mathematical modeling for EEG time series.

The results of the two methods used in this paper are quite different; we didn't discuss which method is more accurate, since the purpose of this paper is to study the difference of embedding dimension between normal and epileptic EEG signals. However, it is recommended to set the embedding dimension of epileptic EEG signals 17 and embedding dimension of normal EEG signals 8 for the general processing of EEG time series, we are apt to believe that the results obtained by Cao's method are closer to the true values, since the embedding dimension represents the minimal number of differential equations demanded for mathematical modeling of NDS, the embedding dimension results of differential entropy method are mainly around 2 or 3, which seems too small for mathematical modeling of a real-world signal.

For the EEG data we used, not all the embedding dimension values of epileptic EEG segments are larger than those of normal EEG segments. We obtained 256 pairs of embedding dimension values of normal and epileptic EEG segments for the two methods respectively; we compared each pair of embedding dimension values E_{ij} ($i, j = 1, 2, \ldots, 16$) (the definition of E_{ij} can be found in "Results of Cao's method" Section in this paper) that are of the same indices i and j, and it is found that for the results of Cao's method, there are 237 pairs $ED_{epileptic} > ED_{normal}$, 9 pairs $ED_{epileptic} = ED_{normal}$ and 10 pairs $ED_{epileptic} < ED_{normal}$. In a similar way, for the results of differential entropy method, the statistical results are 96, 152 and 8 respectively (the embedding dimension values in detail obtained by the two methods are shown in Table 1 and Table2 respectively). There exists the phenomenon that $ED_{epileptic} < ED_{normal}$ for both the methods, however, it is thought that this phenomenon cannot prevent us from getting the impression that the embedding dimension of EEG time series during seizure becomes larger than that of normal EEG time series, since only 3.91% and 3.13% out of 256 pairs of embedding dimension values show $ED_{epileptic} < ED_{normal}$ for Cao's method and differential entropy method respectively.

Based on the results of this paper, normal and epileptic EEG time series show different embedding dimensions and degree of determinism, therefore, we consider that embedding dimension, and even degree of determinism, have potential application value in clinical diagnosis of epilepsy through further research. In future work, we will further investigate the difference of embedding dimensions among EEG data from different types of seizure and different monitored subjects (including different age, sex etc).

CONCLUSIONS

Cao's method and differential entropy method were applied to compute the minimum embedding dimension of normal and epileptic EEG time series respectively. The phenomena that the results of the two methods show are consistent: the embedding dimension of epileptic EEG signals is much larger than that of normal EEG signals, even up to 2 times according to the results of Cao's method; the embedding dimension of EEG signals varies fiercely during seizure, whereas the embedding dimension of normal EEG signals keeps stable basically; and the embedding dimension of epileptic EEG signals has a bigger variation interval than that of normal EEG signals. The variation of the embedding dimension of EEG signals during seizure also means the variation of the freedom degree of the human brain NDS, namely that the freedom degree of human brain NDS increases during seizure. In addition, based on the results of Cao's method, it is found that normal EEG signals are of some degree of randomness, whereas epileptic EEG signals are of strong determinism, which implies that epileptic EEG signals can be predicted well. Therefore, it is proposed that the embedding dimension can be a supplementary parameter for the epileptic seizure characterization.

APPENDIX

1. Cao's method

Suppose that $x_1, x_2, \cdots, x_N$ is a time series, the embedding dimension can be determined by Cao's method as follows:

Reconstruct the time series like time delay vectors in phase space:

$$Y_i(d) = [x(i), x(i+\tau), x(i+2\tau), \cdots, x(i+(d-1)\tau)], \quad \text{i} = 1, 2, \cdots, N-(d-1)\tau \qquad \text{(A.1)}$$

where d is the embedding dimension and τ is the time delay. $Y_i(d)$ is the *i*th reconstructed vector and $Y_i^{NN}(d)$ as the nearest neighbor of $Y_i(d)$ in embedding dimension d as follows:

$$Y_i^{NN}(d) = [x^{NN}(i), x^{NN}(i+\tau), x^{NN}(i+2\tau), \cdots, x^{NN}(i+(d-1)\tau)] \qquad \text{(A.2)}$$

Define

$$a_2(i,d) = \frac{\left\| Y_i(d+1) - Y_i^{NN}(d+1) \right\|}{\left\| Y_i(d) - Y_i^{NN}(d) \right\|} \qquad \text{(A.3)}$$

where $\|\cdot\|$ is the Euclidian distance and is given by the maximum norm here. $Y_i(d)$ is the *i*th reconstructed vector and $Y_i^{NN}(d+1)$ is its nearest neighbor in embedding dimension $d+1$. The mean value of all $a_2(i,d)$'s is defined as:

$$E(d) = \frac{1}{N-d\tau} \sum_{i=1}^{N-d\tau} a_2(i,d) \qquad \text{(A.4)}$$

$E(d)$ is only dependent on the dimension d and lag τ. To investigate its variation from d to $d+1$, define

$$E_1(d) = E(d+1)/E(d) \qquad \text{(A.5)}$$

$E_1(d)$ will stop changing when d is greater than some value d_0, which is the minimum embedding dimension we look for. In Section 4.2, we take the first plot in the left side in Fig. 2 as an example for explaining how to determining the embedding dimension value of a time series by Cao's method, at this time the embedding dimension value of the EEG signal segment is $d_0 = 10$.

Another quantity is defined to distinguish deterministic signals from stochastic signals. Let

$$E^*(d) = \frac{1}{N - d\tau} \sum_{i=1}^{N-d\tau} \left| x(i + d\tau) - x^{NN}(i + d\tau) \right| \qquad \text{(A.6)}$$

$$E_2(d) = E^*(d+1)/E^*(d) \qquad \text{(A.7)}$$

$E_1(d)$ is calculated for determining the minimum embedding dimension of time series, and $E_2(d)$ for distinguishing deterministic data from random data.

2. Differential entropy method

The differential entropy method can be summarized as below:

a) For the given signal $x(t)(t = 1,2,\cdots,N)$, generate its N_s surrogates $x_{s,i}(t), i = 1,\cdots,N$ by performing a random permutation of the time samples.

b) The Kozachenko-Leonenko estimates for the time delay embedded versions of the original time series $H(x,m,\tau)$ and its surrogates $H(x_{s,i},m,\tau)$ are computed using Eq.8 for increasing m and τ.

$$H(x) = \sum_{j=1}^{N} \ln(N\rho_j) + \ln 2 + C_E \quad \text{(A.8)}$$

c) Minimize the ratio

$$I(m,\tau) = \frac{H(x,m,\tau)}{\langle H(x_{s,i},m,\tau) \rangle_i} \qquad \text{(A.9)}$$

to determine the optimal embedding parameters, where $\langle \cdot \rangle_i$ denotes the average

over i.

d) To penalize for higher embedding dimensions, the minimum description length (MDL) method is superimposed, yielding the entropy ratio (ER):

$$R_{ent}(m,\tau) = I(m,\tau)(1 + \frac{m \ln N}{N}) \qquad \text{(A.10)}$$

where N is the number of delay vectors.

e) The minimum of the plot of the entropy ratio yields the optimal set of embedding parameters, namely optimal embedding dimension m and delay time τ.

The authors thank Ralph G. Andrzejak for many helpful discussions. This research is supported by the National Nature Science Foundation of China under Grant No. 40574051 and 40774054.

REFERENCES

1. Grogan NM. Neural network detection of epileptic seizures in the electroencephalogram. Oxford University. 1999. Available online: http://citeseer.ist.psu.edu/625725.html.
2. Iasemidis LD, Principe JC, Sackellares JC. Measurement and quantification of spatio-temporal dynamics of human epileptic seizures. In: Akay M, editor. Nonlinear signal processing in medicine. IEEE Press; 1999. p. 1-27.
3. Niedermeyer E, Silva FLD. Electroencephalography—Basic principles, clinical applications, and related fields. Baltimore: Lippincott Williams & Wilkins; 1993.
4. Bauer M, Heng H, Martienssen W. Characterization of spatiotemporal chaos from time series. Phys Rev Lett. 1993;71:521-4.
5. Richman JS, Moorman JR. Physiological time-series analysis using approximate entropy and sample entropy. Am J Physiol Heart Circ Physiol. 2000;278:2039-49.
6. Gautama T, Mandic DP, Marc MH. Indications of nonlinear structures in brain electrical activity. Phys Rev E. 2003;67:046204.
7. Saito K, Suzuki H, Kawakami Y. Power spectrum density of EEGs of sleeping epilepsy-prone El mice and their non-epileptic mother strain. J Physiol Sci. 2006;56:313-6.
8. Phothisonothai M, Nakagawa M. Fractal-based EEG data analysis of body parts movement imagery tasks. J Physiol Sci. 2007;57:217-26.
9. Li XL, Ouyang G. Nonlinear similarity analysis for epileptic seizures prediction. Nonlinear Analysis. 2006;64:1666-78.
10. Diambra L, Malta CP. Nonlinear models for detecting epileptic spikes. Phys Rev E. 1999;59:929-37.
11. Silva C, Pimentel IR, Andrade A, Foreid JP, Ducla-Soares E. Correlation dimension maps of EEG from epileptic absences. Brain Topogr. 1999;11:201-9.
12. Kannathal N, Puthusserypady SK, Min LC. Complex dynamics of epileptic EEG. IEEE Eng Med Biol Soc. 2004;1:204-7.
13. Frank GW, Lookman T, Nerenberg MAH, Essex C, Lemieux J, Blume W. Chaotic time series analyses of epileptic seizures. Physica D. 1990;46:427-38.
14. Martinerie J, Adam C, Quyen MLV, Baulac M, Clemenceau S, Renault B, Varela FJ. Epileptic seizures can be anticipated by non-linear analysis. Nat Med. 1998;4:1173-6.
15. Cao LY. Practical method for determining the minimum embedding dimension of a scalar time series. Physica D. 1997;110:43-50.
16. Gautama T, Mandic DP, Marc MH. A differential entropy based method for determining the optimal embedding parameters of a signal. Proceedings of the 1nt Conf on Acoustics, Speech and Signal Processing [C], Hong Kong. 2003;35:29-32.
17. Fraser AM, Swinney HL. Independent coordinates for strange attractors from mutual information. Phys Rev A. 1986;33:1134-40.
18. Jeong J, Kim MS, Kim SY. Test for low-dimensional determinism in electroencephalograms. Phys Rev E. 1999;60:831-7.

The Journal of Physiological Sciences Advance Publication **by J-STAGE**
doi:10.2170/physiolsci.RP004708
This version is to be replaced by the final version after page-setting and proofing.

Table 1. Embedding dimension results obtained by Cao's method.

SE	Embedding dimension																AV_i	Var_i
	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}		
Fp1	22	20	11	15	16	19	14	13	12	10	12	10	17	18	19	12	15	14
	6	7	11	8	5	10	12	10	4	7	5	11	9	13	10	7	8	7
F3	16	23	17	10	28	18	25	23	25	17	11	22	25	12	21	16	19	30
	10	9	12	6	6	7	13	11	11	9	5	7	6	14	7	7	9	7
C3	22	15	13	14	21	20	22	25	15	10	21	24	16	19	29	25	19	26
	5	8	8	6	5	11	11	7	7	8	10	6	7	7	8	8	8	3
P3	20	15	13	18	13	18	23	16	10	15	15	12	12	21	10	15	15	14
	5	13	7	10	11	8	4	5	10	7	9	14	7	5	10	7	8	8
O1	16	13	18	16	15	18	13	13	18	12	12	11	11	17	13	14	14	6
	5	10	9	7	8	5	6	10	7	6	5	5	14	12	11	9	8	7
F7	25	17	14	19	25	24	19	17	17	28	11	15	24	23	16	14	19	24
	12	7	5	6	11	6	5	10	5	8	8	7	11	5	7	9	8	5
T3	20	15	15	16	19	14	22	24	21	25	25	12	26	23	16	21	20	19
	7	6	7	8	8	6	9	5	10	5	8	9	10	14	11	7	8	5
T5	24	16	18	12	17	15	21	10	11	12	12	13	15	23	14	12	15	18
	5	7	5	4	8	5	10	7	7	9	8	6	7	5	7	5	7	3
Fp2	24	13	11	19	12	21	14	19	16	7	8	11	14	16	14	15	15	20
	4	10	9	4	9	7	13	6	10	7	14	7	9	16	10	9	9	10
F4	28	29	22	22	18	23	28	21	15	22	17	16	16	20	27	19	21	21
	13	6	11	9	10	7	7	7	6	7	5	9	9	16	3	9	8	10
C4	16	19	14	16	12	28	25	25	19	23	15	20	27	20	16	16	19	23
	9	12	7	8	11	5	11	8	7	05	8	13	7	13	10	10	9	6
P4	14	14	10	11	20	20	20	14	16	9	14	12	16	10	17	21	15	15
	7	11	6	5	10	5	11	8	11	10	12	8	7	18	10	12	9	10
O2	18	14	14	13	17	17	17	12	14	15	12	12	18	17	15	15	15	5
	7	5	6	9	7	8	5	7	14	5	11	10	5	7	7	6	7	6
F8	18	18	13	14	11	25	15	14	11	17	11	10	22	22	18	14	16	19
	11	04	09	06	14	11	09	11	07	07	10	05	08	06	15	6	9	10
T4	21	25	23	10	20	11	15	16	16	29	14	14	27	15	16	29	19	38
	3	4	14	10	14	9	9	13	11	9	12	10	3	9	7	8	9	12
T6	16	15	11	11	13	12	12	10	11	15	10	10	13	10	11	15	12	4
	13	10	10	5	5	8	8	7	7	11	11	9	13	10	16	10	10	9
AV_j	20	18	15	15	17	19	19	17	15	17	14	14	19	18	17	17		
	8	8	9	7	9	7	9	8	8	8	9	9	8	11	9	8		
Var_j	15	21	14	12	23	21	24	27	16	47	18	19	31	19	26	22		
	10	7	6	3	8	4	8	5	7	3	8	7	7	19	10	3		

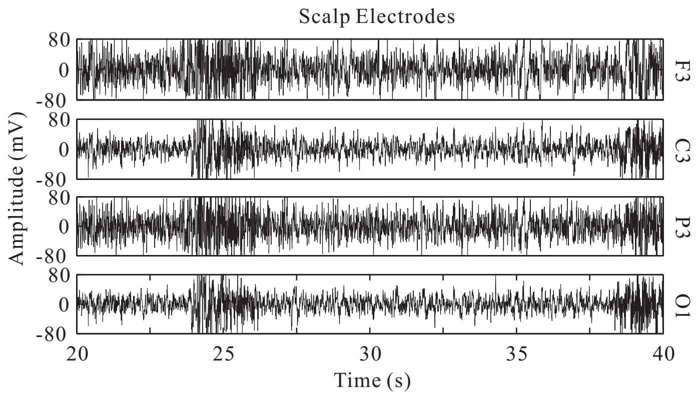
Note: SE is short for scalp electrode. For each cell of the table, the upper number corresponds to epileptic EEG time series and the lower number corresponds to normal EEG time series. AV_i stands for the average value of *i*th row embedding dimensions and AV_j stands for the average value of the *j*th line embedding dimensions. Var_i stands for the variance of *i*th row embedding dimensions and Var_j stands for the variance of the *j*th line embedding dimensions.

***The Journal of Physiological Sciences Advance Publication* by J-STAGE**
doi:10.2170/physiolsci.RP004708
This version is to be replaced by the final version after page-setting and proofing.

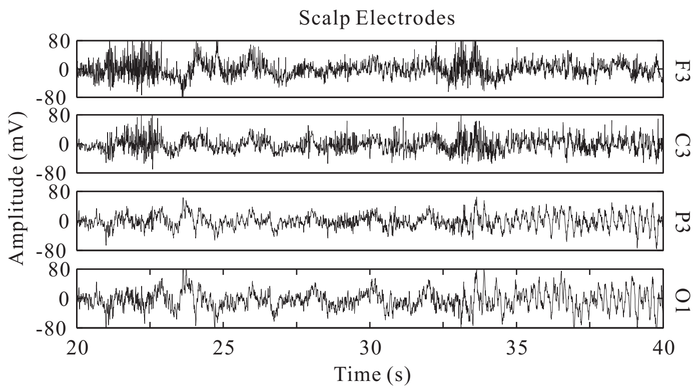
Table 2. Embedding dimension results obtained by differential entropy method.

SE	Embedding dimension																AV_i	Var_i
	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}		
Fp1	2 2	2 3	3 2	2 2	2 2	2 2	2 2	2 2	3 2	3 2	3 2	4 2	3 2	3 2	3 2	3 2	3 2	0.4 0.1
F3	2 2	2 2	2 2	5 3	3 2	4 2	4 2	4 2	4 2	4 2	4 2	4 2	5 2	2 2	4 2	2 2	3 2	1 0.1
C3	2 2	2 2	2 2	2 2	4 2	4 2	4 2	4 2	4 2	4 2	4 2	4 2	4 2	3 2	4 2	2 2	3 2	0.9 0
P3	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	3 2	2 2	2 2	2 2	2 2	0.1 0
O1	3 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	3 2	2 2	0.1 0
F7	2 2	2 2	2 3	4 2	3 2	3 2	3 2	3 3	3 2	4 2	4 2	4 2	4 2	2 2	5 2	2 2	3 2	0.9 0.1
T3	2 2	2 2	2 2	4 2	4 2	4 2	3 2	3 2	4 2	3 2	3 2	4 2	3 2	3 2	4 2	4 3	3 2	0.6 0.1
T5	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 3	4 2	2 2	2 2	3 2	2 2	0.3 0.1
Fp2	2 2	2 3	3 2	2 3	2 2	2 2	2 2	2 2	2 3	4 2	3 2	3 2	4 2	3 2	2 2	3 2	3 2	0.5 0.2
F4	2 2	2 2	2 3	2 2	2 2	2 2	2 2	2 2	2 2	4 2	2 2	2 2	5 2	3 2	5 2	2 2	3 2	1 0.1
C4	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	4 2	2 2	5 2	4 2	2 2	5 2	3 2	3 2	1 0
P4	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	0 0
O2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 3	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	0 0.1
F8	2 2	2 2	2 2	2 2	2 2	4 3	3 2	3 2	4 2	4 2	2 2	2 2	4 2	2 2	5 2	3 2	3 2	1 0.1
T4	2 2	2 2	3 2	2 2	2 2	4 2	4 2	4 2	4 2	4 2	4 2	4 2	3 2	2 2	4 2	4 2	3 2	0.9 0
T6	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	3 2	2 2	2 2	2 2	3 2	2 2	2 2	2 2	2 2	0.1 0
AV_j	2 2	2 2	2 2	2 2	2 2	3 2	3 2	3 2	3 2	3 2	3 2	3 2	3 2	2 2	3 2	3 2		
Var_j	0.1 0	0 0.1	0.2 0.1	0.9 0.1	0.5 0	0.8 0.1	0.6 0	0.6 0.1	0.8 0.1	0.9 0	0.7 0	1 0.1	0.9 0	0.2 0	1 0	0.5 0.1		

Note: SE is short for scalp electrode. For each cell of the table, the upper number corresponds to epileptic EEG time series and the lower number corresponds to normal EEG time series. AV_i stands for the average value of *i*th row embedding dimensions and AV_j stands for the average value of the *j*th line embedding dimensions. Var_i stands for the variance of *i*th row embedding dimensions and Var_j stands for the variance of the *j*th line embedding dimensions.

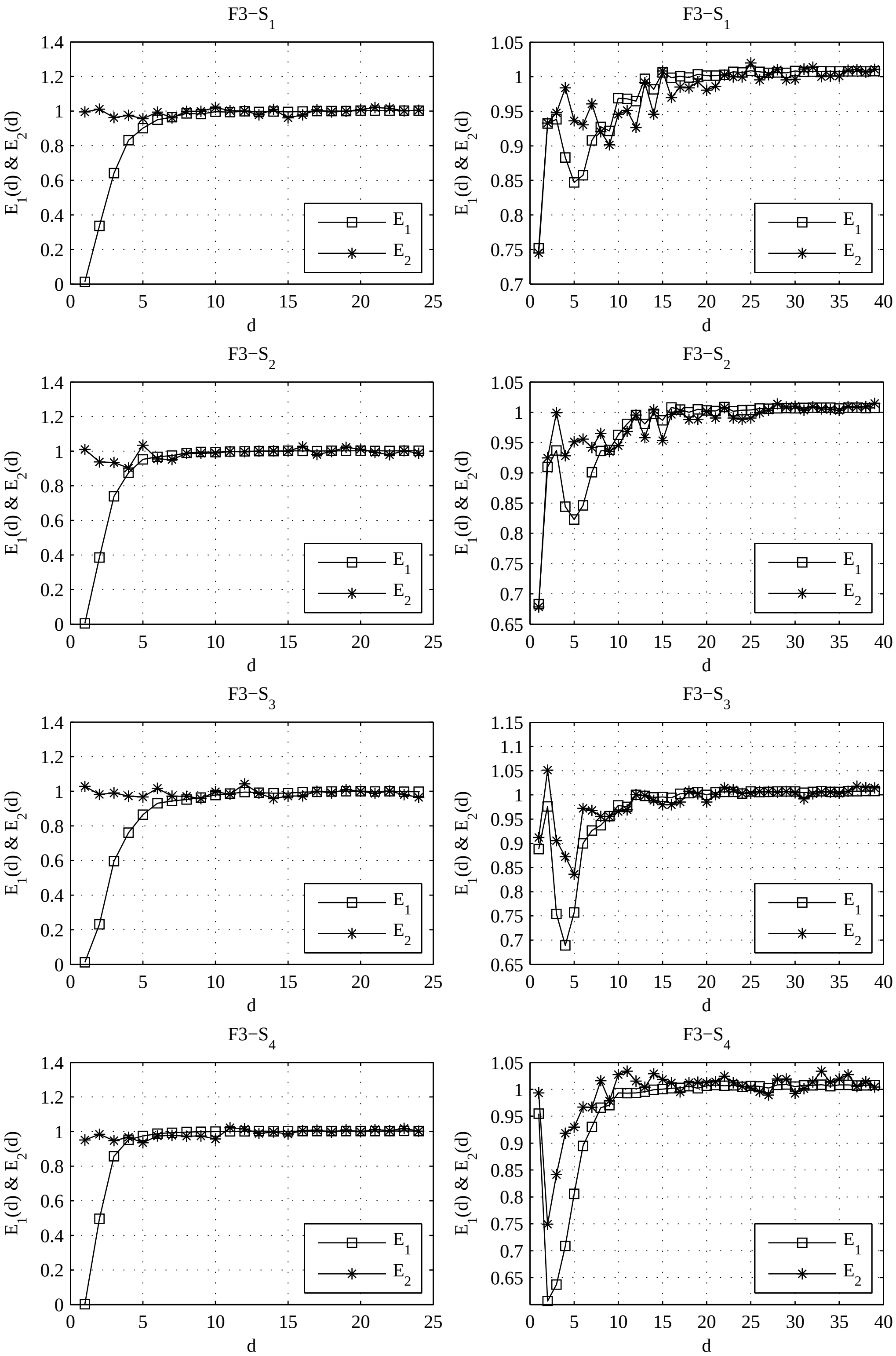


(a)

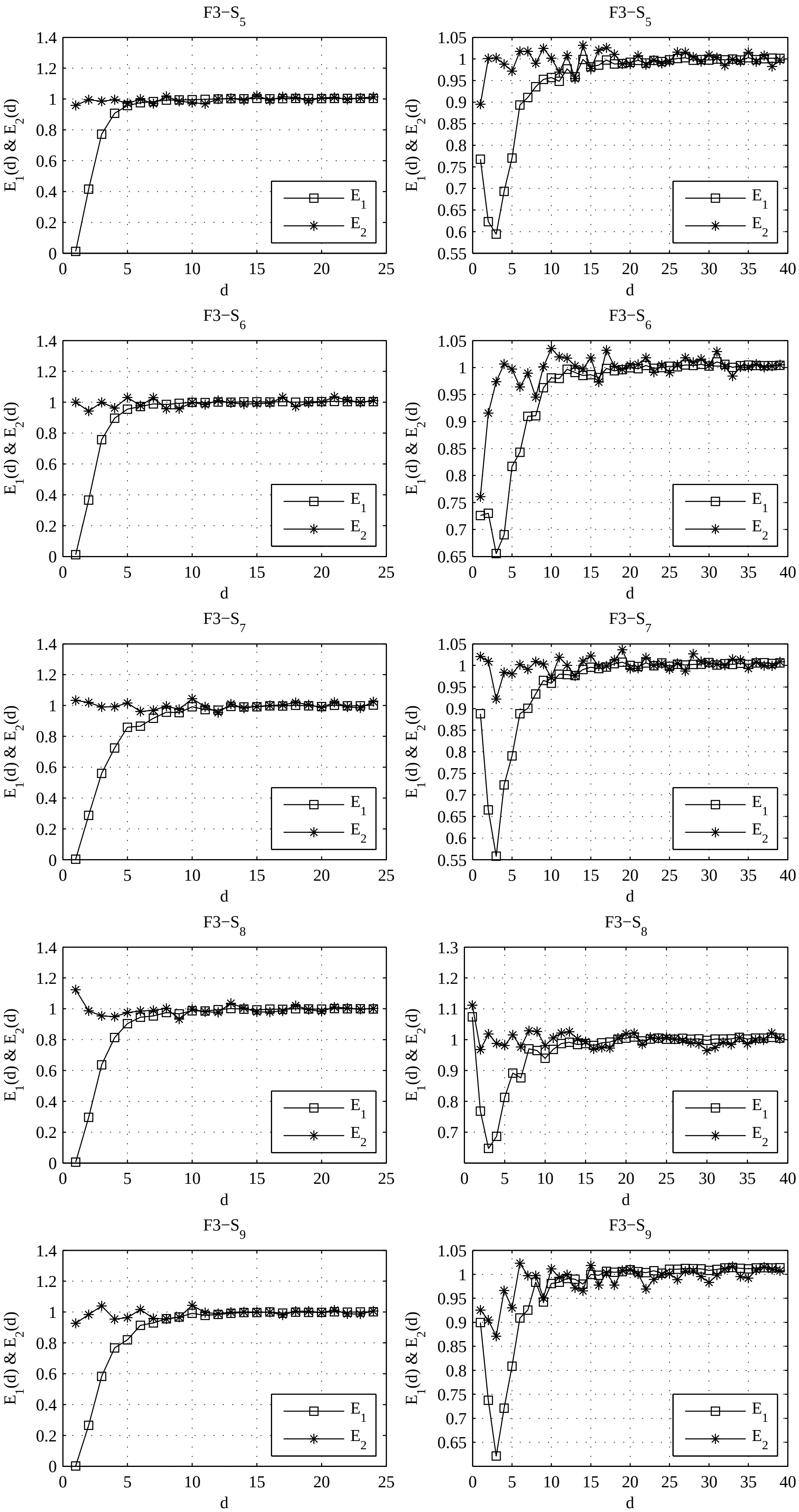


(b)

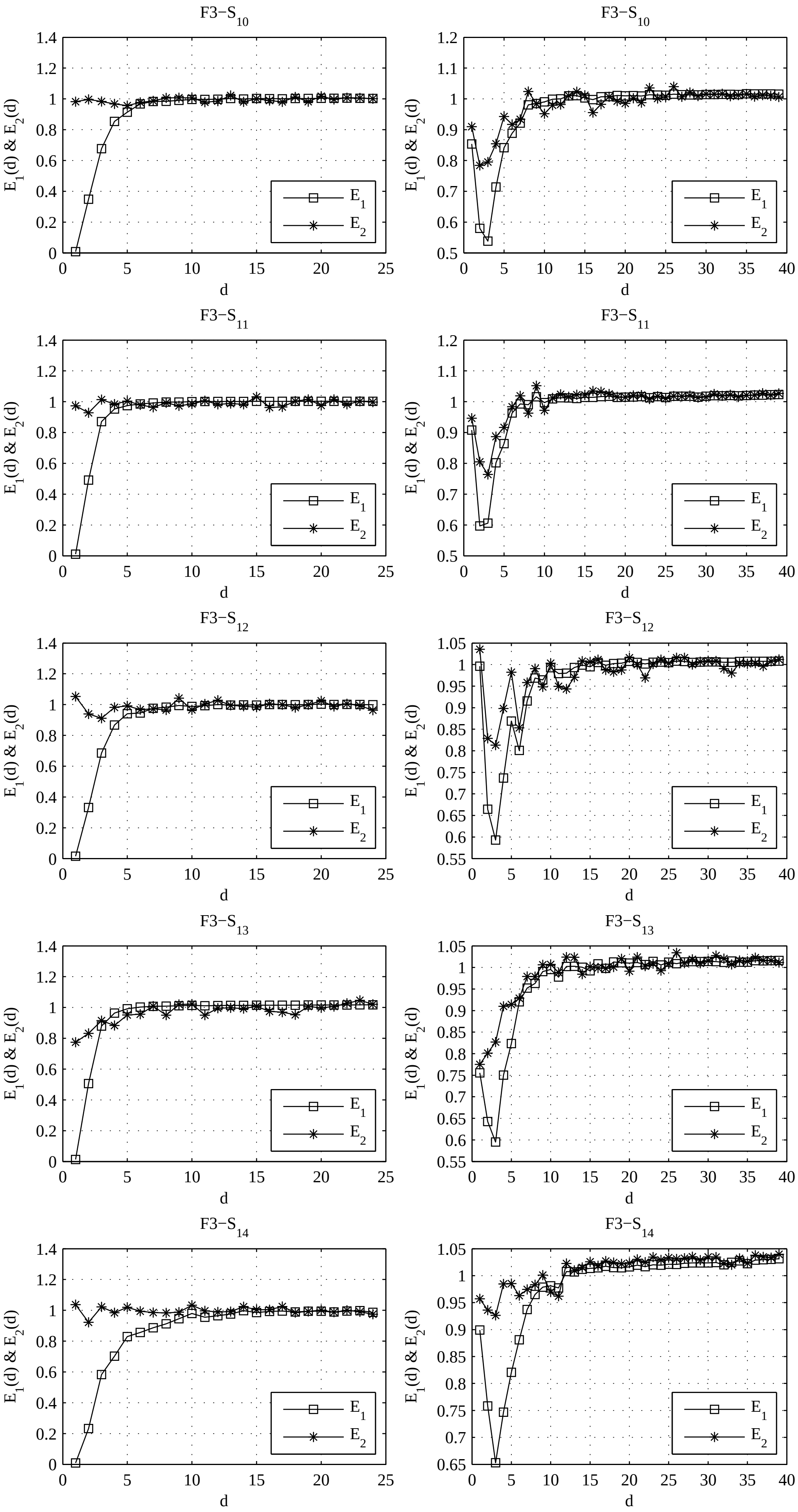
Fig.1. EEG signal examples, (**a**) Normal EEG (**b**) Epileptic EEG.


F3−S1
F3−S1
F3−S2
F3−S2
F3−S3
F3−S3
F3−S4
F3−S4
E1(d) & E2(d)
d
E1
E2

The Journal of Physiological Sciences Advance Publication by J-STAGE
doi:10.2170/physiolsci.RP004708
This version is to be replaced by the final version after page-setting and proofing.



The Journal of Physiological Sciences Advance Publication by J-STAGE
doi:10.2170/physiolsci.RP004708
This version is to be replaced by the final version after page-setting and proofing.



The Journal of Physiological Sciences Advance Publication by J-STAGE
doi:10.2170/physiolsci.RP004708

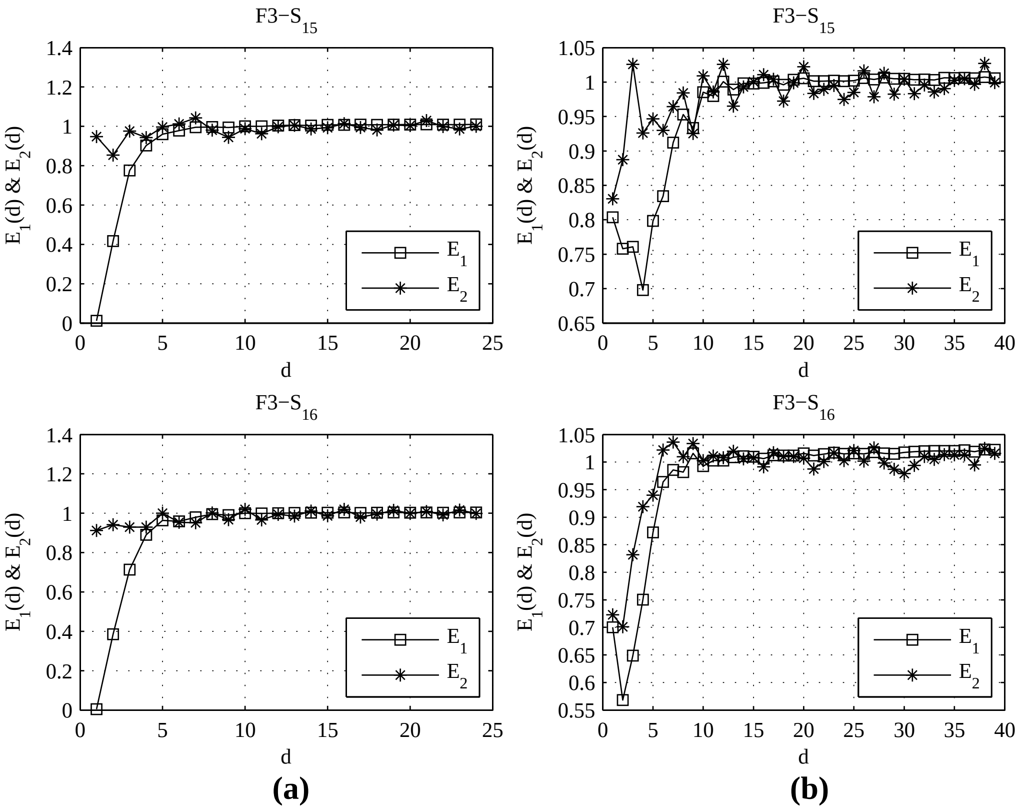
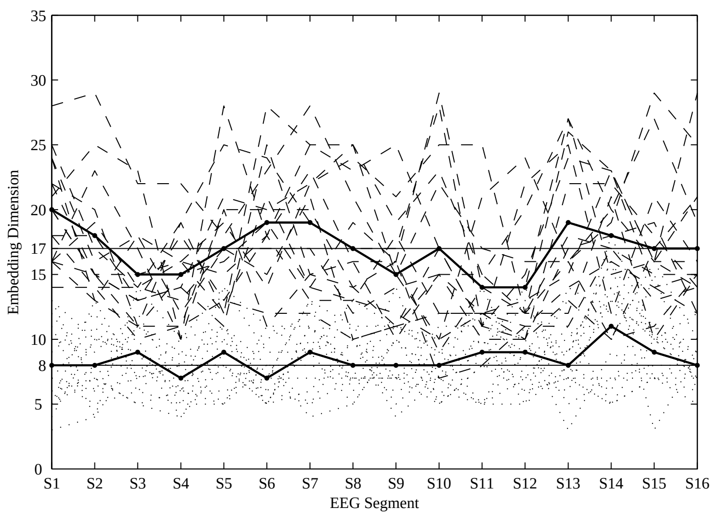


Fig. 2. Embedding dimension results of EEG segments corresponding to scalp electrode F3 by Cao's method, (**a**) Normal EEG (**b**) Epileptic EEG. (Note: the title of each plot F3-Si (I =1,2,3,…,16) means the *i*th EEG segment corresponding to scalp electrode F3.)



The 16 dash curves are plotted by the embedding dimension values of 16-channel epileptic EEG signals. The upper curve marked by black points and the straight line are plotted by AV_j (shown in Table 1) and ED values of epileptic EEG time series.

The 16 dot curves are plotted by the embedding dimension values of 16-channel normal EEG signals. The lower curve marked by black points and the straight line are plotted by AV_j (shown in Table 1) and ED values of normal EEG time series.

Fig. 3. Scheme of variation with time of EEG's embedding dimensions obtained by Cao's method.

doi:10.2170/physiolsci.RP004708

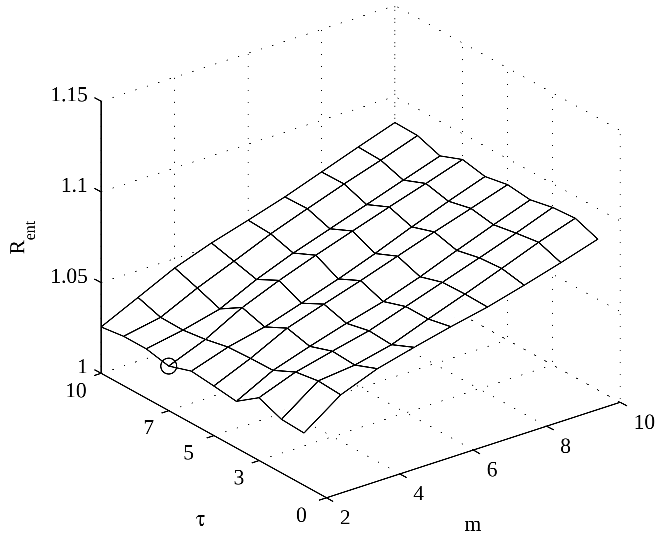


Fig. 4. An example for determining the optimal embedding parameters by differential entropy method. The minimum of the plot indicated by an open circle yields $m_{opt} = 2$ and $\tau_{opt} = 7$.

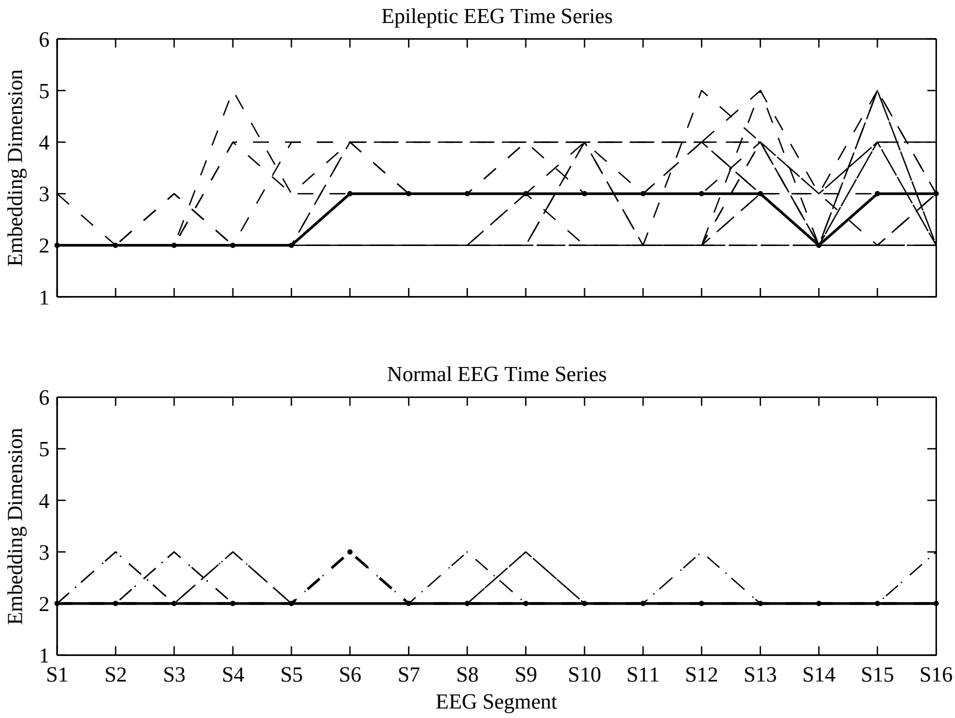


Fig. 5. Scheme of variation with time of EEG's embedding dimensions obtained by differential entropy method. (Note: The curves marked by black points are constructed by AV_j values shown in Table 2).