Image super-resolution with B-Spline kernels

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Abstract

A novel approach to image super-resolution is described in this paper. By modeling our image acquisition system with a Spline sampling kernel, we are able to retrieve from the samples some statistical information about the observed continuous scene before its acquisition (irradiance light-field). This information, called *continuous moments*, allows to register exactly a set of low-resolution images and to ultimately generate a super-resolved image. The novelty of the proposed algorithm resides in its ability to operate entirely on low-resolution images and to enhance the resolution of the entire field of view with a relatively low computational complexity. We ran experiments with real acquired images and obtained super-resolved images with a good level of details although some ringing effects are also noticeable.

1. Introduction

An image super-resolution algorithm aims at creating a single detailed image called super-resolved image (SR) from a set of low-resolution images (LR) taken from different unknown locations and observing the same scene of interest. It is often assumed that the disparity between any two LR images can be described by a parametric form (*e.g.* an affine transform) that needs to be estimated in the *image registration* step. Most registration algorithms start by detecting a large number of features (*e.g.* corners) in the LR images and then find their correspondences among the LR images. Thus a subpixel accuracy can be obtained in the estimation of the registration parameters once a sufficient set of feature correspondences has been correctly established.

The second step of super-resolution is called *image fusion* and consists in calculating the SR image from the registered data as well as restoring it from any blur or noise introduced by the acquisition system. When less pixels from the set of LR images are available than in the SR image, fusion can be expressed as a linear system whose matrix obtained from the registration step is underdetermined and ill-conditioned. Any noise and/or errors in the linear system make the search for a suitable solution increasingly difficult. The use of regularization methods to circumvent this problem implies that a smoothened SR image is retrieved whose certain details may be lost and where ringing effects may occur.

The accuracy of image registration is thus critical to get a SR image of good quality. Many super-resolution algorithms need input LR images with a fairly good resolution in order to have successful features extraction and correspondences steps. Then, for memory requirements, a small region of interest (ROI), *e.g.* 64 x 64px, is selected and its resolution enhanced by the desired magnification factor. In our proposed approach, we consider input LR images whose size is similar to the ROI previously mentioned.

In this research, new results from the sampling theory of signals with finite rate of innovation (FRI) (see Dragotti *et al* 2006) are extended to application for real images. Combined to the theory of B-Spline processing, we are able to register LR images with a greater accuracy and apply this method to a new image super-resolution algorithm.

2. Shift-invariant Spline space

Let $V(\varphi)$ be the following shift-invariant function space generated by the function φ:

$$V(\varphi) : \left\{ g_V(x) = \sum_{k=-\infty}^{\infty} p[k]\varphi(x-k) : p \in l^2 \right\}$$

The integer shifts of φ form a basis of $V(\varphi)$ and $p[k]$ are the coordinates of $g_V(x)$ in this basis. $V(\varphi)$ is a closed-subspace of L^2 when φ is a Riesz basis (Aldroubi & Unser 1994):

$$A\|p\|_{l^2}^2 \leq \left\| \sum_{k=-\infty}^{\infty} p[k]\varphi(x-k) \right\|^2 \leq B\|p\|_{l^2}^2, \qquad B > A > 0.$$

Let $g(x) \in L^2$, its least-square approximation $\hat{g}(x)$ in $V(\varphi)$ is its orthogonal projection onto $V(\varphi)$:

$$\hat{g}(x) = \sum_{k=-\infty}^{\infty} p[k]\varphi(x-k) \quad \text{where} \quad p[k] = \langle g(\tau), \widetilde{\varphi}(k-\tau) \rangle$$

The function $\widetilde{\varphi}$ is the dual basis of φ and satisfies:$\langle \varphi(x-i), \widetilde{\varphi}(x-j) \rangle = \delta_{ij}$, $i, j \in \mathbb{Z}$.

A Spline $\gamma^\rho(x)$ is a piecewise polynomial curve of degree ρ whose pieces are smoothly connected together at equidistant joining points called knots. A fundamental result from Schoenberg (1946) shows that any Spline $\gamma^\rho(x)$ can be characterized by a unique expansion with a B-Spline basis denoted $\beta^\rho(x)$. Thus the function $\beta^\rho(x)$ generates a shift-invariant Spline space $V(\beta^\rho)$. B-Spline functions are obtained from successive convolution of the box B-Spline $\beta^0(x)$:

$$\beta^\rho(x) = \underbrace{\beta^0(x) * \beta^0(x) * \ldots * \beta^0(x)}_{\rho+1 \text{ times}} \quad \text{where} \quad \beta^0(x) = \begin{cases} 1, & -\frac{1}{2} < x < \frac{1}{2} \\ \frac{1}{2}, & |x| = \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

B-Splines have many interesting properties. In particular, they have a compact support, form a Riesz Basis and satisfy also Strang-Fix conditions (Strang & Fix 1994):

$$\beta^\rho(x) \Leftrightarrow \mathbf{B}^\rho(f) = [\text{sinc}(f)]^{\rho+1} \rightarrow \begin{cases} \mathbf{B}^\rho(0) \neq 0 \\ D^j\mathbf{B}^\rho(2\pi k) = 0, \quad k \in \mathbb{Z}, \quad j = 0, 1, \ldots, \rho. \end{cases}$$

where $\mathbf{B}^\rho(f)$ is the Fourier transform of $\beta^\rho(x)$ and D^j is the differential operator. These conditions ensure that a linear combination of B-Splines can reproduce polynomials. Interestingly, any Spline function also satisfy Strang-Fix conditions. Discrete B-Splines are defined as:

$$b^\rho(k) = \beta^\rho(x)\big|_{x=k}$$

A dual Spline function $\widetilde{\gamma}^\rho(x)$ corresponds to any Spline function $\gamma^\rho(x)$ (Unser *et al* 1993):

$$\gamma^\rho(x) = \sum_{k=-\infty}^{\infty} p[k] \cdot \beta^\rho(x-k)$$

$$\widetilde{\gamma}^\rho(x) = \sum_{k=-\infty}^{\infty} (p * b^{2\rho+1})^{-1}[k] \cdot \beta^\rho(x-k)$$

3. B-Spline sampling kernels and continuous moments

In our camera model, the incoming irradiance light-field $f(x, y)$ of the observed scene is blurred and sampled by a sampling kernel $\gamma^\rho(x, y)$ which models the point-spread function (PSF) of the camera lens and the sensor on the image plane. The PSF $\gamma^\rho(x, y)$ is supposed to be known and to be a Spline function of compact support. The obtained samples $S_{m,n}$ are the pixels of the acquired image:

$$S_{m,n} = \langle f(x,y), \gamma^\rho(x/T - mT, y/T - nT) \rangle \quad \text{with} \quad x, y \in \mathbb{R}, \quad m, n \in \mathbb{Z}.$$

Mathematically, the samples $S_{m,n}$ are the coordinates of $f(x,y)$ in the shift-invariant Spline space with respect to the dual Spline basis $\widetilde{\gamma}^\rho(x,y)$. Since $\gamma^\rho(x,y)$ satisfies Strang-Fix conditions, there exists a set of coefficients $\left\{c_{m,n}^{(p,q)}\right\}$ such that:

$$\sum_{m\in\mathbb{Z}}\sum_{n\in\mathbb{Z}} c_{m,n}^{(p,q)} \gamma^\rho(x-m, y-n) = x^p y^q, \quad \text{where} \quad c_{m,n}^{(p,q)} = \langle x^p y^q, \widetilde{\gamma}^\rho(x-m, y-n) \rangle$$

where $p, q = 0, \ldots, \rho - 1$ and $T = 1$ for clarity. The knowledge of these particular coefficients allows us to retrieve *exactly* the geometric moments $\mathbf{m}_{p,q}$ of the irradiance light-field $f(x,y)$ from a simple linear combination of the samples:

$$\begin{aligned}
\mathbf{m}_{p,q} &= \int\int f(x,y)\, x^p y^q dxdy \\
&= \int\int f(x,y) \sum_m \sum_n c_{m,n}^{(p,q)} \gamma^\rho(x-m, y-n)\, dxdy \\
&= \sum_m \sum_n c_{m,n}^{(p,q)} \int\int f(x,y)\, \gamma^\rho(x-m, y-n)\, dxdy \\
&= \sum_m \sum_n c_{m,n}^{(p,q)} S_{m,n}
\end{aligned}$$

These moments are called *continuous moments* to distinguish them from the discrete moments $\hat{\mathbf{m}}_{p,q} = \sum_m \sum_n S_{m,n}(mT)^p(nT)^q$ which only approximate the true moments of $f(x,y)$. The main benefit comes when one considers images of low-resolution. As the resolution decreases, the continuous moments remain exact whereas discrete moments diverge rapidly (see also Baboulaz & Dragotti 2006). The required minimum number of samples is related to the size of the support of $\gamma^\rho(x,y)$. From the geometric continuous moments, it then becomes possible to compute a whole variety of other continuous moments like central or complex moments using adequate linear combinations. In the next section, $\gamma^\rho(x,y)$ is chosen to be a cubic B-Spline $\beta^3(x,y)$ (*i.e.* $p[k] = \delta[k]$). This is a reasonable choice as the PSF of the lens is often modeled by a Gaussian pulse which is very similar to a cubic B-Spline. The PSF of the sensor is assumed to be a box function.

4. Image super-resolution

The disparity between any two images $\mathbf{g}_1$ and $\mathbf{g}_2$ is assumed to be a global affine transform which consists of a 2 x 2 matrix $\mathbf{A}$ and one vector of translation $\mathbf{t}$. It is straightforward to retrieve $\mathbf{t}$ by difference of the barycenters calculated from the continuous moments. To find the matrix $\mathbf{A}$, we use the method described by Heikkilä (2004). Let $\mathbf{F}_i$ be the Cholesky decomposition of the covariance matrix of $\mathbf{g}_i$:

$$\mathbf{F}_i = \begin{bmatrix} \sqrt{\mu_{2,0}^{(i)}} & 0 \\ \frac{\mu_{1,1}^{(i)}}{\sqrt{\mu_{2,0}^{(i)}}} & \sqrt{\mu_{0,2}^{(i)} - \frac{\mu_{1,1}^{(i)2}}{\mu_{2,0}^{(i)}}} \end{bmatrix}$$

where $\mu_{p,q}^{(i)}$ denotes the central moments. It can be shown that $\mathbf{A}$ can be written as $\mathbf{A} = \mathbf{F}_2 \cdot \mathbf{R} \cdot \mathbf{F}_1^{-1}$ where $\mathbf{R}$ is a matrix of rotation. By applying a whitening transform to each image $\bar{\mathbf{g}}_i = \mathbf{F}_i^{-1}(\mathbf{g}_i - \mathrm{E}\{\mathbf{g}_i\})$, the affine disparity between $\mathbf{g}_1$ and $\mathbf{g}_2$ reduces to a rotational disparity with matrix $\mathbf{R}$ between $\bar{\mathbf{g}}_1$ and $\bar{\mathbf{g}}_2$ (Sprinzak & Werman 1994). The rotation angle α can then be retrieved by looking at the complex moments of $\bar{\mathbf{g}}_1$ and $\bar{\mathbf{g}}_2$ since:

$$\bar{\kappa}_{p,q}^{(1)} = e^{j(p-q)\alpha} \cdot \bar{\kappa}_{p,q}^{(2)}$$

FIGURE 1. Image super-resolution with 40 cameras. (a) Original image 512x512px (left) - (b) LR image 64x64px (middle) - (c) SR image 512x512px (right)

In Heikkilä (2004), the complex moments $\bar{\kappa}^{(i)}_{p,q}$ have been expressed in function of the central moments $\mu^{(i)}_{p,q}$. It is thus possible in theory to register exactly images that are related by an affine transform using continuous moments.

The chosen image fusion procedure is simple: after registration of the LR images, the data on the grid of the HR image is estimated by a 2-D interpolation; the obtained image is then restored using a Wiener filter.

Figure 1 shows results of an image super-resolution experiment. 40 images of the same scene (a tiger plush) have been acquired from different locations (random horizontal and vertical translations) using a digital camera. Figure 1(a) shows the ground-truth image of reference (512 x 512 px). Each image has been downsampled and blurred to 64 x 64 px with a cubic B-Spline sampling kernel to create the set of LR images (see Figure 1(b)). Finally, Figure 1(c) shows the reconstructed SR image with our algorithm (512 x 512 px, PSNR = 24.2dB). It is worth noticing that the system is underdetermined, *i.e.* there are less pixels in the set of LR images than in the SR image (ratio is $\sim$ 62.5%).

5. Conclusion

We have presented in this paper an image super-resolution algorithm based on the results of the theory of FRI signals and the theory of B-Spline processing. We use the notion of continuous moments to accurately register images. In future works, we would like to estimate reconstruction quality bounds from noisy samples.

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