$$\varphi(t) = g(t)\cos(\omega_0 t)$$

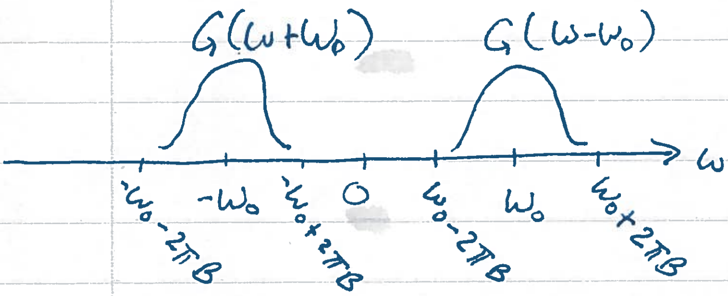
Slide 28 : $E_\varphi = \frac{1}{2} E_g$ ←

To prove

Slide 44 : $g(t)\cos(\omega_0 t) \Longleftrightarrow \frac{1}{2}\left[G(\omega+\omega_0) + G(\omega-\omega_0)\right]$

$$E_\varphi = \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} \frac{1}{4}\left|G(\omega+\omega_0)+G(\omega-\omega_0)\right|^2 d\omega$$

$$= \frac{1}{4}\cdot\frac{1}{2\pi}\cdot\int_{\omega=-\infty}^{\infty}\left[\left|G(\omega+\omega_0)\right|^2 + \left|G(\omega-\omega_0)\right|^2\right] . d\omega$$



$\because$ $G(\omega+\omega_0)$ & $G(\omega-\omega_0)$ do not overlap in the Spectrum because $\omega_0 \geq 2\pi B$ where B is the bandwidth of $g(t)$ in Hz.

$$\Rightarrow \quad E_\varphi = \frac{1}{4}\cdot\frac{1}{2\pi}\cdot 2\cdot\int_{\omega=-\infty}^{\infty}\left|G(\omega)\right|^2 d\omega$$

$$\Rightarrow \quad E_\varphi = \frac{1}{2}\cdot\frac{1}{2\pi}\int_{\omega=-\infty}^{\infty}\left|G(\omega)\right|^2 d\omega$$

$$\Rightarrow \quad E_\varphi = \frac{1}{2}E_g .$$

QED