

Communications I

Problem Sheet One

1. Find the mean value (time average) and the power for the following signals

(a) $v(t) = 3 \cos(2\pi 10^3 t),$

[0, 9/2]

(b) $v(t) = 3,$

[3, 9]

(c) $v(t) = 3 \sin(40t) + 4 \cos(\pi 10^4 t).$

[0, 25/2]

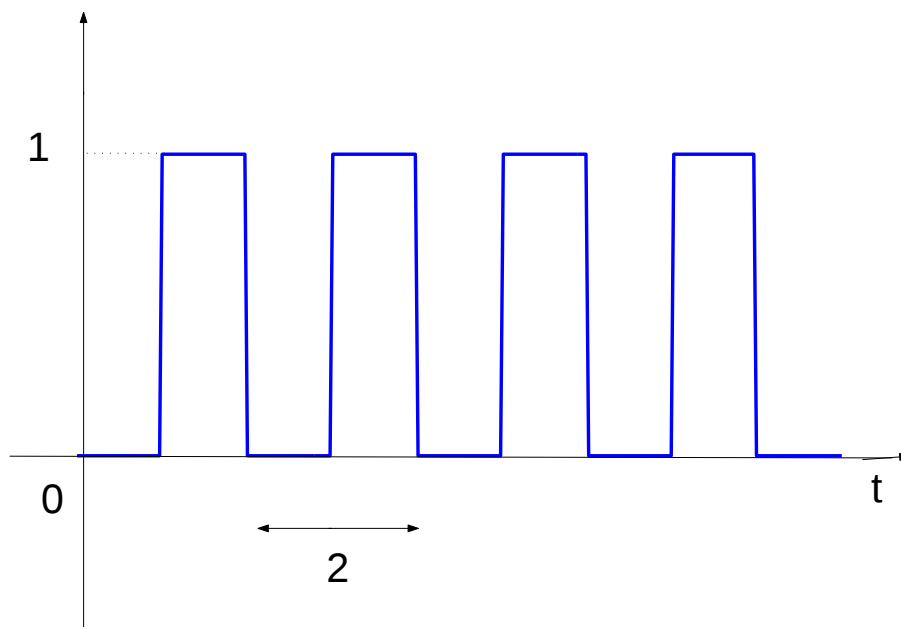
2. (a) Find the energy of the signal

$$g(t) = \begin{cases} \sin t & \text{for } 0 \leq t \leq 2\pi \\ 0 & \text{otherwise} \end{cases}$$

[π]

- (b) Comment on the effect on energy of sign change or time shifting. What is the effect on the energy if the signal is multiplied by k ?

3. Find the power for the following periodic signal.



[1/2]

Problem Sheet Two

1. Find the energy E_x and E_y of the signals $x(t)$ and $y(t)$ shown in Figure 1. Find the correlation coefficient between $x(t)$ and $y(t)$. Sketch the signal $x(t) + y(t)$ and show that the energy of this signal is equal to $E_x + E_y$. (Why?)

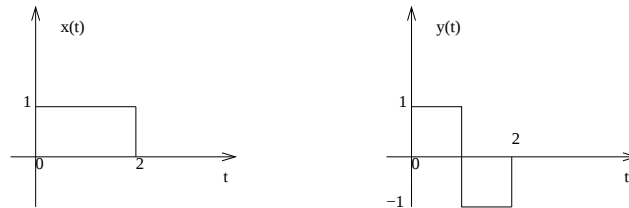


Figure 1:

2. Repeat the procedure for the signal pair shown in Figure 2. That is, compute:
 - Signal energies.
 - Correlation coefficient.
 - Energy of the signal $z(t) = x(t) + y(t)$. (Is $E_z \neq E_x + E_y$? Why?)

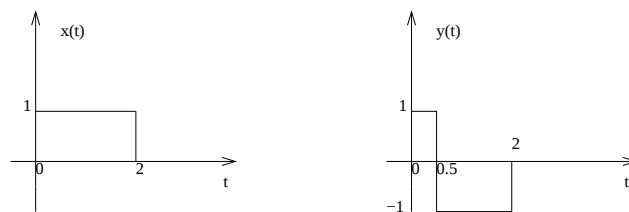


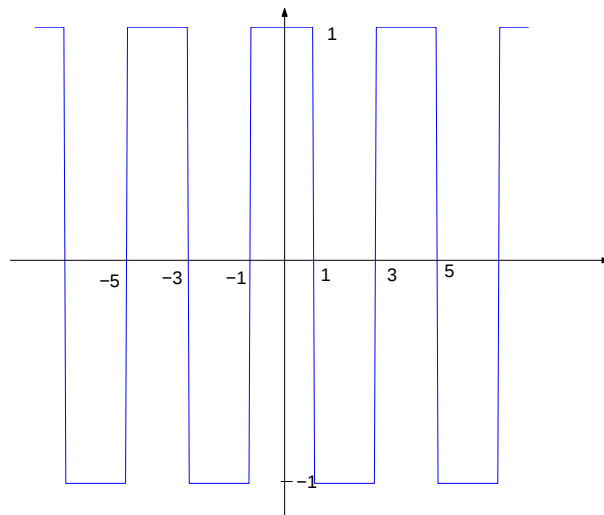
Figure 2:

3. If a periodic signal satisfies certain symmetry conditions, the evaluation of the Fourier series component is somewhat simplified. Show that:

- (a) If $g(t) = g(-t)$ (even symmetry), then all the sine terms in the trigonometric Fourier series vanish ($b_n = 0$).
- (b) If $g(t) = -g(-t)$ (odd symmetry), then the dc and all the cosine terms in the Fourier series vanish ($a_0 = a_n = 0$).

4. Show that the trigonometric Fourier series of the signal shown below is

$$x(t) = \frac{4}{\pi} \left(\cos \frac{\pi t}{2} - \frac{1}{3} \cos \frac{3\pi t}{2} + \frac{1}{5} \cos \frac{5\pi t}{2} - \frac{1}{7} \cos \frac{7\pi t}{2} + \dots \right)$$



Problem Sheet Three

1. Show that the Fourier transform of $g(t)$ may be expressed as

$$G(\omega) = \int_{-\infty}^{\infty} g(t) \cos \omega t dt - j \int_{-\infty}^{\infty} g(t) \sin \omega t dt$$

Hence, show that

- if $g(t)$ is a real and even function of t , then $G(\omega)$ is real and even.
- if $g(t)$ is a real and odd function of t , then $G(\omega)$ is imaginary and odd.

2. Using the definition of the Fourier transform:

$$G(\omega) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$$

show that if $g(t)$ is real then $G^*(\omega) = G(-\omega)$.

(An important consequence of the above property is that if $g(t)$ is real then $|G(\omega)|$ is an even function of ω and $\angle G(\omega)$ is an odd function of ω).

3. From the definition of the Fourier transform, find the Fourier transform of $\text{rect}(t - 5)$.
4. Using time shift property, compute again the Fourier transform of $\text{rect}(t - 5)$ and compare the two results.

Problem Sheet Four

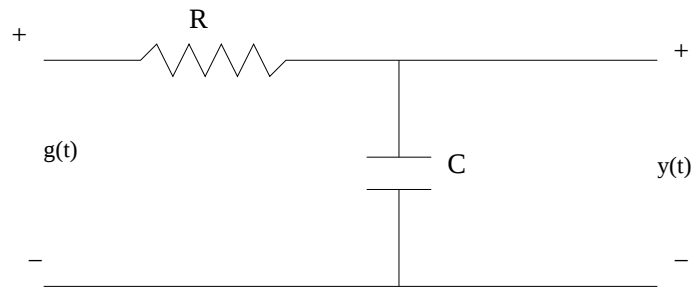
- Find the Power Spectral Density $S_g(\omega)$ of the power signal $g(t) = \cos \omega_0 t$.
(Hint: Compute the autocorrelation function first, and then use the property $\mathcal{R}_g(\tau) \iff S_g(\omega)$).

$$\left[\frac{\pi}{2} [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]\right]$$

- Find the power of the output signal $y(t)$ of the RC circuit shown below with $RC = 1$ if the input PSD $S_g(\omega)$ is given by:

- $\text{rect}(\omega/2)$
- $\delta(\omega + 1) + \delta(\omega - 1)$

(Hint: recall that $S_y(\omega) = |H(\omega)|^2 S_g(\omega)$ and that $P_y = 1/2\pi \int_{-\infty}^{\infty} S_y(\omega) d\omega$)



$$[1/4, 1/2\pi]$$

Problem Sheet Five

1. Consider the two baseband signals $x_1(t) = \cos 2000t$ and $x_2(t) = \cos 1900t$. Plot the magnitude spectrum of the signal $s(t) = x_1(t) \cos 10000t + x_2(t) \cos 20000t$.

- (a) The signal $s(t)$ is multiplied by $\cos 10000t$ and fed to the filter with frequency response

$$H(\omega) = \begin{cases} 1 & \text{for } |\omega| \leq 2050 \text{ rad/s} \\ 0 & \text{otherwise} \end{cases}$$

Plot the magnitude spectrum of the output time waveform $y(t)$ (see Figure 3)

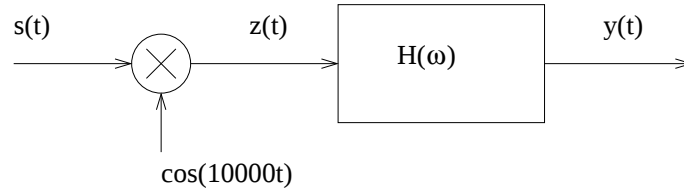


Figure 3: DSB-SC receiver

- (b) What happens if $s(t)$ is multiplied by $\cos 20000t$ and fed to the same filter? Plot the amplitude spectrum of this new output.
2. The signals $m_1(t) = \frac{2}{\pi} \text{sinc}(2t)$ and $m_2(t) = \frac{4}{\pi^2} \text{sinc}^2(2t)$ are to be transmitted simultaneously over a channel. Call $y(t) = (m_1(t) + x(t)) \cos(1000t)$ the signal transmitted over the channel, where $x(t) = m_2(t) \cos 6t$.
- (a) Sketch the spectra of $x(t)$ and $y(t)$ [Hint: recall that a product in the time domain is equivalent to a convolution in the frequency domain].
- (b) What is the bandwidth of $m_1(t) + x(t)$? [$5/\pi$ Hz]
- (c) Can $m_1(t)$ and $m_2(t)$ be recovered from $y(t)$?
- (d) Design (only the block diagram) a synchronous receiver to recover $m_2(t)$.

Problem Sheet Six

1. Consider the amplitude modulated signal $s(t) = (A + m(t)) \cos \omega_c t$, where $A = 2$, $\omega_c = 10000 \text{ rad/s}$ and $m(t) = \cos 100t + \sin 100t$. Compute:
 - (a) the peak amplitude m_p of $m(t)$.
(Hint: use the identity $a \cos \omega_0 t + b \sin \omega_0 t = c \cos(\omega_0 t + \theta)$ with $c = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}(-b/a)$).
 - (b) Compute the modulation index $\mu = \frac{m_p}{A}$.
 - (c) Compute the power efficiency $\eta = \frac{P_s}{P_c + P_s}$

2. The output signal from an AM modulator is

$$u(t) = 5 \cos(1800\pi t) + 20 \cos(2000\pi t) + 5 \cos(2200\pi t).$$

Determine:

- (a) the modulating signal $m(t)$ and the carrier $c(t)$,
 - (b) the modulation index,
 - (c) the ratio of the power in the sidebands to the power in the carrier.
3. Consider the modulating signal $m(t) = \cos 100t$.
 - (a) Sketch the spectrum of $m(t)$
 - (b) Find and sketch the spectrum of the DSB-SC signal $\phi(t) = 2m(t) \cos 1000t$
 - (c) From the spectrum of $\phi(t)$, suppress the LSB spectrum to obtain the USB spectrum.
 - (d) From the USB spectrum, write the expression of $\phi_{USB}(t)$.

Problem Sheet Seven

1. Over an interval $0 \leq t \leq 1$, an angle modulated signal is given by

$$\phi(t) = 10 \cos 13000t$$

The carrier frequency is $\omega_c = 10000$.

- (a) If this were a PM signal with $k_p = 1000$, determine $m(t)$ over $0 \leq t \leq 1$.
 - (b) If this were an FM signal with $k_f = 1000$, determine $m(t)$ over $0 \leq t \leq 1$.
2. An angle modulated signal has the form $u(t) = 100 \cos[2\pi f_c t + 4 \sin(2000\pi t)]$ where $f_c = 10$ MHz.
- (a) Determine the average transmitted power.
 - (b) Is this an FM or PM signal? Explain.
 - (c) Determine the frequency deviation Δf . [4000Hz]
 - (d) Using Carson's rule, find the bandwidth of the modulated signal. [10kHz]
3. The message signal $m(t) = 10 \text{sinc}(400\pi t)$ frequency modulates the carrier $c(t) = 100 \cos(2\pi f_c t)$. The modulation index is $\beta = 6$.
- (a) Write an expression for the modulated signal $u(t)$. [Hint: you need to find the value of k_f]
 - (b) What is the maximum frequency deviation of the modulation signal? [1200Hz]
 - (c) Using Carson's rule, find the bandwidth of the modulated signal. [2800Hz]

Problem Sheet Eight

1. Consider the FM signal

$$\varphi(t) = 10 \cos[2\pi f_0 t + k_f \int_{-\infty}^t x(\alpha) d\alpha]$$

where $k_f = 10\pi$. The message $x(t)$ is given by

$$x(t) = \sum_{n=0}^2 m_n(t)$$

with

$$m_n(t) = \frac{2^n}{\pi} \text{sinc}(t) \cos(2nt).$$

- (a) Sketch and dimension the Fourier transform of $m_1(t)$.
- (b) Sketch and dimension the Fourier transform of $x(t)$.
- (c) Using Carson's rule, determine the bandwidth of $\varphi(t)$.

[75/ π Hz]

- (d) Assume now that $x(t) = Ae^{-10t}u(t)$. Using Carson's rule, the bandwidth of $\varphi(t)$ is 50.4 Hz. Find the amplitude A of $x(t)$. Select the bandwidth, B, of the baseband message $x(t)$ so that it contains 95% of the signal energy.

[$A = 1$]