Communications I
Solutions to problem sheet seven

1. (a)
$$\phi_{PM}(t) = A\cos[\omega_c t + k_p m(t)] = 10\cos[10000t + k_p m(t)]$$
We know that $\phi_{PM}(t) = 10\cos(13000t)$ with $k_p = 1000$. Therefore, $m(t) = 3t$.

   (b)
$$\phi_{FM}(t) = A\cos[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha]$$
Since $k_f = 1000$, $3t = \int_0^t m(\alpha)d\alpha$. Therefore, $m(t) = 3$.

2. (a) Since an angle modulated signal is essentially a sinusoidal signal with constant amplitude, we have
$$P = \frac{A^2}{2} = \frac{100^2}{2} = 5000$$

   (b) The angle modulated signal can be interpreted both as a PM and an FM signal. It is a PM signal with $k_p = 4$ and message signal $m(t) = \sin(2000\pi t)$ and it is an FM signal with $k_f = 8000\pi$ and message signal $m(t) = \cos(2000\pi t)$

   (c) The instanteneous frequency is:
$$f_i = f_c + \frac{1}{2\pi}\frac{d}{dt}\theta(t) = f_c + \frac{4}{2\pi}\cos(2000\pi t)2000\pi = f_c + 4000\cos(2000\pi t)$$
Hence, $\Delta f = 4000$.

   (d) Baseband signal bandwidth $B = 1000$Hz, therefore $B_{EM} = 2(\Delta f + B) = 2(4000+1000) =$ 10kHz

3. (a) Since
$$\text{sinc}(400\pi t) \Longleftrightarrow \frac{1}{400}\text{rect}(\frac{\omega}{800\pi}),$$
the bandwidth of the message signal is $B = 400\pi/(2\pi) = 200Hz$ and the resulting modulation index
$$\beta_f = \frac{k_f max\{|m(t)|\}}{2\pi B} = \frac{k_f 10}{2\pi B} = 6 \Rightarrow k_f = 240\pi.$$
Hence, the modulated signal is
$$\begin{aligned} u(t) &= A\cos(2\pi f_c t + k_f \textstyle\int_{-\infty}^{t} m(\tau)d\tau) \\ &= 100\cos(2\pi f_c t + 2400\pi \textstyle\int_{-\infty}^{t} sinc(400\tau)d\tau) \end{aligned}$$

   (b) The maximum deviation of the modulated signal is
$$\Delta f_{max} = \beta_f B = 6 \times 200 = 1200Hz$$

   (c) Using Carson's rule, the effective bandwidth of the modulated signal can be approximated by
$$B_{FM} = 2(\beta_f + 1)B = 2(6+1)200 = 2800Hz$$