Solutions to Problem Sheet Four

Problem 1. The trigonometric identities used

1. $\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$
2. $\sin(\alpha)\cos(\beta) = \frac{1}{2}\sin(\alpha-\beta) + \frac{1}{2}\sin(\alpha+\beta)$

$$\begin{aligned} R_g(\tau) &= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\cos(\omega_0 t)\cos(\omega_0 t+\omega_0\tau)dt = \quad Using\ identity\ (1) \\ &= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\cos^2(\omega_0 t)cos(\omega_0\tau)dt - \underbrace{\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\sin(\omega_0\tau)\cos(\omega_0 t)\sin(\omega_0 t)dt}_{=0\ use\ identity\ 2} \\ &= \cos(\omega_0\tau)\underbrace{\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\cos^2(\omega_0 t)dt}_{=1/2} = \frac{\cos(\omega_0\tau)}{2} \end{aligned} \tag{1}$$

$$R_g(\tau) = \frac{\cos(\omega_0\tau)}{2} \Longleftrightarrow S_g(\omega) = \frac{\pi}{2}\Big[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\Big] \tag{2}$$

Problem 2. $|H(\omega)| = \frac{1}{1+\omega^2}$

(a)

$$\begin{aligned} P_g &= \frac{1}{2\pi}\int_{-1}^{+1} d\omega = \frac{1}{\pi} \\ P_y &= \frac{1}{2\pi}\int_{-1}^{+1}\frac{1}{1+\omega^2}\,d\omega = \frac{1}{4} \end{aligned} \tag{3}$$

(b)

$$\begin{aligned} P_g &= \frac{1}{2\pi}\int_{-\infty}^{+\infty}[\delta(\omega+1)+\delta(\omega-1)]d\omega = \frac{1}{\pi} \\ P_y &= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{1}{1+\omega^2}[\delta(\omega+1)+\delta(\omega-1)]\,d\omega \\ &= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{\delta(\omega+1)}{1+\omega^2}\,d\omega + \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{\delta(\omega-1)}{1+\omega^2}\,d\omega = \frac{1}{2\pi} \end{aligned} \tag{4}$$