EE1: Introduction to Signals and Communications

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Lecture Three

Lecture Aims

- To examine modulation process
- Baseband and bandpass signals
- Double Sideband Suppressed Carrier (DSB-SC)
  - Modulation
  - Demodulation
- One of various modulators
  - Switching modulator

Modulation

- Modulation is a process that causes a shift in the range of frequencies in a signal.

- Two types of communication systems
  - Baseband communication: communication that does not use modulation
  - Carrier modulation: communication that uses modulation

- The baseband is used to designate the band of frequencies of the source signal. (e.g., audio signal 4kHz, video 4.3MHz)

Modulation (continued)

In analog modulation the basic parameter such as amplitude, frequency or phase of a sinusoidal carrier is varied in proportion to the baseband signal $m(t)$. This results in amplitude modulation (AM) or frequency modulation (FM) or phase modulation (PM).

The baseband signal $m(t)$ is the modulating signal.

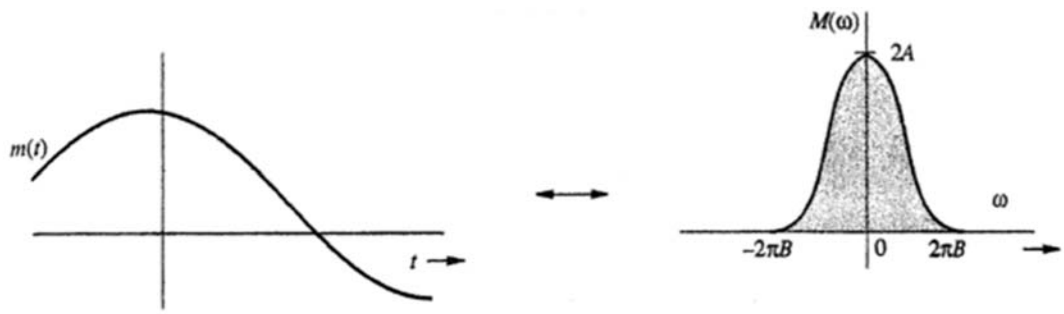
The sinusoid is the carrier or modulator.

Why modulation?

- To use a range of frequencies more suited to the medium
- To allow a number of signals to be transmitted simultaneously (frequency division multiplexing)
- To reduce the size of antennas in wireless links

Amplitude Modulation

- Carrier $A\cos(\omega_c t + \theta_c)$
  - Phase is constant $\theta_c = 0$
  - Frequency is constant
- Modulating signal $m(t)$

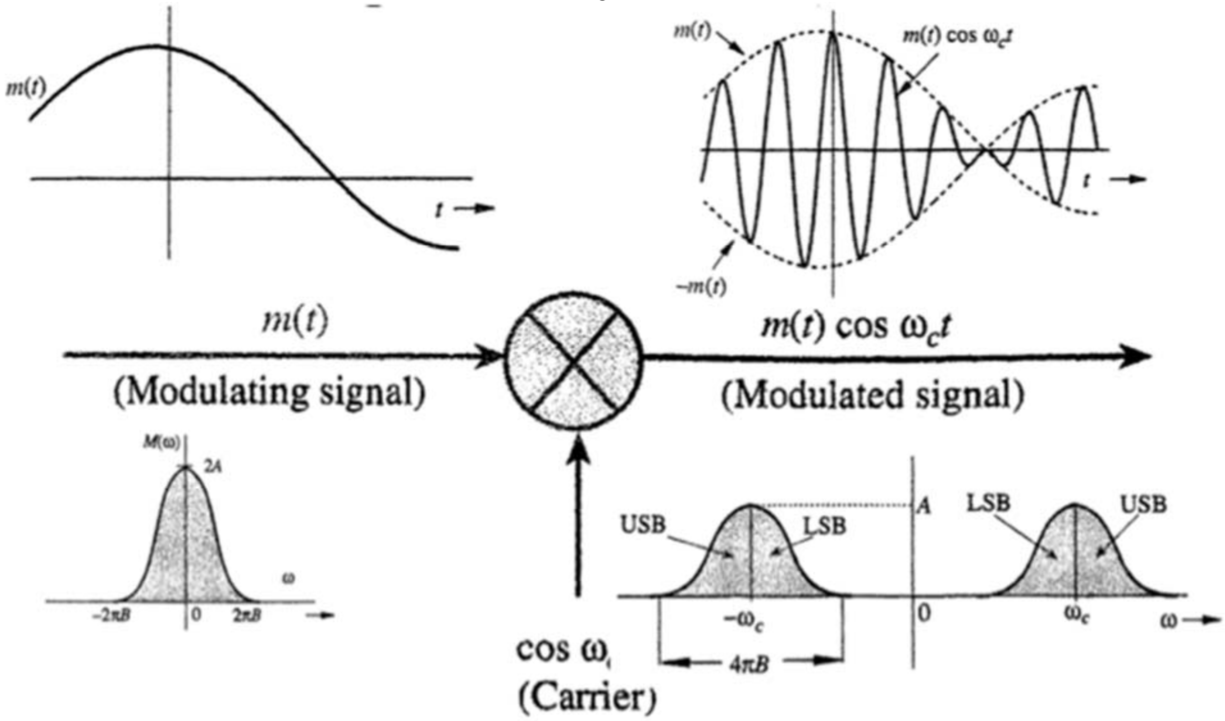


- With amplitude spectrum

$$m(t) \Leftrightarrow M(\omega)$$

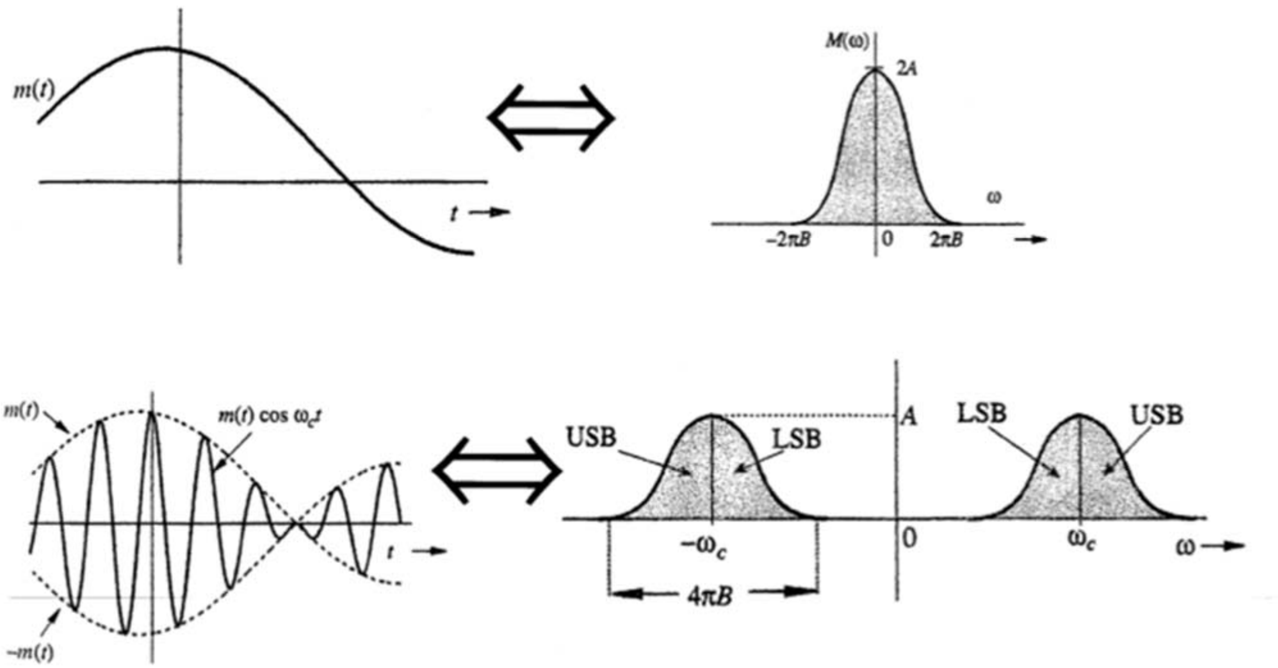
Modulated signal

- Modulated signal: $m(t)\cos\omega_c t$



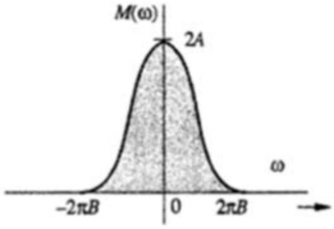
Modulated signal

- Modulated signal: $m(t)\cos\omega_c t$

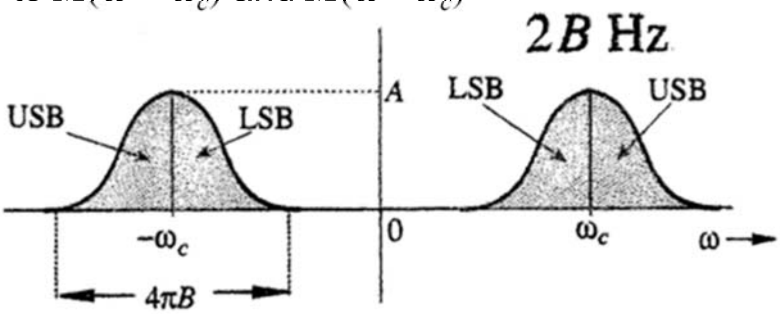


Modulated signal

- Baseband spectrum: $B\,Hz$



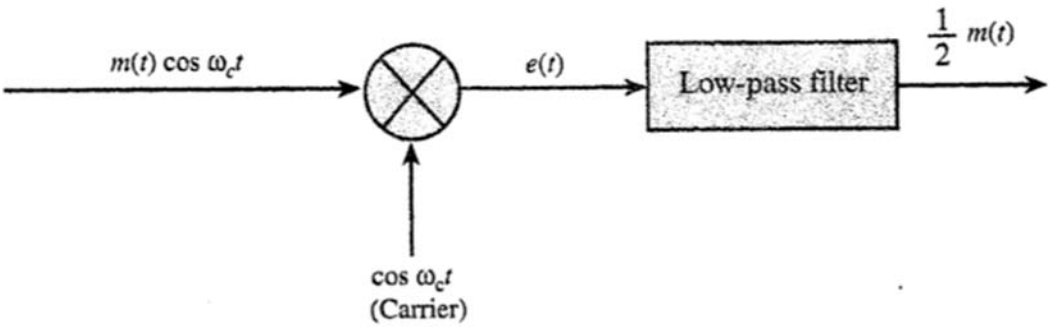
- $M(\omega)$ is shifted to $M(\omega + \omega_c)$ and $M(\omega - \omega_c)$



$$m(t)\cos\omega_c t \Leftrightarrow \frac{1}{2}\left[M(\omega+\omega_c)+M(\omega-\omega_c)\right]$$

Demodulation of DSB signal

- Process modulated signal $m(t)\cos\omega_c t$



- Multiply modulated signal with $\cos\omega_c t$

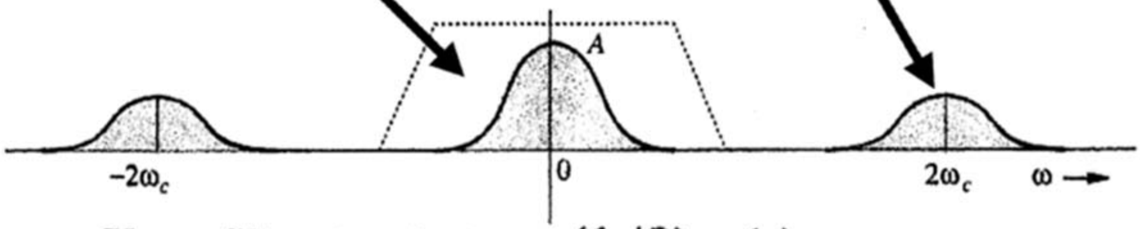
$$e(t) = m(t)\cos^2\omega_c t = \frac{1}{2}\left[m(t)+m(t)\cos 2\omega_c t\right]$$

$$E(\omega) = \frac{1}{2}M(\omega)+\frac{1}{4}\left[M(\omega+2\omega_c)+M(\omega-2\omega_c)\right]$$

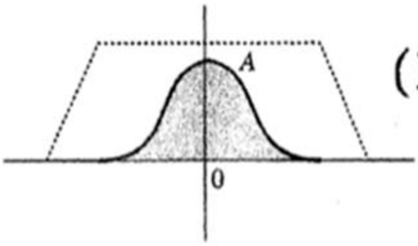
Demodulation of DSB signal

- Process modulated signal $m(t)\cos\omega_c t$

$$E(\omega) = \frac{1}{2}M(\omega) + \frac{1}{4}[M(\omega + 2\omega_c) + M(\omega - 2\omega_c)]$$



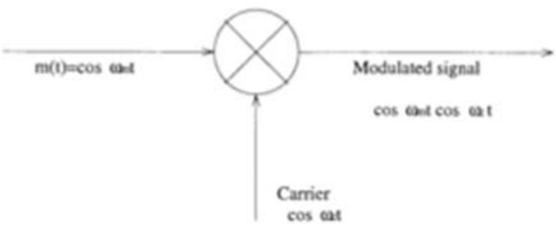
  - Use a filter to select $(1/2)m(t)$ remove



$(1/2)m(t)\cos 2\omega_c t$

Example: AM of a cosine signal

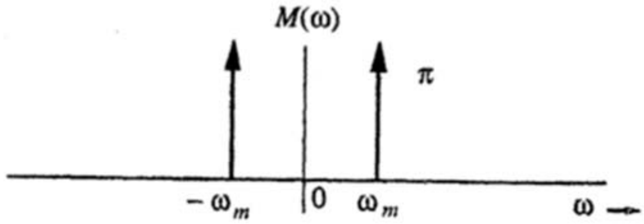
- Modulating signal $m(t)=\cos \omega_m t$
- Carrier $\cos \omega_c t$
- Modulated signal $\phi(t) = m(t) \cos \omega_c t = \cos \omega_m t \cos \omega_c t$



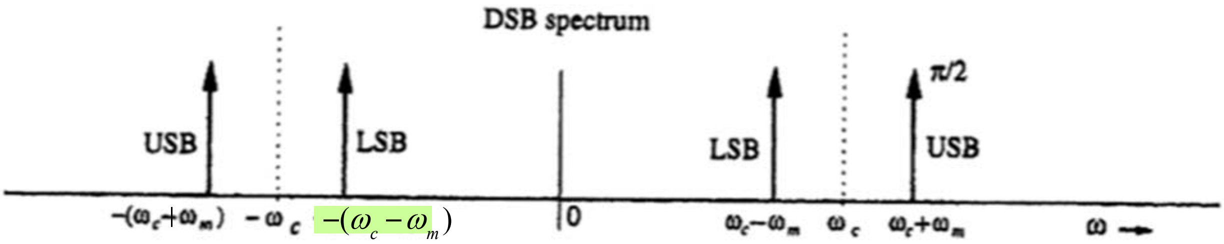
Amplitude spectrum

- Baseband signal

$$M(\omega)=\pi\left[\delta(\omega-\omega_m)+\delta(\omega+\omega_m)\right]$$

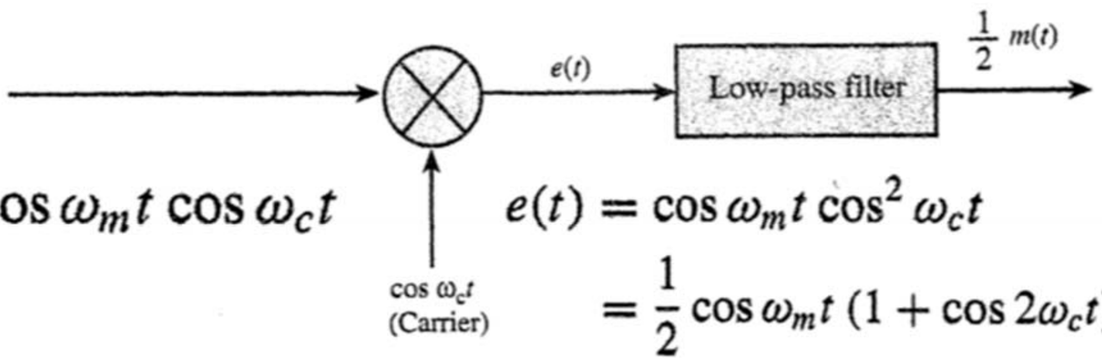


$$\varphi_{DSB-SC}(t)=\frac{1}{2}\left[\cos(\omega_c+\omega_m)t+\cos(\omega_c-\omega_m)t\right]$$



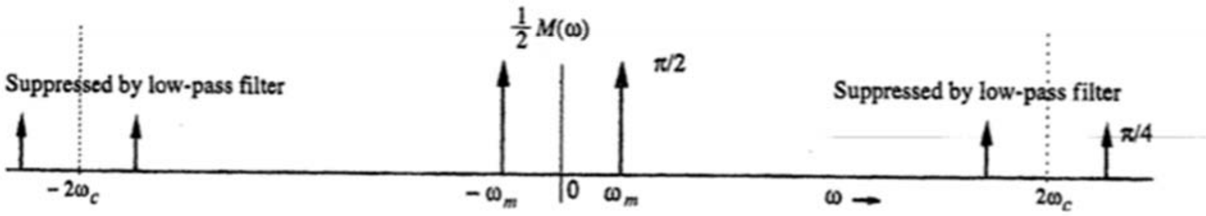
Demodulation of DSB signal

- Process modulated signal $m(t)\cos\omega_c t$



$\cos\omega_m t\cos\omega_c t$

$$e(t)=\cos\omega_m t\cos^2\omega_c t$$
$$=\frac{1}{2}\cos\omega_m t\,(1+\cos 2\omega_c t)$$



Modulators

- We need to implement multiplication $m(t)\cos\omega_c t$
- Among various methods, we can use
  - Switching modulators
- Switching modulators can be implemented using diodes
  - (not included in your exam)

Switching modulator using square pulses

- Consider a square pulse train

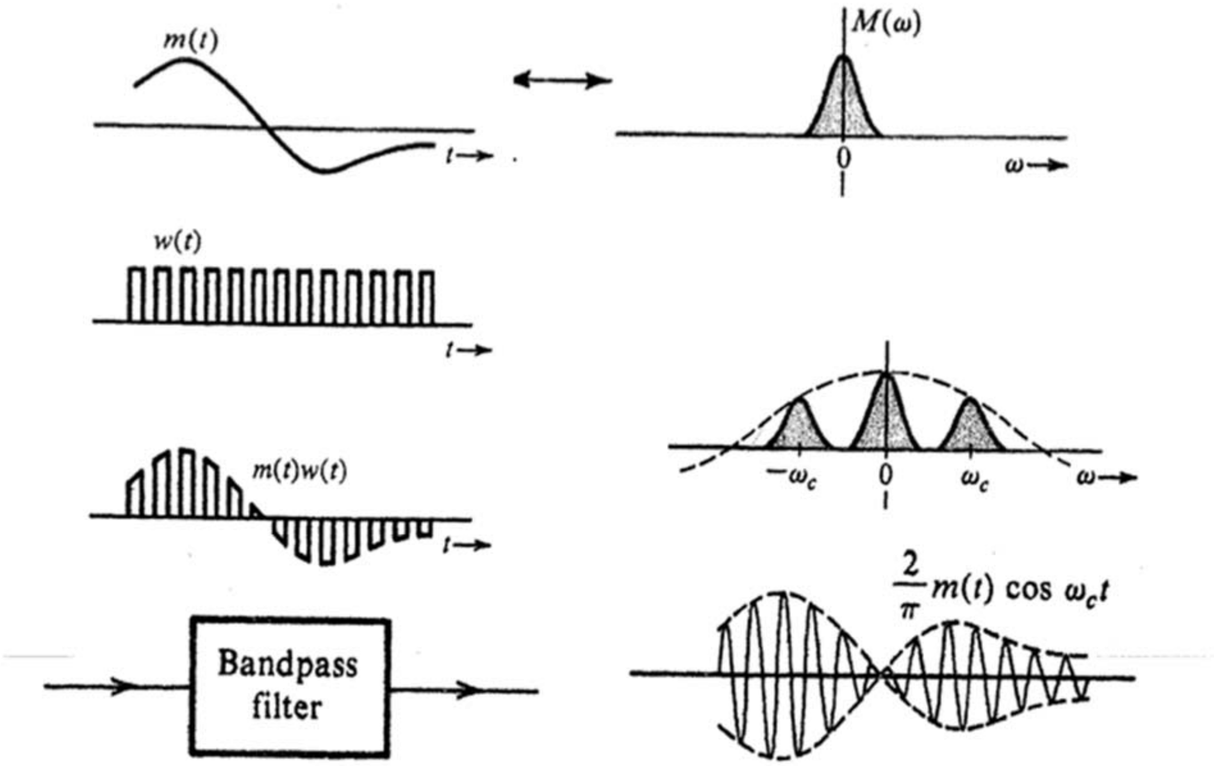
- The Fourier series for this periodic waveform is

$$w(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \cdots\right)$$

- The signal $m(t)w(t)$ is

$$m(t)w(t) = \frac{1}{2}m(t) + \frac{2}{\pi}\left[m(t)\cos\omega_c t - \frac{1}{3}m(t)\cos 3\omega_c t + \cdots\right.$$

Switching modulator



Double Sideband Suppressed Carrier

- A receiver must generate a carrier in frequency and phase synchronism with the carrier at the transmitter
- This calls for sophisticated receiver and could be quite costly
- An alternative is for the transmitter to transmit the carrier along with the modulated signal
- In this case the transmitter needs to transmit much larger power

Amplitude Modulation

- Carrier $A \ \cos(\omega_c t + \theta_c)$
  - Phase is constant $\theta_c = 0$
  - Frequency is constant.
- Modulation signal $m(t)$
- With amplitude spectrum $m(t) \Leftrightarrow M(\omega)$
- Full AM signal is

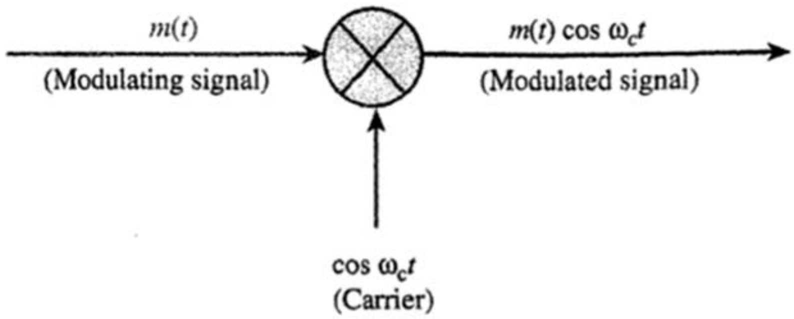
$$\varphi_{AM}(t) = A\cos\omega_c t + m(t)\cos\omega_c t$$
$$= \left[A + m(t)\right]\cos\omega_c t$$

- Spectrum of full AM signal

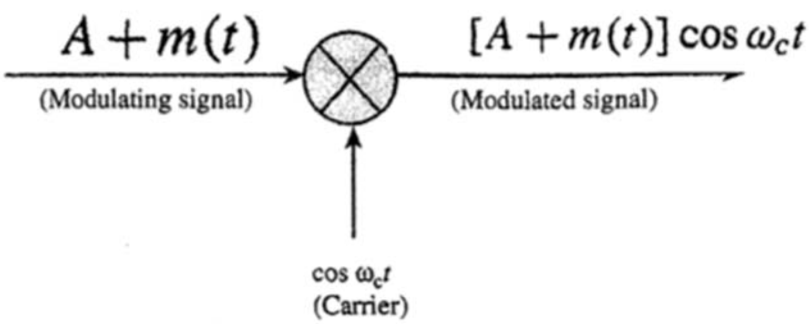
$$\varphi_{AM}(t) \Leftrightarrow \frac{1}{2}\left[M(\omega+\omega_c) + M(\omega-\omega_c)\right] + \pi A\left[\delta(\omega+\omega_c) + \delta(\omega-\omega_c)\right]$$

Full AM Modulated signal

- DSB Modulated signal:

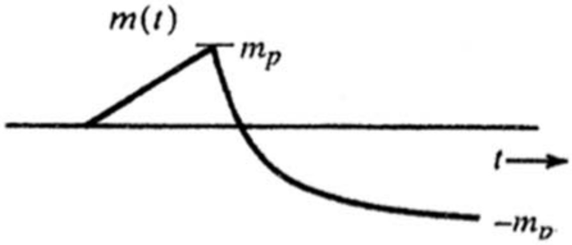


- Full AM signal



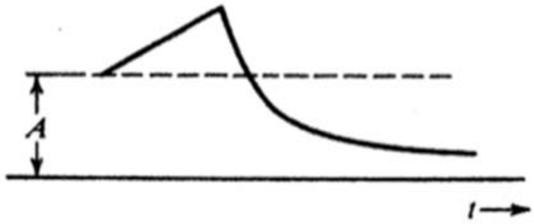
Full AM Modulated signal

- Signal



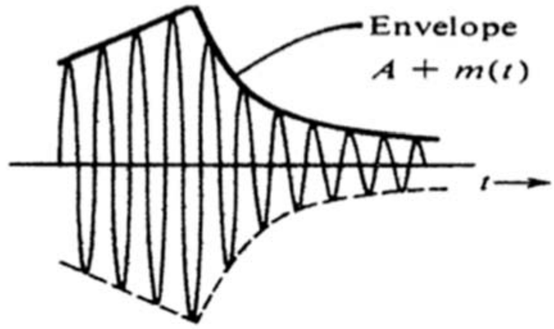
A + m(t) > 0 for all t

- Modulating signal



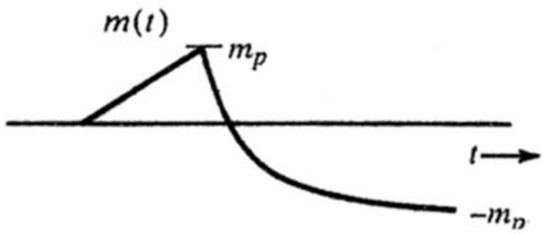
- Modulated signal:

$\left[A+m(t)\right]\cos\omega_c t$



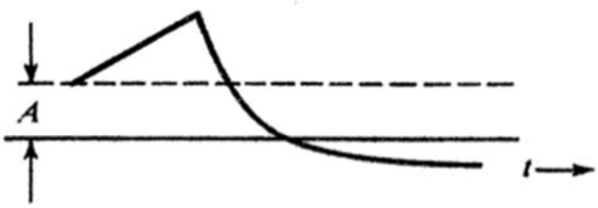
Envelope detection is not possible when

- Signal



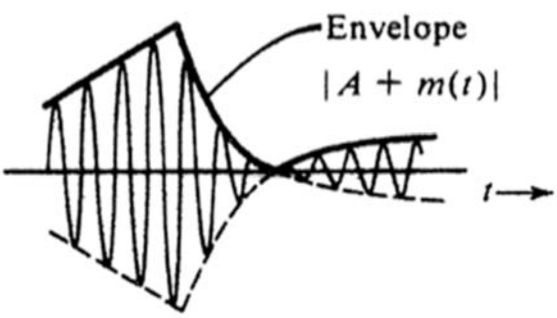
A + m(t) ≯ 0 for all t

- Modulating signal



- Modulated signal:

$\left[A+m(t)\right]\cos\omega_c t$



Envelope detection condition

- Detection condition $A + m(t) \geq 0$
- Let m_p be the maximum negative value of $m(t)$. This means that $m(t) \geq -m_p$
- When we have $A \geq m_p$, we can use envelope detector
- The parameter $\mu = \frac{m_p}{A}$ is called the modulation index
- When $0 \leq \mu \leq 1$, we can use an envelope detector

Envelope detection example

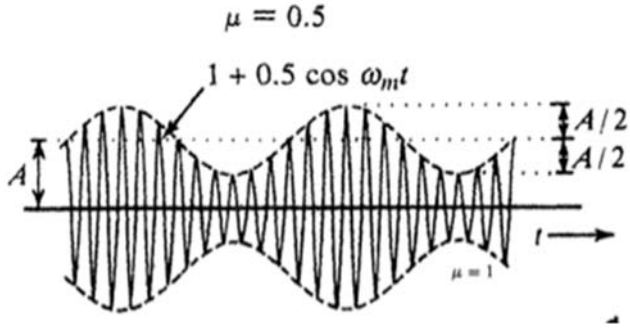
- Modulating signal $m(t) = B\cos\omega_m t$
- Modulating signal amplitude is $m_p = B$
- Hence $\mu = \frac{B}{A}$ and $B = \mu A$
- Modulating and modulated signals are

$$m(t) = B\cos\omega_m t = \mu A\cos\omega_m t$$

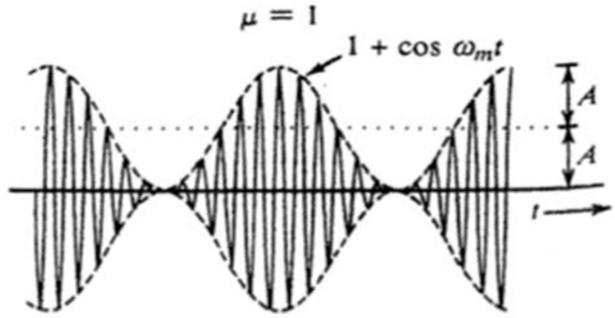
$$\varphi_{AM}(t) = \left[A + m(t)\right]\cos\omega_c t = A\left[1 + \mu\cos\omega_m t\right]\cos\omega_c t$$

Demodulation of DSB signal

- Consider modulation index to be $\mu = 0.5$



- For modulation index $\mu = 1$



Sideband and Carrier power

- Consider full AM signal

$$\varphi_{AM}(t) = \underbrace{A\cos\omega_c t}_{\text{carrier}} + \underbrace{m(t)\cos\omega_c t}_{\text{sidebands}}$$

- Power P_c of the carrier $A \cos \omega_c t$

$$A^2/2$$

- Power P_s of the sideband signals

$$0.5\,\widetilde{m^2(t)}$$

- Power efficiency

$$\eta = \frac{\text{useful power}}{\text{total power}} = \frac{P_s}{P_c + P_s} = \frac{\widetilde{m^2(t)}}{A^2 + \widetilde{m^2(t)}} 100\%$$

Maximum power efficiency of Full AM

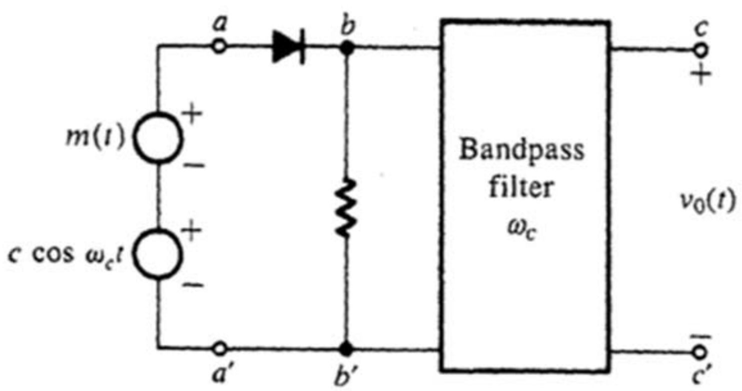
- When we have $m(t) = \mu A \cos \omega_m t$
- Signal power is $\widetilde{m^2(t)} = \frac{(\mu A)^2}{2}$
- When $0 \le \mu \le 1$
- When modulation index is unity, the efficiency is

  $\eta_{\max} = 33\%$
- When μ=0.3 the efficiency is

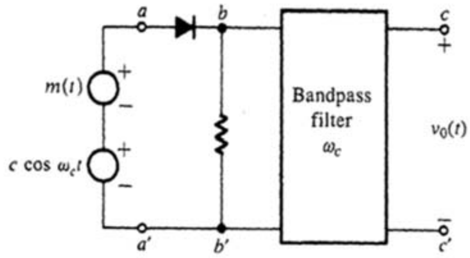
$$\eta = \frac{(0.3)^2}{2+(0.3)^2} 100\% = 4.3\%$$

Generation of AM signals

- Full AM signals can be generated using DSB-SC modulators
- But we do not need to suppress the carrier at the output of the modulator, hence we do not need a balanced modulators
- Use a simple diode



Simple diode modulator design



- Input signal $c\cos\omega_c t + m(t)$
- Consider the case $c >> m(t)$
- Switching action of the diode is controlled by

  $c\cos\omega_c t$

- A switching waveform

  is generated. The diode open and shorts periodically with $w(t)$

- The signal is generated

$$v_{bb'}(t) = \left[c\cos\omega_c t + m(t)\right] w(t)$$

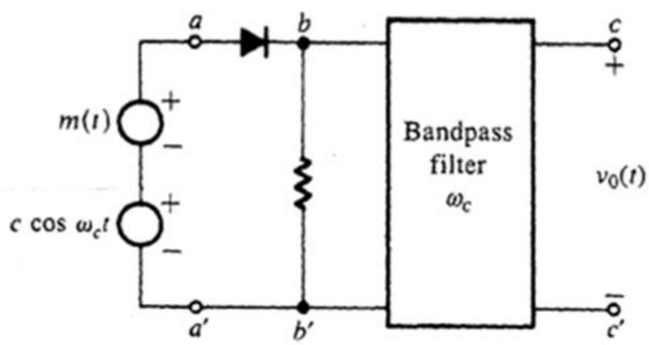
Diode Modulator

- Diode acts as a multiplier

$$v_{bb\text{n}}(t) = \left[c\cos\omega_c t + m(t)\right] w(t)$$

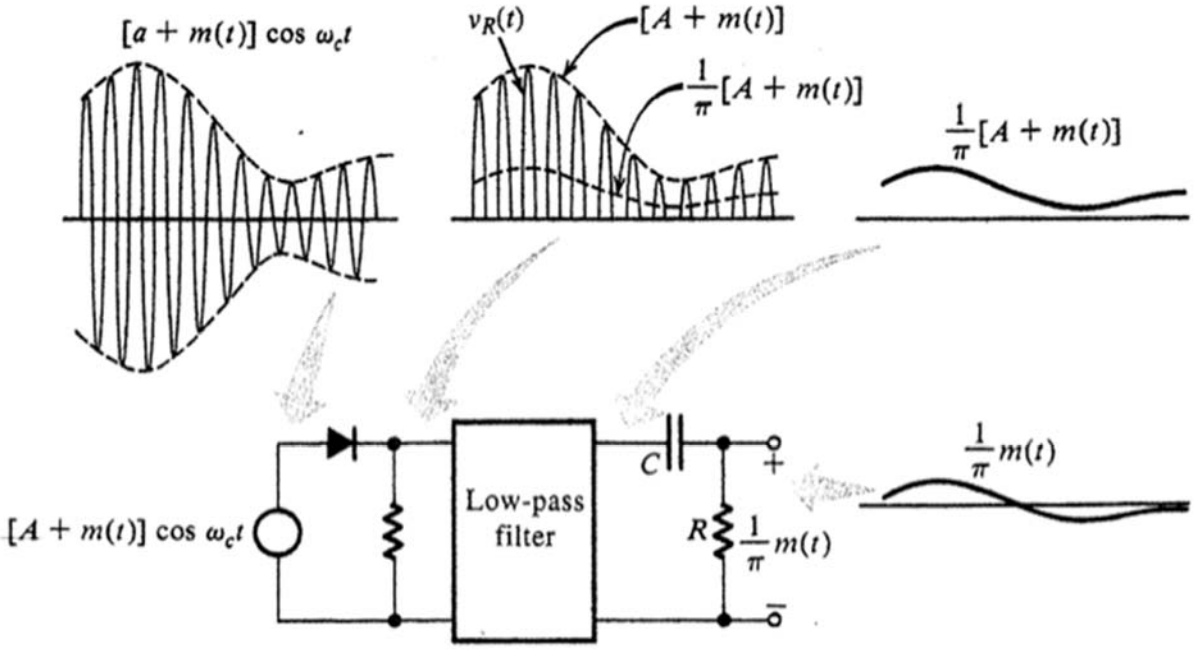
$$= \left[c\cos\omega_c t + m(t)\right]\left[\frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \cdots\right)\right]$$

$$= \underbrace{\frac{c}{2}\cos\omega_c t + \frac{2}{\pi} m(t)\cos\omega_c t}_{\text{AM}} + \underbrace{\text{other terms}}_{\substack{\text{suppressed by}\\ \text{bandpass filter}}}$$



Demodulation of AM signals

- Rectifier detector



Demodulation of AM signals

- Half-wave rectified signal υ_R is given by

$$\upsilon_R = \left\{\left[A + m(t)\right]\cos\omega_c t\right\} w(t)$$

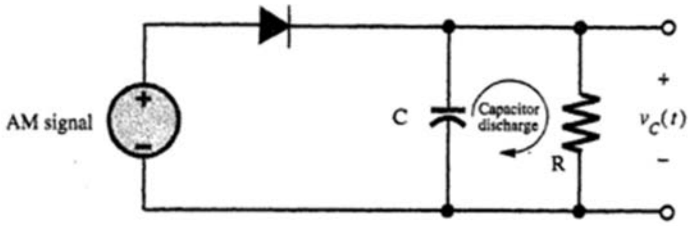
where $w(t)$

$$v_R = \left[A + m(t)\right]\cos\omega_c t\left[\frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \cdots\right)\right]$$

$$= \frac{1}{\pi}\left[A + m(t)\right] + \text{other terms of higher frequencies}$$

Demodulation of AM signals using an envelope detector

- Simple detector



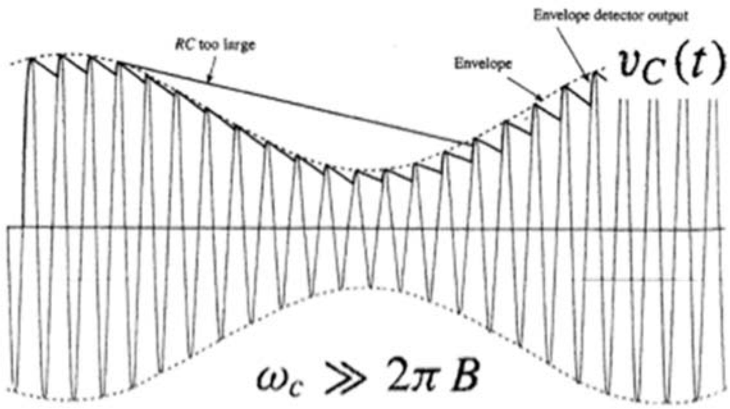
Large

$$RC \gg 1/\omega_c$$

But smaller than

$$1/2\pi B$$

- Detector operation

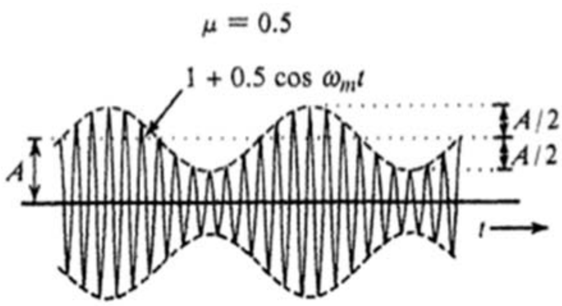


$$v_C(t) = A + m(t)$$

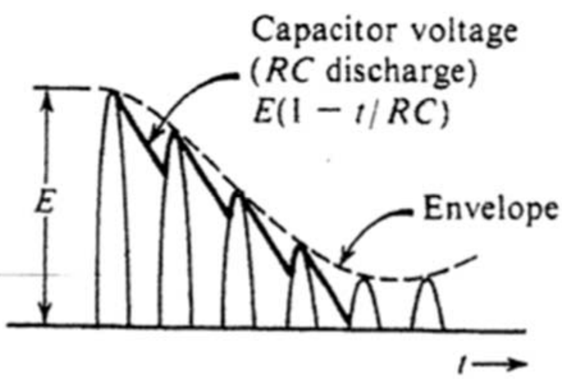
$$\omega_c \gg 2\pi B$$

Envelope detector example

- For the single tone



- Design envelope detector

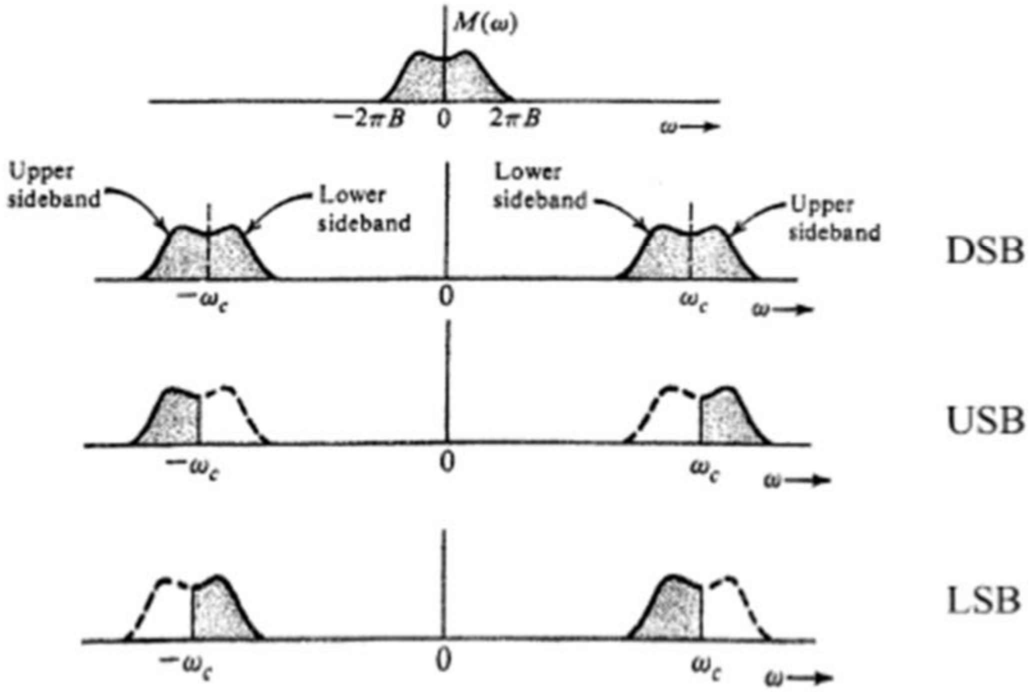


$$v_C = Ee^{-t/RC}$$

$$v_C \simeq E\left(1 - \frac{t}{RC}\right)$$

Double vs Upper/Lower Side Band (USB/LSB)

Modulated Signal



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Lecture Four

Lecture Aims

- Angle Modulation
  - Phase and Frequency modulation
  - Concept of instantaneous frequency
  - Examples of phase and frequency modulation
  - Power of angle-modulated signals

Angle modulation

Consider a modulating signal $m(t)$ and a carrier $v_c(t) = A\cos(\omega_c t + \theta_c)$.

The carrier has three parameters that could be modulated: the amplitude A (AM) the frequency ω_c (FM) and the phase θ_c (PM).

The latter two methods are closely related since both modulate the argument of the cosine.

Instantaneous frequency

- By definition a sinusoidal signal has a constant frequency and phase: $A\cos(\omega_c t + \theta_c)$
- Consider a generalized sinusoid with phase $\theta(t)$: $\phi(t) = A\cos\theta(t)$
- We define the instantaneous frequency ω_i as:

$$\omega_i(t) = \frac{d\theta}{dt}$$

- Hence, the phase is

$$\theta(t) = \int_{-\infty}^{t} \omega_i(\alpha)\,d\alpha.$$

Phase modulation

We can transmit the information of $m(t)$ by varying the angle θ of the carrier. In **phase modulation** (PM) the angle $\theta(t)$ is varied linearly with $m(t)$:

$$\theta(t) = \omega_c t + k_p m(t)$$

where k_p is a constant and ω_c is the carrier frequency. Therefore, the resulting PM wave is

$$\phi_{PM}(t) = A\cos\left[\omega_c t + k_p m(t)\right]$$

The instantaneous frequency in this case is given by

$$\omega_i(t) = \frac{d\theta}{dt} = \omega_c + k_p \dot{m}(t)$$

Frequency modulation

In PM the instantaneous frequency ω_i varies linearly with the **derivative** of $m(t)$. In **frequency modulation (FM)**, ω_i is varied linearly with $m(t)$. Thus

$$\omega_i(t) = \omega_c + k_f m(t).$$

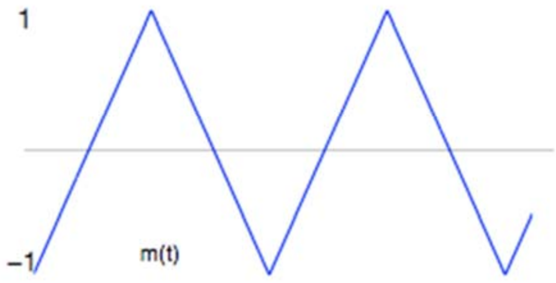
where k_f is a constant. The angle $\theta(t)$ is now

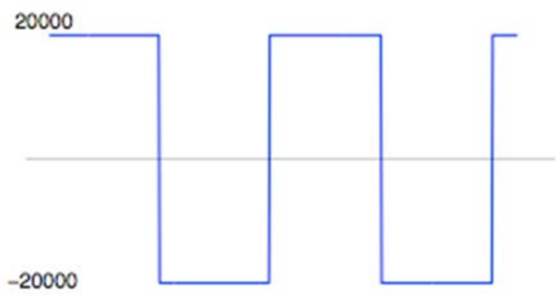
$$\theta(t) = \int_{-\infty}^{t} \left[\omega_c + k_f m(\alpha) \right] d\alpha = \omega_c t + k_f \int_{-\infty}^{t} m(\alpha) d\alpha.$$

The resulting FM wave is

$$\phi_{FM}(t) = A \cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha) d\alpha \right]$$

Example





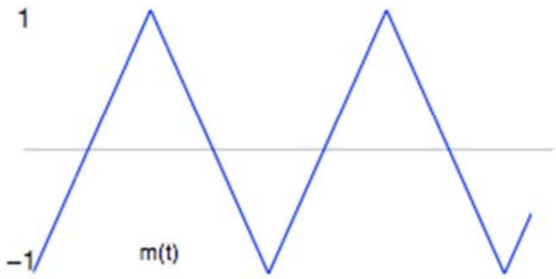
Sketch FM and PM signals if the modulating signal is the one above (on the left). The constants k_f and k_p are $2\pi \times 10^5$ and 10π, respectively, and the carrier frequency f_c =100$M\text{H}_z$.

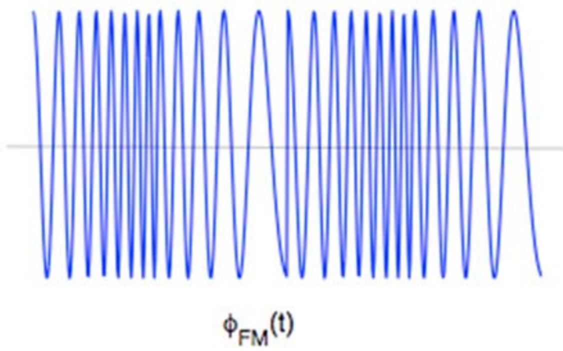
FM example

- Instantaneous angular frequency $\omega_i = \omega_c + k_f m(t)$
- Instantaneous frequency $f_i = f_c + \dfrac{k_f}{2\pi} m(t) = 10^8 + 10^5 m(t)$

$$(f_i)_{\min} = 10^8 + 10^5 \left[m(t)\right]_{\min} = 99.9 MHz$$

$$(f_i)_{\max} = 10^8 + 10^5 \left[m(t)\right]_{\max} = 100.1 MHz$$



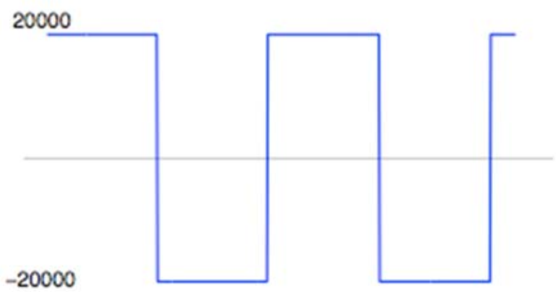


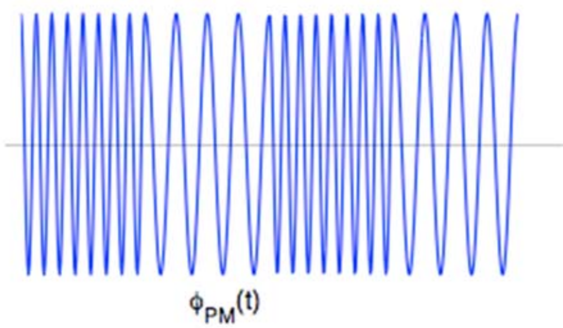
PM example

- Instantaneous frequency $f_i = f_c + \dfrac{k_p}{2\pi} \dot{m}(t) = 10^8 + 5\dot{m}(t)$

$$(f_i)_{\min} = 10^8 + 5\left[\dot{m}(t)\right]_{\min} = 10^8 - 10^5 = 99.9 MHz$$

$$(f_i)_{\max} = 10^8 + 5\left[\dot{m}(t)\right]_{\max} = 10^8 + 10^5 = 100.1 MHz$$





Bandwidth of angle modulated waves

In order to study bandwidth of FM waves, define

$$a(t) = \int_{-\infty}^{t} m(\alpha)d\alpha$$

and

$$\hat{\phi}_{FM}(t) = Ae^{j\left[\omega_c t + k_f a(t)\right]} = Ae^{jk_f a(t)} e^{j\omega_c t}$$

The frequency modulated signal is

$$\phi_{FM}(t) = \mathrm{Re}\left\{\hat{\phi}_{FM}(t)\right\}$$

Bandwidth of angle modulated waves

Expanding the exponential $e^{jk_f a(t)}$ in power series yields

$$\hat{\phi}_{FM}(t) = A\left[1 + jk_f a(t) - \frac{k_f^2}{2!}a^2(t) + \cdots + j^n \frac{k_f^n}{n!}a^n(t) + \cdots\right]e^{j\omega_c t}$$

and

$$\begin{aligned}\phi_{FM}(t) &= \mathrm{Re}\left\{\hat{\phi}_{FM}(t)\right\} \\ &= A\left[\cos\omega_c t - k_f a(t)\sin\omega_c t - \frac{k_f^2}{2!}a^2(t)\cos\omega_c t + \frac{k_f^3}{3!}a^3(t)\sin\omega_c t + \cdots\right]\end{aligned}$$

Narrow-Band Angle Modulation

The signal $a(t)$ is the integral of $m(t)$. It can be shown that if $M(\omega)$ is band limited to B, $A(\omega)$ is also band limited to B.

If $|k_f a(t)| \ll 1$ then all but the first term are negligible and

$$\phi_{FM}(t) \sim A\left[\cos\omega_c t - k_f a(t)\sin\omega_c t\right]$$

This case is called **narrow-band FM**.

Similarly, the **narrow-band PM** is given by

$$\phi_{PM}(t) \sim A\left[\cos\omega_c t - k_p m(t)\sin\omega_c t\right]$$

Narrow-Band Angle Modulation

Comparison of narrow band FM with Full AM.

Narrow band FM

$$\phi_{FM}(t) \sim A\left[\cos\omega_c t - k_f a(t)\sin\omega_c t\right]$$

Full AM

$$\left[A + m(t)\right]\cos\omega_c t = A\cos\omega_c t + m(t)\cos\omega_c t$$

Narrow band FM and full AM require a transmission bandwidth equal to $2B$ Hz. Moreover, the above equations suggest a way to generate narrowband FM or PM signals by using DSB-SC modulator

Wide-Band FM

- Assume that $|k_f\, a(t)| \ll 1$ is not satisfied.
- Cannot ignore higher order terms, but power series expansion analysis becomes complicated.
- The precise characterization of the FM bandwidth is mathematically intractable.
- Use an empirical rule (Carson's rule) which applies to most signals of interests.

Bandwidth equation

- Take the angular frequency deviation as $\Delta\omega = k_f\, m_p$ where $m_p = \max_t |m(t)|$ and frequency deviation as $\Delta f = \dfrac{k_f m_p}{2\pi}$.
- The transmission bandwidth of an FM signal is, with good approximation, given by

$$B_{FM} = 2(\Delta f + B) = 2\left(\frac{k_f m_p}{2\pi} + B\right)$$

Carson's rule

- The formula

$$B_{FM} = 2(\Delta f + B) = 2\left(\frac{k_f m_p}{2\pi} + B\right)$$

goes under the name of **Carson's rule**.

- If we define frequency deviation ratio as $\beta = \frac{\Delta f}{B}$
- Bandwidth equation becomes

$$B_{FM} = 2B(\beta + 1)$$

Wide-Band PM

- All results derived for FM can be applied to PM.
- Angular frequency deviation $\Delta\omega = k_p \dot{m}_p$ and frequency deviation $\Delta f = \frac{k_p \dot{m}_p}{2\pi}$

  where we assume $\dot{m}_p = \max_t |\dot{m}(t)|$

- The bandwidth for the PM signal will be

$$B_{PM} = 2\left(\frac{k_p \dot{m}_p}{2\pi} + B\right) = 2(\Delta f + B)$$

Verification of FM bandwidth

- To verify Carson's rule

$$B_{FM} = 2(\Delta f + B) = 2\left(\frac{k_f m_p}{2\pi} + B\right)$$

- Consider a single tone modulating sinusoid

$$m(t) = \alpha \cos \omega_m t \qquad a(t) = \int_{-\infty}^{t} m(\tau) d\tau = \frac{\alpha}{\omega_m} \sin \omega_m t$$

- We can express the FM signal as

$$\hat{\varphi}_{FM}(t) = Ae^{j(\omega_c t + \frac{k_f \alpha}{\omega_m} \sin \omega_m t)}$$

Verification of FM bandwidth

- The angular frequency deviation is $\Delta\omega = k_f m_p = \alpha k_f$

- Since the bandwidth of $m(t)$ is $B = f_m Hz$, the frequency deviation ratio (or modulation index) is

$$\beta = \frac{\Delta f}{f_m} = \frac{\Delta\omega}{\omega_m} = \frac{\alpha k_f}{\omega_m}$$

- Hence the FM signal become

$$\hat{\varphi}_{FM}(t) = Ae^{(j\omega_c t + j\beta \sin \omega_m t)} = Ae^{j\omega_c t}\left(e^{j\beta \sin \omega_m t}\right)$$

Verification of FM bandwidth

The exponential term $e^{j\beta \sin \omega_m t}$ is a periodic signal with period $2\pi/\omega_m$ and can be expanded by the exponential Fourier series:

$$e^{j\beta \sin \omega_m t} = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_m t}$$

where

$$C_n = \frac{\omega_m}{2\pi} \int_{-\pi/\omega_m}^{\pi/\omega_m} e^{j\beta \sin \omega_m t} e^{-jn\omega_m t} dt$$

Bessel functions

By changing variables $\omega_m t = x$, we get

$$C_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{(j\beta \sin x - jnx)} dx$$

This integral is denoted as the Bessel function $J_n(\beta)$ of the first kind and order n. It cannot be evaluated in closed form but it has been tabulated.

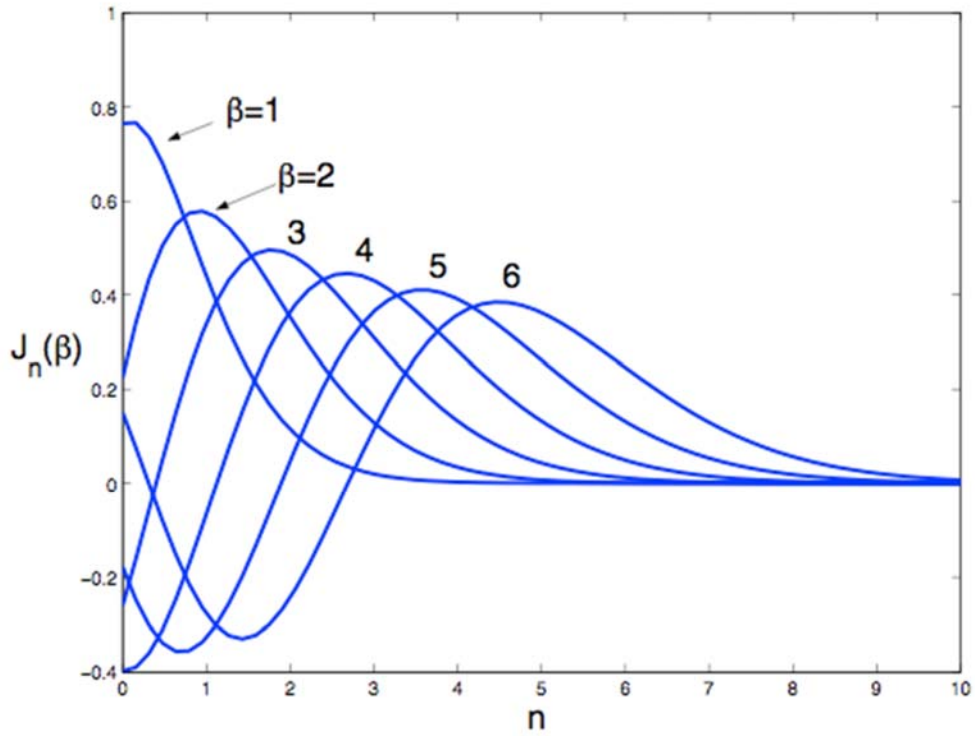
Hence the FM waveform can be expressed as

$$\hat{\varphi}_{FM}(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) e^{(j\omega_c t + jn\omega_m t)}$$

and

$$\varphi_{FM}(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t$$

Bessel functions of the first kind



Bandwidth calculation for FM

The FM signal for single tone modulation is

$$\varphi_{FM}(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t.$$

The modulated signal has 'theoretically' an infinite bandwidth made of one carrier at frequency ω_c and an infinite number of sidebands at frequencies $\omega_c \pm \omega_m$, $\omega_c \pm 2\omega_m$, ..., $\omega_c \pm n\omega_m$, ... However

- for a fixed β, the amplitude of the Bessel function $J_n(\beta)$ decreases as n increases. This means that for any fixed β there is only a finite number of significant sidebands.
- As $n > \beta + 1$ the amplitude of the Bessel function becomes negligible. Hence, the number of significant sidebands is $\beta + 1$.

This means that with good approximation the bandwidth of the FM signal is

$$B_{FM} = 2nf_m = 2(\beta + 1) f_m = 2(\Delta f + B).$$

Example

Estimate the bandwidth of the FM signal when the modulating signal is the one shown in Fig. 1 with period $T = 2 \times 10^{-4}$ sec, the carrier frequency is $f_c = 100\text{MHz}$ and $k_f = 2\pi \times 10^5$.

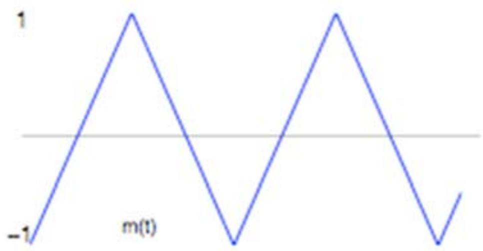


Figure 1: The modulating signal $m(t)$

Repeat the problem when the amplitude of $m(t)$ is doubled.

Example

- Peak amplitude of $m(t)$ is $m_p = 1$.
- Signal period is $T = 2 \times 10^{-4}$, hence fundamental frequency is $f_0 = 5\text{kHz}$.
- We assume that the essential bandwidth of $m(t)$ is the third harmonic. Hence the modulating signal bandwidth is $B = 15\text{kHz}$.
- The frequency deviation is:

$$\Delta f = \frac{1}{2\pi} k_f m_p = \frac{1}{2\pi}\left(2\pi \times 10^5\right)\left(1\right) = 100kHz.$$

- Bandwidth of the FM signal:

$$B_{FM} = 2\left(\Delta f + B\right) = 230kHz.$$

Example

- Doubling amplitude means that $m_p = 2$.
- The modulating signal bandwidth remains the same, i.e., $B = 15\text{kHz}$.
- The new frequency deviation is :

$$\Delta f = \frac{1}{2\pi} k_f m_p = \frac{1}{2\pi}\left(2\pi \times 10^5\right)\left(2\right) = 200kHz.$$

- The new bandwidth of the FM signal is :

$$B_{FM} = 2\left(\Delta f + B\right) = 430kHz.$$

Example

Now estimate the bandwidth of the FM signal if the modulating signal is time expanded by a factor 2.

- The time expansion by a factor 2 reduces the signal bandwidth by a factor 2. Hence the fundamental frequency is now $f_0 = 2.5\text{kHz}$ and $B = 7.5\text{kHz}$.
- The peak value stays the same, i.e., $m_p = 1$ and

$$\Delta f = \frac{1}{2\pi} k_f m_p = \frac{1}{2\pi}\left(2\pi \times 10^5\right)\left(1\right) = 100kHz.$$

- The new bandwidth of the FM signal is:

$$B_{FM} = 2\left(\Delta f + B\right) = 2\left(100 + 7.5\right) = 215kHz.$$

Second Example

An angle modulated signal with carrier frequency $\omega_c = 2\pi \times 10^5$ rad/s is given by:

$$\varphi_{FM}(t) = 10\cos\left(\omega_c t + 5\sin 3000t + 10\sin 2000\pi t\right).$$

- Find the power of the modulated signal
- Find the frequency deviation Δf
- Find the deviation ration $\beta = \frac{\Delta f}{B}$
- Estimate the bandwidth of the FM signal

Second Example

- The carrier amplitude is 10 therefore the power is $P = 10^2 / 2 = 50$.
- The signal bandwidth is $B = 2000\pi / 2\pi = 1000\text{Hz}$.
- To find the frequency deviation we find the instantaneous frequency:

$$\omega_i = \frac{d}{dt}\theta(t) = \omega_c + 15,000\cos 3000t + 20,000\pi\cos 2000\pi t.$$

The angle deviation is the maximum of $15,000 \cos 3000t + 20,000\pi \cos 2000\pi t$. The maximum is: $\Delta\omega = 15,000 + 20,000\pi$rad/s. Hence, the frequency deviation is

$$\Delta f = \frac{\Delta\omega}{2\pi} = 12,387.32Hz.$$

- The modulation index is

$$\beta = \frac{\Delta f}{B} = 12.387.$$

- The bandwidth of the FM signal is: $B_{FM} = 2\left(\Delta f + B\right) = 26,774.65Hz.$

Achieve angle modulation by use of non-linearity

- FM signals are constant envelope signals, therefore they are less susceptible to non-linearity
- Example: a non-linear device whose input $x(t)$ and output $y(t)$ are related by

$$y(t) = a_1 x(t) + a_2 x^2(t)$$

- if $x(t) = \cos\left[\omega_c t + \psi(t)\right]$
- Then

$$y(t) = a_1 \cos\left[\omega_c t + \psi(t)\right] + a_2 \cos^2\left[\omega_c t + \psi(t)\right]$$
$$= \frac{a_2}{2} + a_1 \cos\left[\omega_c t + \psi(t)\right] + \frac{a_2}{2} \cos\left[2\omega_c t + 2\psi(t)\right]$$

Angle modulation and non-linearity

- For FM wave

$$\psi(t) = k_f \int m(\alpha) d\alpha$$

- The output waveform is

$$y(t) = \frac{a_2}{2} + a_1 \cos\left[\omega_c t + k_f \int m(\alpha) d\alpha\right] + \frac{a_2}{2} \cos\left[2\omega_c t + 2k_f \int m(\alpha) d\alpha\right]$$

- Unwanted signals can be removed by means of a bandpass filter

Higher order non-linearity

- Consider higher order non-linearities

$$y(t) = a_0 + a_1 x(t) + a_2 x^2(t) + \cdots + a_n x^n(t)$$

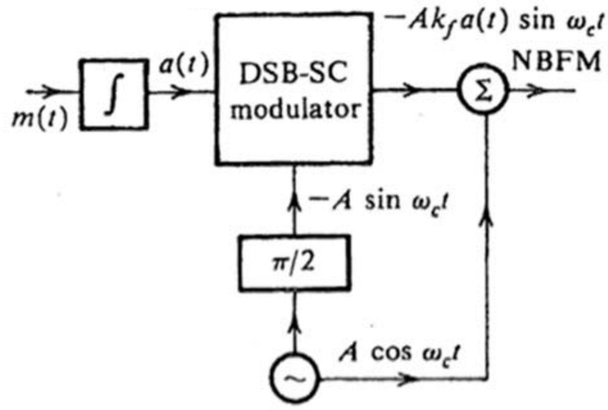
- If the input signal is an FM wave, $y(t)$ will have the form

$$y(t) = c_0 + c_1 \cos\left[\omega_c t + k_f \int m(\alpha)d\alpha\right] + c_2 \cos\left[2\omega_c t + 2k_f \int m(\alpha)d\alpha\right] + \cdots + c_n \cos\left[n\omega_c t + nk_f \int m(\alpha)d\alpha\right]$$

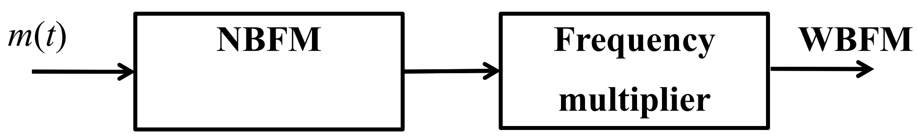
- The deviations are Δf, $2\Delta f$, ..., $n\Delta f$

From Narrowband to Wideband Frequency Modulation (NBFM to WBFM)

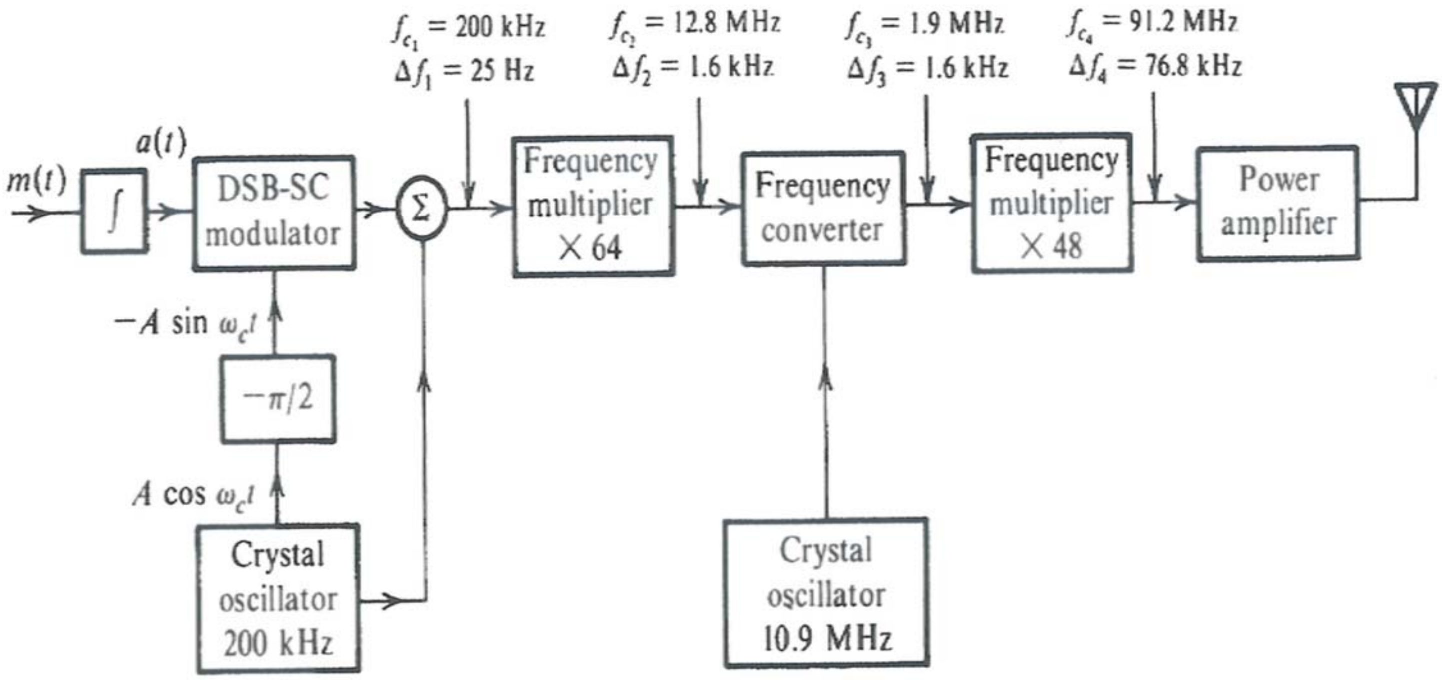
- Narrowband signal is generated using



- NBFM signal is then converted to WBFM using



Armstrong indirect FM transmitter



Direct method of FM generation

- The modulating signal $m(t)$ can control a voltage controlled oscillator to produce instantaneous frequency

$$\omega_i(t) = \omega_c + k_f m(t)$$

- A voltage controlled oscillator can be implemented using an LC parallel resonant circuit with centre frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

- If the capacitance is varied by $m(t)$

$$C = C_0 - km(t)$$

Direct method of FM generation

- The oscillator frequency is given by

$$\omega_i(t) = \frac{1}{\sqrt{LC_0\left[1-\frac{km(t)}{C_0}\right]}} = \frac{1}{\sqrt{LC_0}\left[1-\frac{km(t)}{C_0}\right]^{1/2}}$$

- If $\frac{km(t)}{C_0} \ll 1$, the binomial series expansion gives

$$\omega_i(t) \sim \frac{1}{\sqrt{LC_0}}\left[1+\frac{km(t)}{2C_0}\right]$$

- This gives the instantaneous frequency as a function of the modulating signal.

Demodulation of FM signals

- The FM demodulator is given by a differentiator followed by an envelope detector

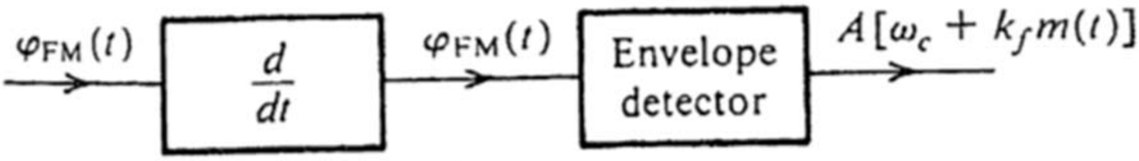
- Output of the ideal differentiator

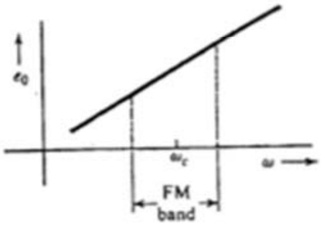
$$\varphi_{FM}(t) = \frac{d}{dt}\left\{A\cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right]\right\}$$
$$= A\left[\omega_c + k_f m(t)\right]\sin\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right]$$

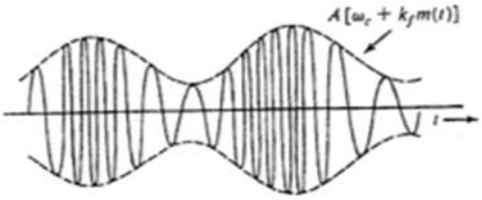
- The above signal is both amplitude and frequency modulated. Hence, an envelope detector with input $\varphi_{FM}(t)$ yields an output proportional to

$$A\left[\omega_c + k_f m(t)\right]$$

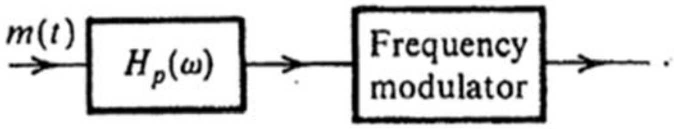
● As $\Delta\omega = k_f m_p < \omega_c$ and $\omega_c + k_f m(t) > 0$ for all t. The modulating signal $m(t)$ can be obtained using and envelope detector



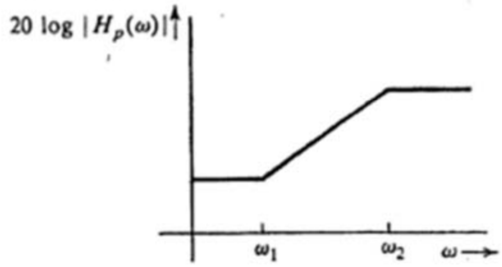


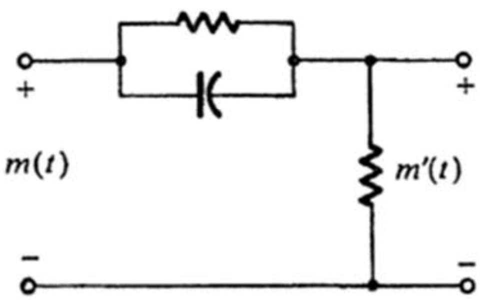


● To improve noise immunity of FM signals we use a pre-emphasis circuit ant transmitter

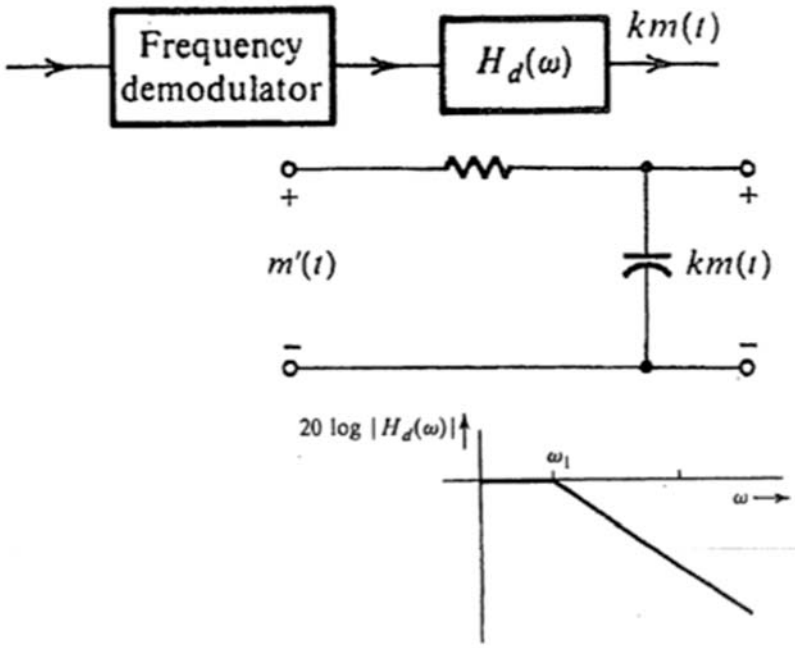


$$H_p(\omega) = K\frac{j\omega + \omega_1}{j\omega + \omega_2}$$

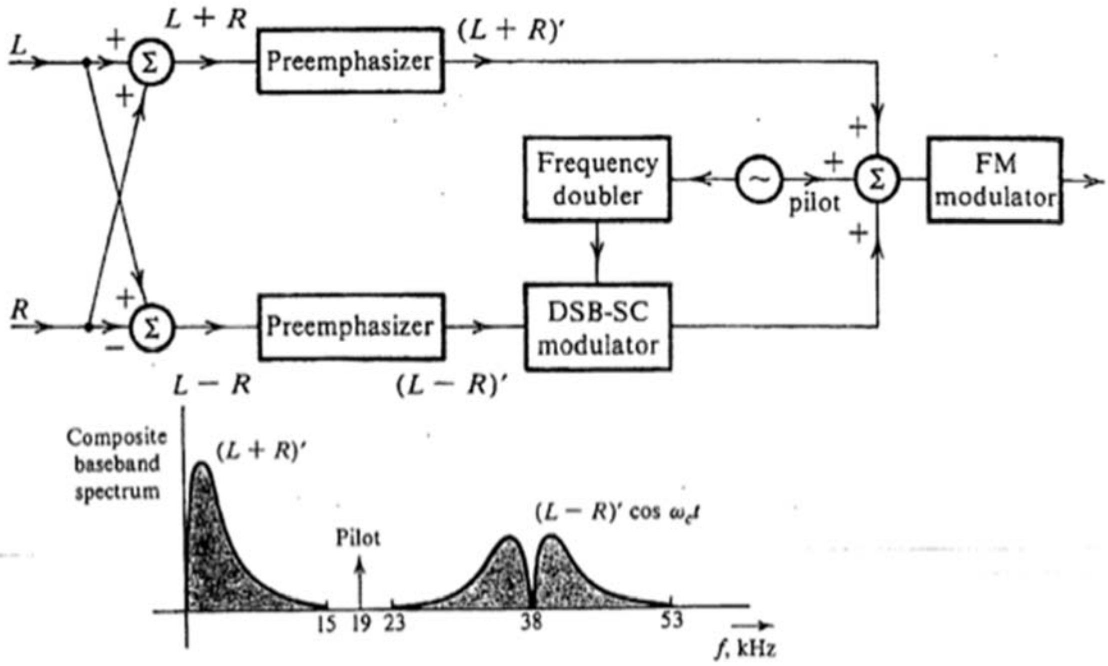




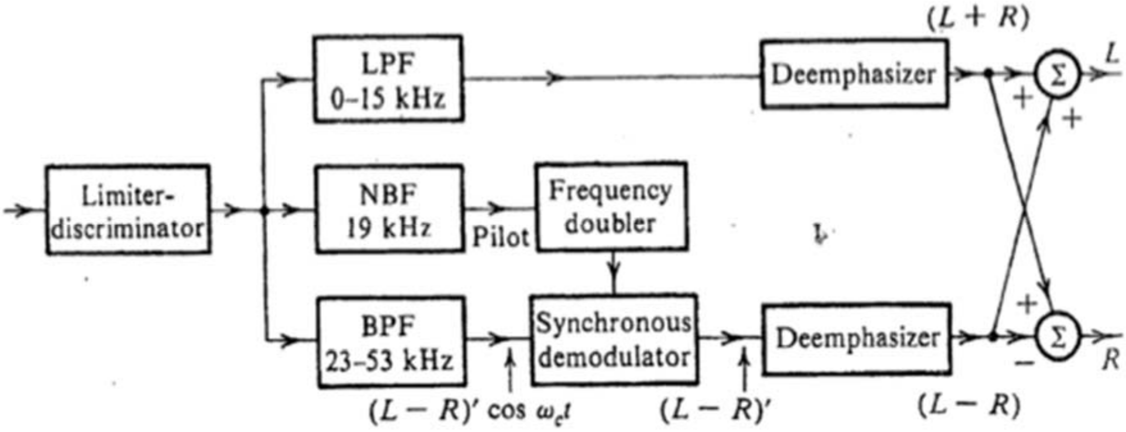
- Receiver de-emphasis circuit



- Transmitter



- Receiver



EE1: Introduction to Signals and Communications

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Lecture Five

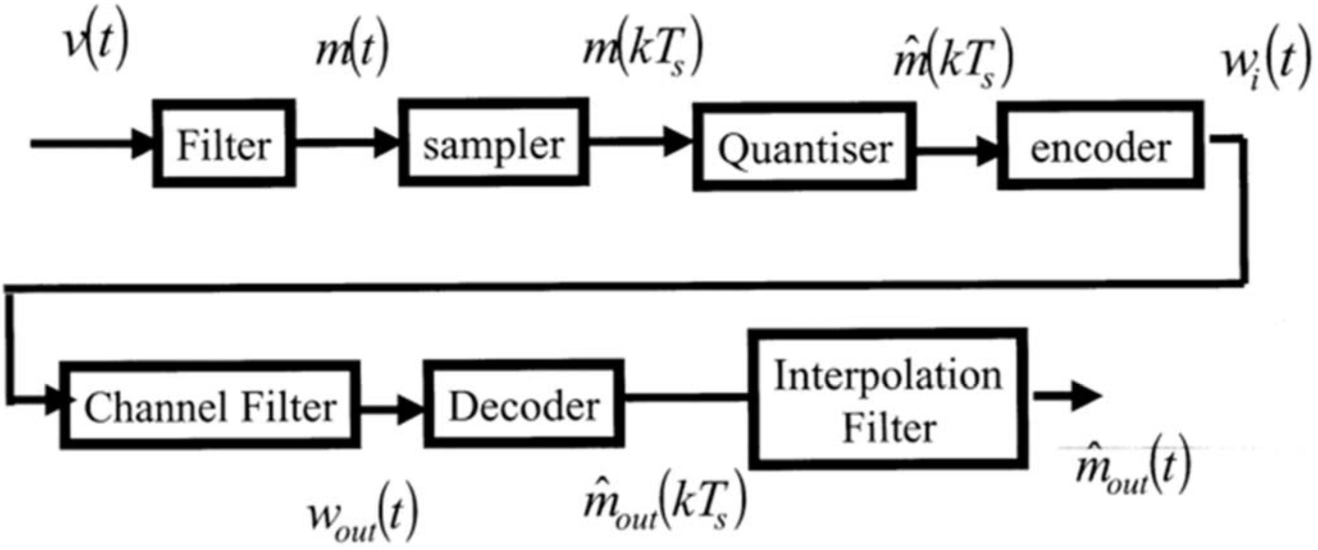
Lecture Aims

- Outline digital communication systems

Why digital modulation?

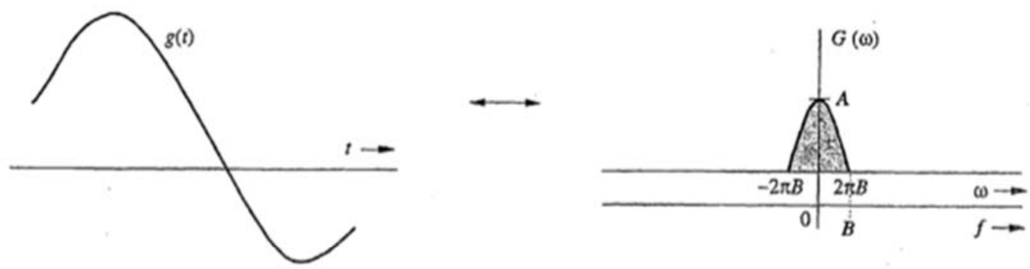
- More resilient to noise
- Viability of regenerative repeaters
- Digital hardware more flexible
- It is easier to multiplex digital signals

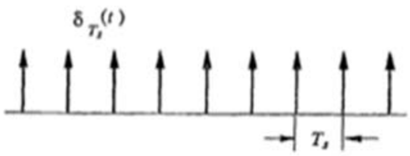
Digital transmission system



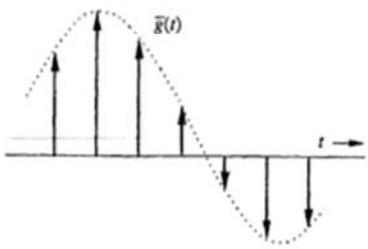
Analogue waveform

- Analogue waveform and its spectrum

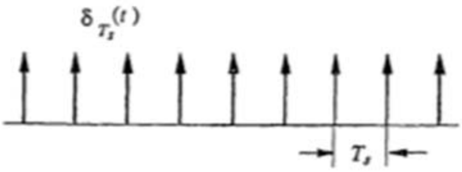




Sampling waveform

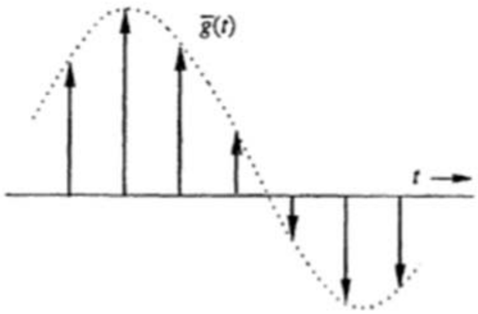


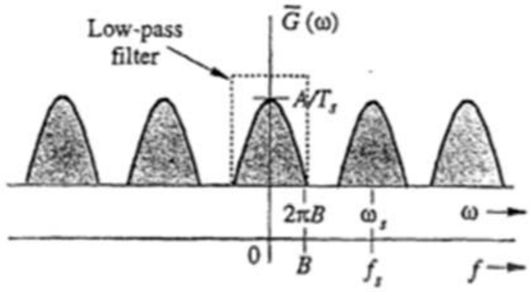
Sampled waveform



$$\omega_s = \frac{2\pi}{T_s} = 2\pi f_s$$

$$\delta_{T_s}(t) = \frac{1}{T_s}\left[1 + 2\cos\omega_s t + 2\cos 2\omega_s t + 2\cos 3\omega_s t + \cdots\right]$$

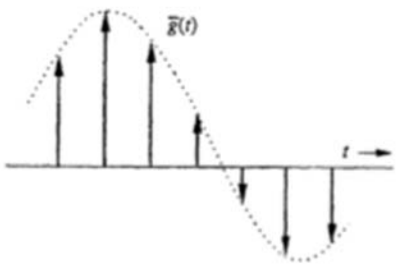


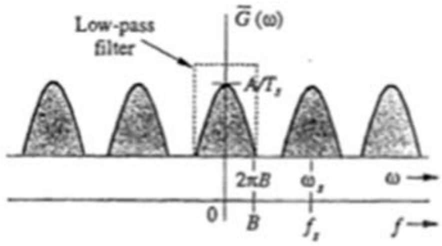


$$\bar{g}(t) = g(t)\delta_{T_s}(t)$$

$$= \frac{1}{T_s}\left[g(t) + 2g(t)\cos\omega_s t + 2g(t)\cos 2\omega_s t + 2g(t)\cos 3\omega_s t + \cdots\right]$$

Sampled signal spectrum





Sampling frequency

$$f_s = 1/T_s \, Hz$$

Sampling time

$$T_s = 1/f_s$$

Also have

Sampled signal spectrum

$$\bar{G}(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} G(\omega - n\omega_s)$$

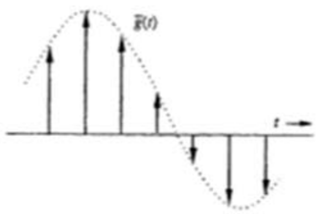
Sampling frequency must satisfy

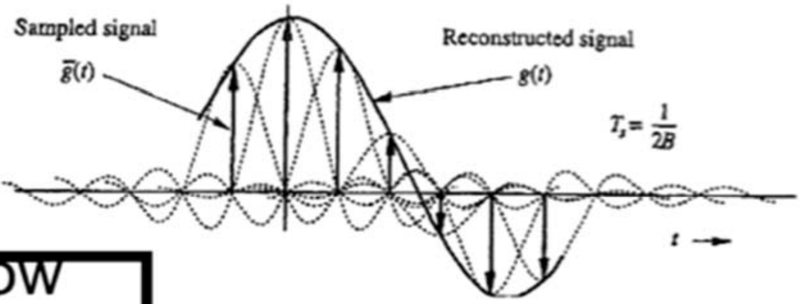
$$f_s > 2B$$

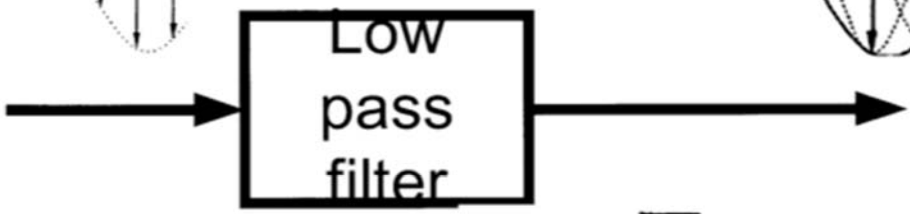
$$T_s < \frac{1}{2B}$$

Signal construction using better filter

$$2BT_s = 1$$





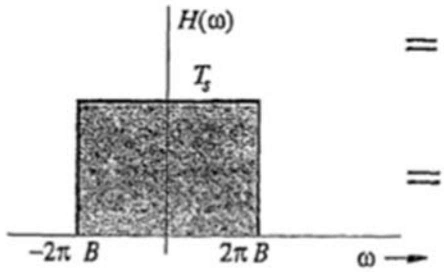


$$h(t) = \operatorname{sinc}(2\pi Bt)$$

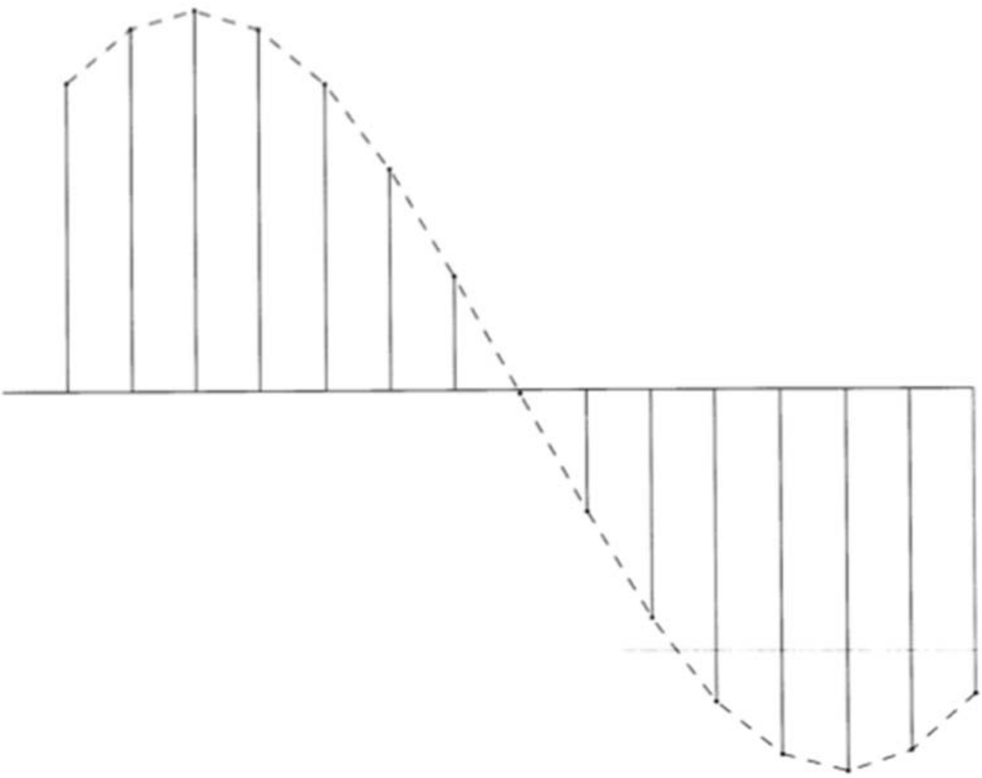
$$g(t) = \sum_k g(kT_s)h(t - kT_s)$$

$$= \sum_k g(kT_s)\operatorname{sinc}[2\pi B(t - kT_s)]$$

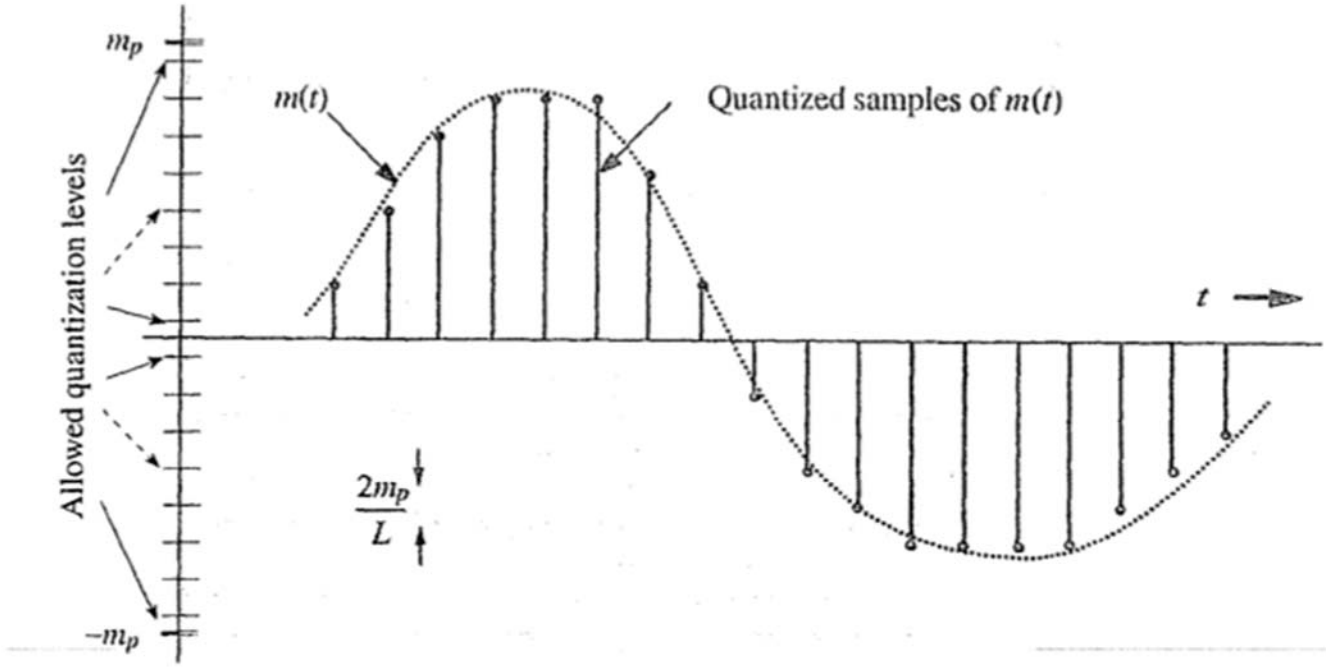
$$= \sum_k g(kT_s)\operatorname{sinc}(2\pi Bt - k\pi)$$



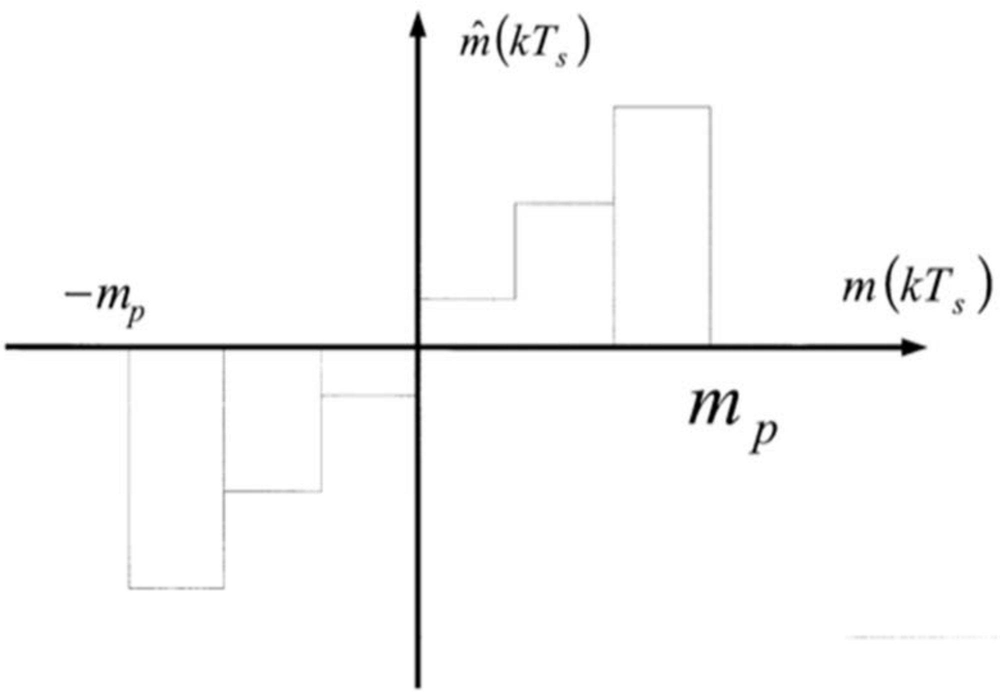
Sampled waveform



Quantized waveform



Uniform quantizer



Minimum and maximum voltage

$$\max\left(m(t)\right) = m_p$$
$$\min\left(m(t)\right) = -m_p$$
$$n = \text{number of bits}$$
$$L = 2^n = \text{number of levels}$$
$$m_i = \text{voltage boundaries}$$
$$i = 0, 1, 2 \cdots L$$
$$m_0 = -m_p \qquad m_L = m_p$$

Voltage range values

$$\Delta = \text{step size} = \frac{\max\left(m(t)\right) - \min\left(m(t)\right)}{L}$$
$$m_0 = \min\left(m(t)\right) = -m_p$$
$$m_i = \min\left(m(t)\right) + i\Delta$$
$$m_L = \max\left(m(t)\right) = m_p$$
$$\Delta = \frac{m_p - \left(-m_p\right)}{L} = \frac{2m_p}{L}$$
$$m_i > m\left(kT_s\right) \geq m_{m-1}$$
$$\hat{m}\left(kT_s\right) = \frac{m_i + m_{i-1}}{2}$$

Quantization and binary representation

- Assume the amplitude of the analog signal $m(t)$ lie in the range $(-m_p, m_p)$.

- with quantization, this interval is partitioned into L sub-intervals, each of magnitude $\delta u = 2m_p / L$.

- Each sample amplitude is approximated by the midpoint value of the subinterval in which the sample falls.

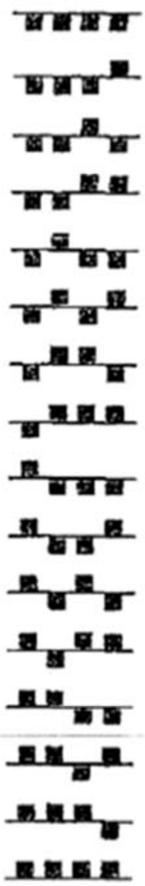
- Thus, each sample of the original signal can take on only one of the L different values.

- Such a signal is known as an L-ary digital signals

- In practice, it is better to have binary signals

Alternatively we can use A sequence of four binary pulses to get 16 distinct patterns

Digit	Binary equivalent	Pulse code waveform
0	0000	
1	0001	
2	0010	
3	0011	
4	0100	
5	0101	
6	0110	
7	0111	
8	1000	
9	1001	
10	1010	
11	1011	
12	1100	
13	1101	
14	1110	
15	1111	



Examples of digital, audio signals

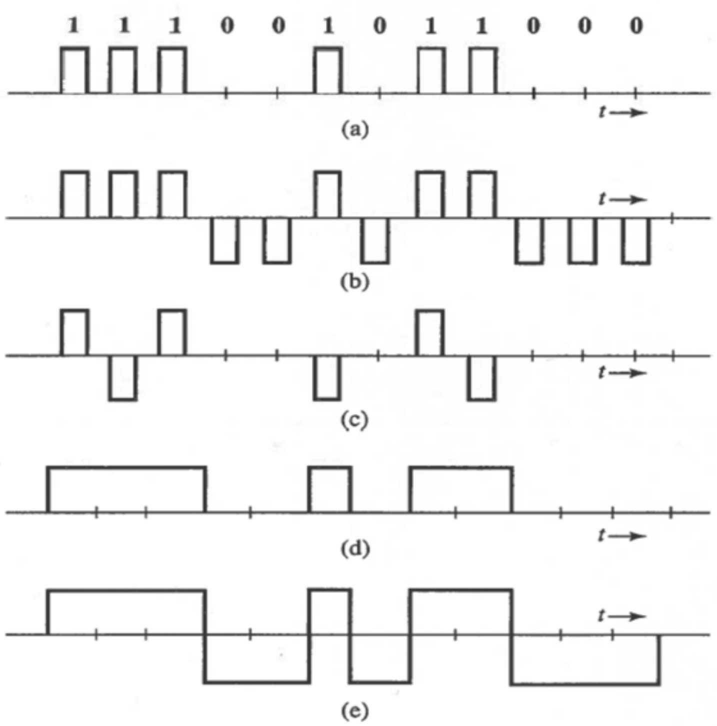
1. Audio Signal (Low Fidelity, used in telephone lines).

- Audio signal frequency from 0 to 15 kHz. Subjective tests show signal articulation (intelligibility) is not affected by components above 3.4 kHz. So, assume bandwidth B = 4 kHz.
- Sampling frequency $f_s = 2B = 8$ kHz that means 8,000 samples per second.
- Each sample is quantized with L = 256 levels, that is a group of 8 bits to encode each sample $2^8 = 256$
- Thus a telephone line requires 8 x 8,000 = 64,000 bits per second (64 kbps).

2. Audio Signal (High Fidelity, used in CD)

- Bandwidth 20 kHz, we assume a bandwidth of B = 22.05kHz.
- Sampling frequency $f_s = 2B = 44.1$ kHz, this means 44,100 samples per seconds.
- Each sample is quantized with L = 65,536 levels, 16 bits per sample.
- Thus, a Hi-Fi audio signal requires 16 x 44,100 $\simeq$706 kbps.

Transmission or line coding



On-off return-to-zero

Polar return-to-zero

Bi-polar return-to-zero

On-off non-return-to-zero

Polar non-return-to-zero

Desirable properties of line coding

- Transmission bandwidth as small as possible
- Power efficiency
- Error detection and correction capability
- Favorable power spectral density (e.g., avoid dc component for use of ac coupling and transformers)
- Adequate timing content
- Transparency (independent of info bits, to avoid timing problem)

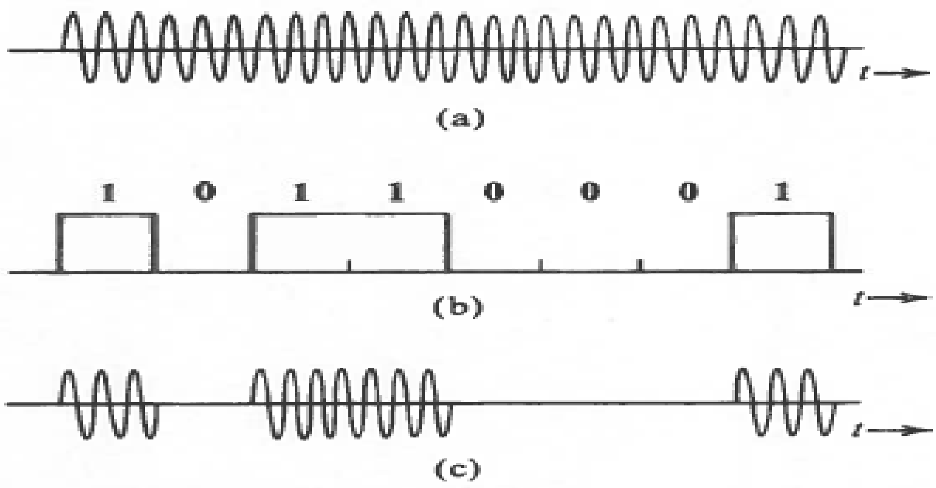
Digital modulation

- The process of modulating a digital signal is called keying
- As for the analogue case, we can choose one of the three parameters of a sine wave to modulate

  1. Amplitude modulation, called *Amplitude Shift Keying* (ASK)
  2. Phase modulation, *Phase Shift Keying* (PSK)
  3. Frequency modulation *Frequency Shift Keying* (FSK)

- In some cases, the data can be sent by simultaneously modulating phase and amplitude, this is called *Quadrature Amplitude Phase Shift Keying* (QASK)

Amplitude Shift Keying (ASK)



On-off non-return-to-zero

Amplitude shift keying (ASK) = **on-off keying (OOK)**

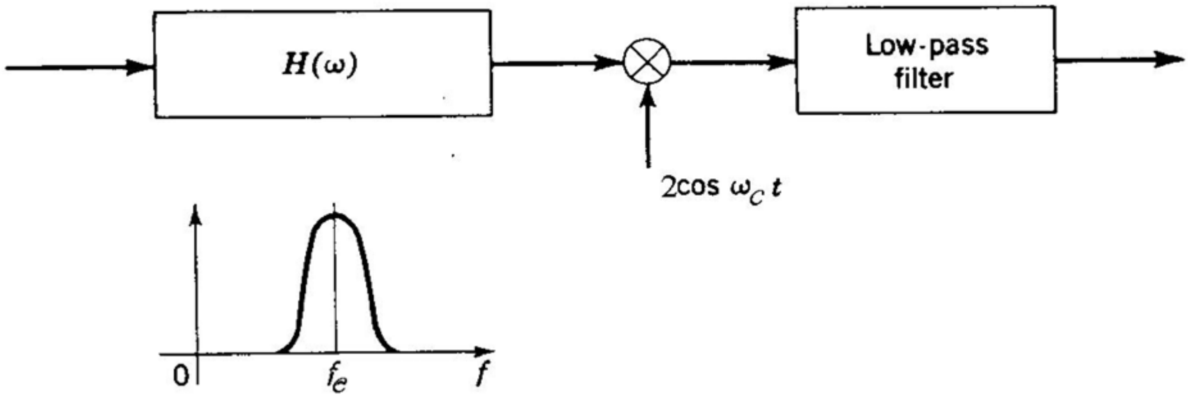
$$\begin{aligned} s_0(t) &= 0 \\ s_1(t) &= A\cos(2\pi f_c t) \end{aligned}$$

or $\quad s(t) = A(t)\cos(2\pi f_c t), \qquad A(t) \in \{0, A\}$

How to recover ASK transmitted symbol?

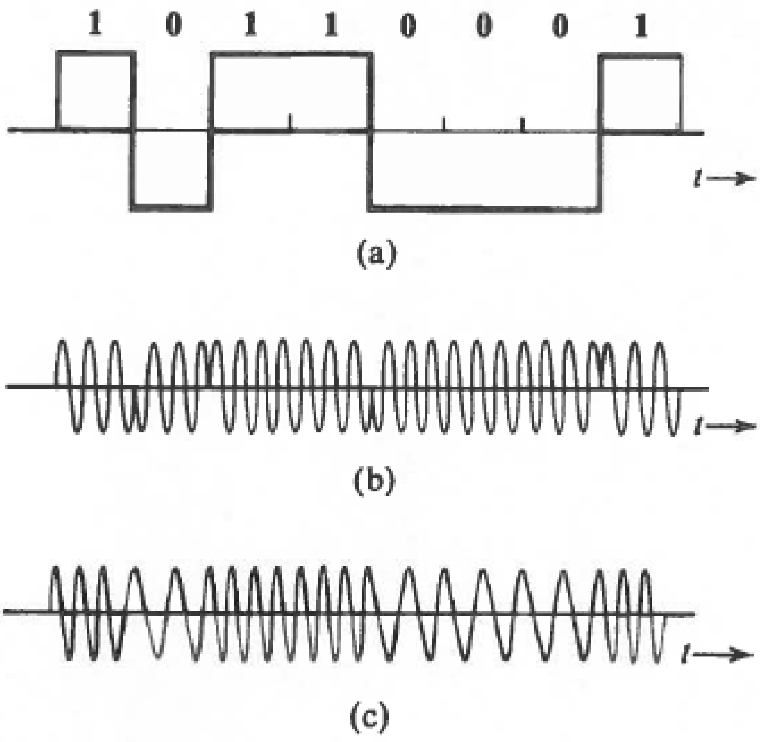
- Coherent (synchronous) detection
  - Use a BPF to reject out-of-band noise
  - Multiply the incoming waveform with a cosine of the carrier frequency
  - Use a LPF
  - Requires carrier regeneration (both frequency and phase synchronization by using a phase-lock loop)
- Noncoherent detection (envelope detection etc.)
  - Makes no explicit efforts to estimate the phase

Coherent Detection of ASK



Assume an ideal band-pass filter with unit gain on $[f_c - W, f_c + W]$. For a practical band-pass filter, $2W$ should be interpreted as the equivalent bandwidth.

Phase and Frequency Shift Keying (PSK, FSK)



m(t): Polar non-return-to-zero

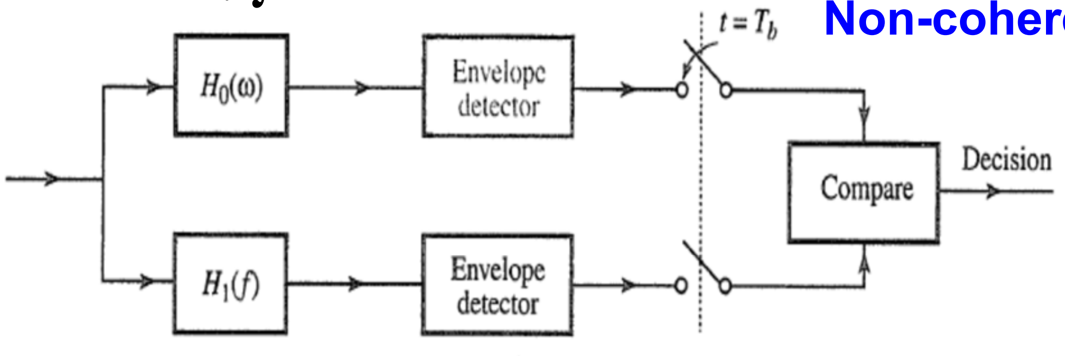
$$\varphi_{PSK} = m(t)\cos(\omega_c t)$$

$$\varphi_{FSK} = \cos[\omega_c t + k_f \int m(t)dt]$$

FSK Non-coherent and Coherent Detection

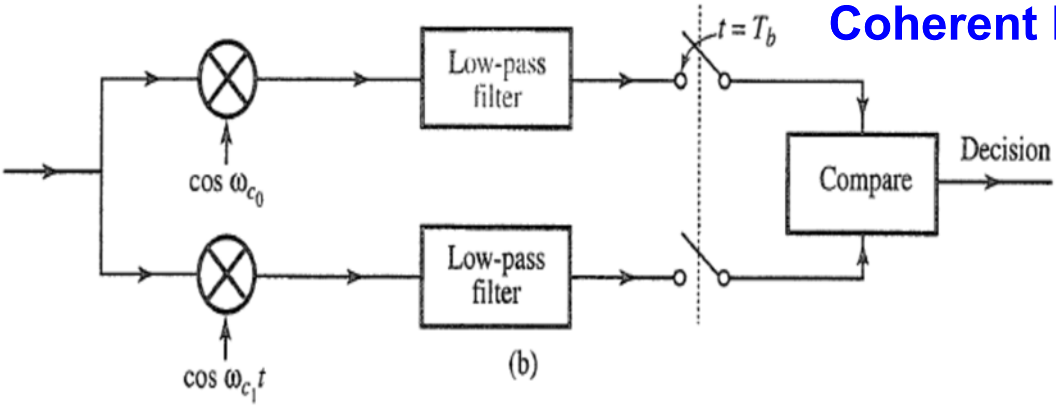
$$\varphi_{FSK} = \cos[\omega_c t + k_f \int m(t)dt]$$

Non-coherent Detection



(a)

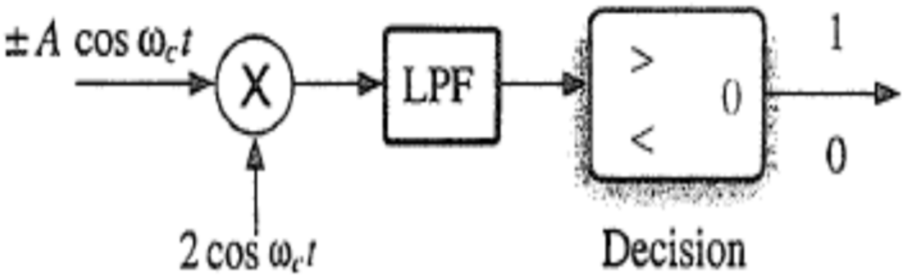
Coherent Detection



(b)

PSK Coherent Detection

$$\varphi_{PSK} = m(t)\cos(\omega_c t)$$

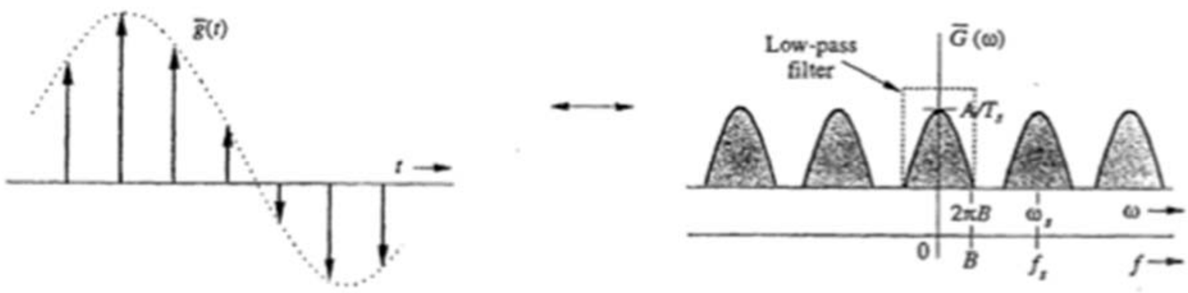


Envelop detection is not applicable to PSK

Signal Bandwidth, Channel Bandwidth & Channel Capacity (Maximum Data Rate)

- Signal bandwidth: the range of frequencies present in the signal
- Channel bandwidth: the range of signal bandwidths allowed (or carried) by a communication channel without significant loss of energy or distortion
- Channel capacity (maximum data rate): the maximum rate (in bits/second) at which data can be transmitted over a given communication channel

Two Views of Nyquist Rate



- Nyquist rate: 2 times of the bandwidth
- Sampling rate: For a given signal of bandwidth B Hz, the sampling rate must be at least 2B Hz to enable full signal recovery (i.e., avoid aliasing)
- Signaling rate: A noiseless communication channel with bandwidth B Hz can support the maximum rate of 2B symbols (signals, pulses or codewords) per second – so called the "baud rate"

Channel Capacity (Maximum Data Rate) with Channel Bandwidth B Hz

- Noiseless channel
  - Each symbol represents a signal of *M* levels (where *M*=2 and 4 for binary symbol and QPSK, respectively)
  - Channel capacity (maximum data rate): bits/second

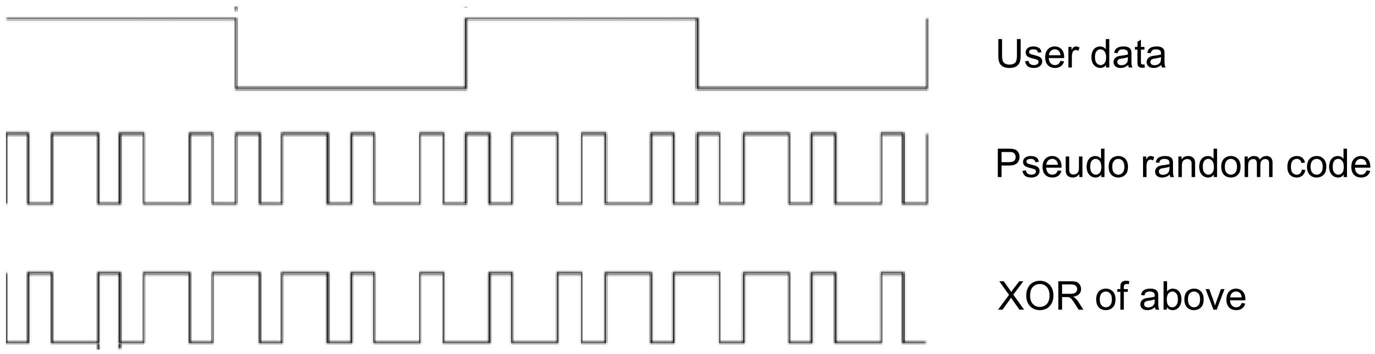
$$C = 2B \log_2 M$$

- Noisy channel
  - Shannon's channel capacity (maximum data rate): bits/second

$$C = B \log_2 (S / N)$$

where *S* and *N* denote the signal and noise power, respectively

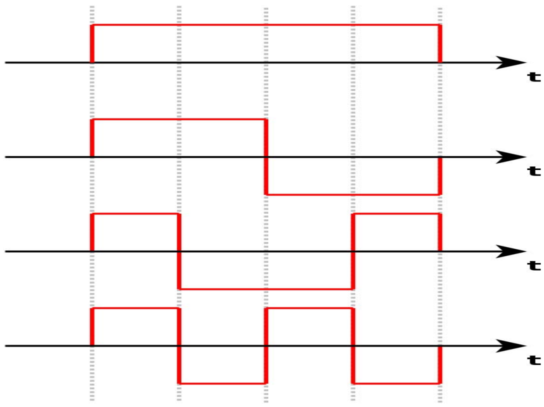
Introduction to CDMA (Code Division Multiple Access)



Courtesy by Marcos Vicente on Wikipedia

- Each user data (bit) is represented by a number of "chips" – pseudo random code – forming a spread-spectrum technique
- Pseudo random codes
  - Appear random but can be generated easily
  - Have close to zero auto-correlation with non-zero time offset (lag)
  - Have very low cross-correlation (almost orthogonal) for simultaneous use by multiple users – thus the name, CDMA

Use of Orthogonal Codes for Multiple Access



- Transmission: Spread each information bit using a code
- Detection: Correlate the received signal with the correspond code
- Orthogonal spreading codes ensure low mutual interference among concurrent transmissions
- Use codes to support multiple concurrent transmissions – Code-division multiple access (CDMA), besides time-division multiple access (TDMA) and frequency-diversion multiplex (FDMA)
- FDMA – 1G, TDMA – 2G, CDMA – 3G, 4G… cellular networks