Solutions to Problem Sheet Two

1

$$
\begin{aligned}
E_x &= \int_0^2 1^2 dt = 2 \\
E_y &= \int_0^1 1^2 dt \; + \; \int_1^2 (-1)^2 dt = 2 \\
c_{xy} &= \frac{1}{2}\int_0^2 x(t)y(t)dt = 0
\end{aligned} \tag{1}
$$

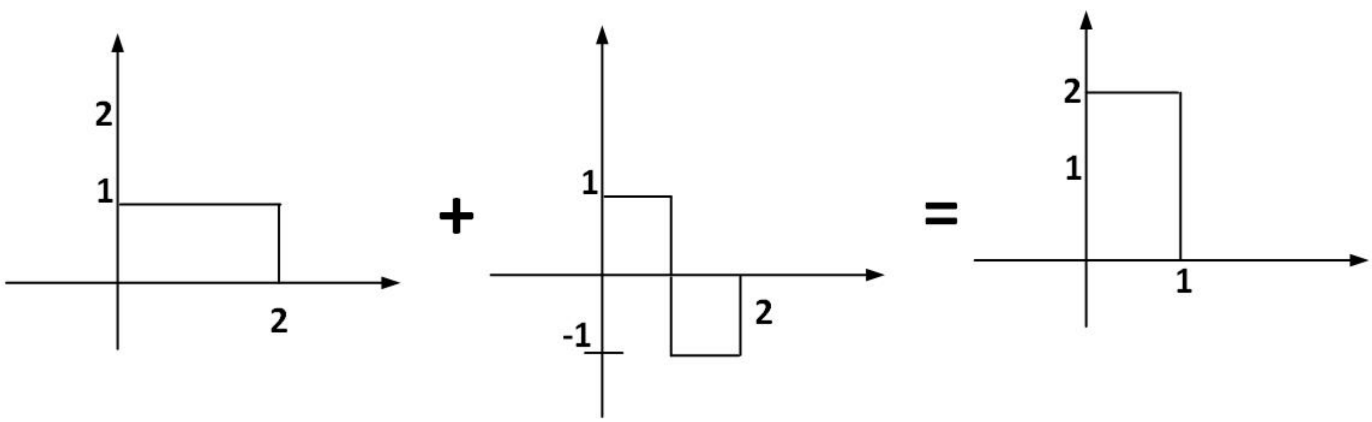

Figure 1:

$$E_{x+y} = \int_0^1 (2)^2 dt = 4$$

$E_{x+y} = E_x + E_y$, because signals are orthogonal.

2

$$
\begin{aligned}
E_x &= 2 \\
E_y &== 2 \\
c_{xy} &= \frac{1}{2}\int_0^2 x(t)y(t)dt = \frac{1}{2}\big[\int_0^{0.5} 1dt - \int_{0.5}^2 1dt\big] = -\frac{1}{2}
\end{aligned} \tag{2}
$$

Energy of $z(t) = x(t) + y(t)$

$$E_{x+y} = \int_0^{0.5} (2)^2 dt = 2 \tag{3}$$

$E_{x+y} \neq E_x + E_y$. This is because the signals are not orthogonal.

In general, $E_{x+y} = E_x + E_y + 2c_{xy}\sqrt{E_x E_y}$

3.a Using the trigonometric Fourier series, we know

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \tag{4}$$

whereas

$$\begin{aligned} g(-t) &= a_0 + \sum_{n=1}^{\infty} a_n \cos(-n\omega_0 t) + b_n \sin(-n\omega_0 t) \\ &= a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) - b_n \sin(n\omega_0 t) \end{aligned} \tag{5}$$

So $g(t) = g(-t)$ means

$$\begin{aligned} a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) &= a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) - b_n \sin(n\omega_0 t) \\ 2\sum_{n=1}^{\infty} b_n \sin(n\omega_0 t) &= 0 \\ \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t) &= 0 \end{aligned}$$

Since $\{sin(n\omega_0 t)\ for\ n = 1, \cdots, \infty\}$ are pairwise orthogonal. It is impossible to have

$$\begin{aligned} \forall m,\ \sin(n\omega_0 t) &= \sum_{k=1}^{\infty} c_k \sin(k\omega_0 t) \\ &\implies b_n = 0 \quad \forall n \end{aligned}$$

3.b

$$\begin{aligned} -g(-t) &= -a_0 - \sum_{n=1}^{\infty} a_n \cos(-n\omega_0 t) - b_n \sin(-n\omega_0 t) \\ &= -a_0 - \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \end{aligned} \tag{6}$$

Now for $g(t) = -g(-t)$, we use Equations (1) and (3) and simplify.

$$\begin{aligned} a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) &= -a_0 - \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \\ 2a_0 + 2\sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) &= 0 \\ a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) &= 0 \end{aligned}$$

$\cos(m\omega_0 t) \perp \cos(n\omega_0 t) \quad \forall m \neq n,$

$$\Longrightarrow a_0 = 0, \quad a_n = 0 \quad \forall \quad n$$

4 $T_o = 4, \ \omega_0 = \pi/2$

By inspection $a_0 = 0$. We also know $b_n = 0$ for all n since the function is even.

$$a_n = \int_0^1 \cos(\frac{n\pi t}{2})dt - \int_1^2 \cos(\frac{n\pi t}{2})dt = \frac{4}{n\pi}\sin(\frac{n\pi}{2})$$