

Communications I

Solutions to Problem Sheet One

1. The power of a sinusoid $g(t) = A \cos(\omega_0 t + \theta)$ is always equal to $\frac{A^2}{2}$. It follows

$$\begin{aligned}
 P_g &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2 \cos^2(\omega_0 t + \theta) dt \\
 &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{A^2}{2} [1 + \cos(2\omega_0 t + 2\theta)] dt \\
 &= \lim_{T \rightarrow \infty} \frac{A^2}{2T} \int_{-T/2}^{T/2} dt + \lim_{T \rightarrow \infty} \frac{A^2}{2T} \int_{-T/2}^{T/2} \cos(2\omega_0 t + 2\theta) dt \\
 &= \frac{A^2}{2} + \lim_{T \rightarrow \infty} \frac{A^2}{4\omega_0 T} \sin(\omega_0 T + 2\theta) + \lim_{T \rightarrow \infty} \frac{A^2}{4\omega_0 T} \sin(2\theta - \omega_0 T) \\
 &= \frac{A^2}{2},
 \end{aligned}$$

where the last identity follows from the fact that

$$\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0.$$

Therefore

$$P_g = \frac{A^2}{2}.$$

- (a) $v_{dc} = 0, \quad P_{v(t)} = \frac{9}{2}.$
- (b) $v_{dc} = 3, \quad P_{v(t)} = 9.$
- (c) $v_{dc} = 0, \quad P_{v(t)} = \frac{25}{2}.$

2. (a)

$$E_g = \int_0^{2\pi} \sin^2 t dt = \frac{1}{2} \int_0^{2\pi} dt - \frac{1}{2} \int_0^{2\pi} \cos 2t dt = \pi + 0 = \pi.$$

- (b) Sign change and time shift do not affect the signal energy. The energy of $kg(t)$ is $k^2 E_g$.

- 3.

$$P_v = \frac{1}{2} \int_1^2 (1)^2 dt = \frac{1}{2}$$