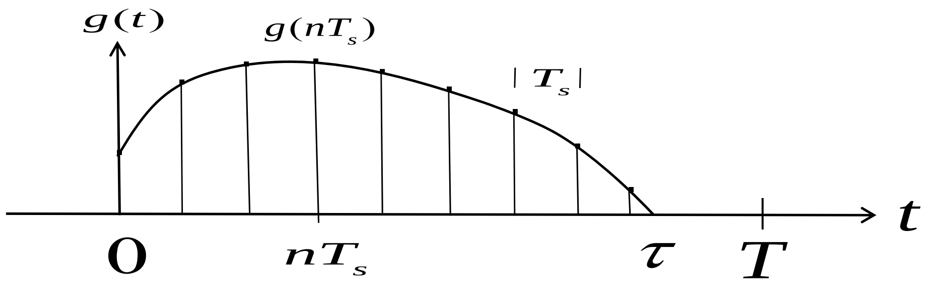
Discrete Fourier Transform (DFT)

- Discrete Fourier Transform (DFT)
  - Numerical computation of Fourier Transform

- Given a signal $g(t)$, which is non-zero for $0 \le t \le \tau$
  - Select $T \ge \tau$ and set $g(t) = 0$ for $\tau \le t \le T$



$$G(\omega) = \int_0^T g(t) e^{-j\omega t}\, dt$$

$$= \lim_{T_s \to 0} \sum_{n=0}^{N-1} g(nT_s) e^{-j\omega n T s} T_S \qquad \text{where} \quad N = T / T_S$$

Discrete Fourier Transform (DFT) and Fast Fourier Transform (FTT)

$$G(\omega) = \lim_{T_s \to 0} \sum_{n=0}^{N-1} g(nT_s) e^{-j\omega n T s} T_S \quad \text{where} \quad N = T / T_S$$

$$G_m = G(m\omega_0) = \sum_{n=0}^{N-1} g(nT_S) T_s e^{-jm\omega_0 n T s} \quad \text{where} \quad m = 0, 1, \ldots, N-1 \text{ and } \omega_0 = 2\pi / T$$

$$G_m = \sum_{n=0}^{N-1} g_n e^{-jmn\omega_0 T_S} \quad \text{where} \quad g_n \equiv g(nT_S) T_S$$

One can derive for *n=0,1, …, N-1*

$$g_n = \tfrac{1}{N} \sum_{m=0}^{N-1} G_m e^{jmn\omega_0 T_S}$$

$$\{g_0, g_1, \ldots, g_{N-1}\} \Leftrightarrow \{G_0, G_1, \ldots, G_{N-1}\}$$

- DFT computation complex $O(N^2)$
- Fast Fourier transform (FFT) by Tukey and Cooley 1965
- FFT computation complex $O(N \log N)$