Solutions to Problem Sheet Eight

Problem 1.

(a)

$$\begin{aligned} m_1(t) &= \frac{2}{\pi} sinc(t)\cos(2t) \\ &= \frac{2}{\pi} sinc(t) \Longleftrightarrow 2rect(\frac{\omega}{2}) \end{aligned} \tag{1}$$

Therefore

$$\frac{2}{\pi} sinc(t)\cos(t) \Longleftrightarrow rect(\frac{\omega-2}{2}) + rect(\frac{\omega+2}{2}) \tag{2}$$

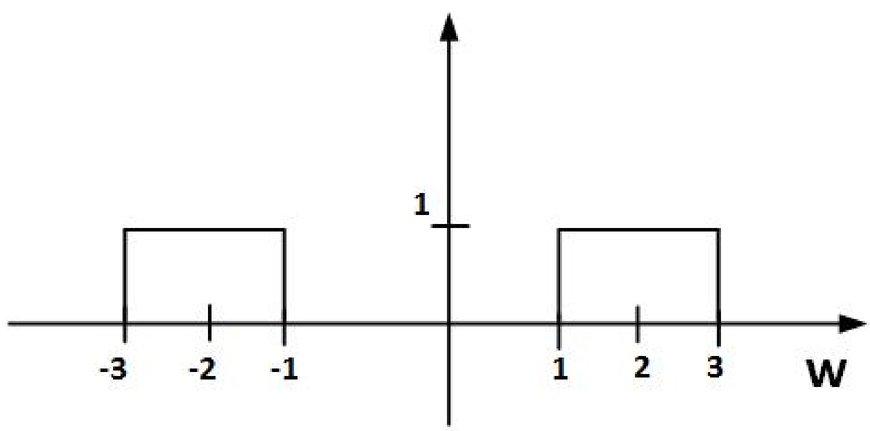

Figure 1:

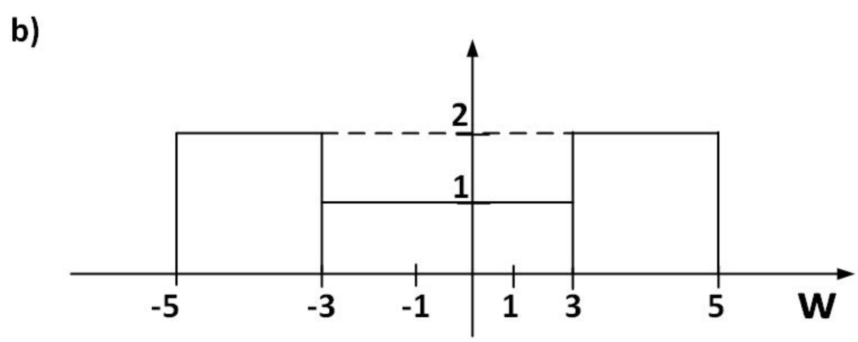

Figure 2:

(c)

$$\begin{aligned} B_{FM} &= 2(\Delta f + B) = 2\big(\frac{K_f X_p}{2\pi} + B\big) \\ B &= \frac{5}{2\pi} Hz, \quad X_p = X(0) = \frac{1}{\pi} + \frac{2}{\pi} + \frac{4}{\pi} = \frac{7}{\pi} \end{aligned} \tag{3}$$

Thus

$$B_{FM} = 2\big(\frac{10\pi . \frac{7}{\pi}}{2\pi} + \frac{5}{2\pi}\big) = \frac{75}{\pi} \tag{4}$$

(d)

$$\begin{aligned} x(t) &= Ae^{-10t}u(t) \\ X(\omega) &= A\int_{-\infty}^{\infty} e^{-10t}u(t)e^{-j\omega t}dt = \frac{A}{10 + j\omega} \end{aligned} \tag{5}$$

Use Parseval's theorem to find the bandwidth of x(t):

$$E_x = \frac{A^2}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = \frac{A^2}{20} \tag{6}$$

Call $\omega = 2\pi B$, ω must be such that

$$\begin{aligned} A^2\frac{0.95}{20} &= \frac{A^2}{2\pi}\int_{-\omega}^{\omega} |X(\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\omega}^{\omega} \frac{d\omega}{\omega^2 + 100} \\ &= \frac{1}{10\pi}\tan^{-1}(\frac{\omega}{10}) \implies \omega = 127.6\ rad/s \end{aligned}$$

and $B = 20.2$ Hz. Thus

$$B_{FM} = 2\left(\frac{AK_f}{2\pi} + B\right) = 2\left(\frac{AK_f}{2\pi} + 20.2\right) = 50.4Hz \implies A = 1$$