Intro to Signals and Communications

Solutions to Problem Sheet One

1. The power of a sinusoid $g(t) = A\cos(\omega_0 t + \theta)$ is always equal to $\frac{A^2}{2}$. It follows

$$\begin{aligned}
P_g &= \lim_{T\to\infty} \tfrac{1}{T} \textstyle\int_{-T/2}^{T/2} A^2 \cos^2(\omega_0 t + \theta) dt \\
&= \lim_{T\to\infty} \tfrac{1}{T} \textstyle\int_{-T/2}^{T/2} \tfrac{A^2}{2}[1 + \cos(2\omega_0 t + 2\theta)] dt \\
&= \lim_{T\to\infty} \tfrac{A^2}{2T} \textstyle\int_{-T/2}^{T/2} dt + \lim_{T\to\infty} \tfrac{A^2}{2T} \textstyle\int_{-T/2}^{T/2} \cos(2\omega_0 t + 2\theta)] dt \\
&= \tfrac{A^2}{2} + \lim_{T\to\infty} \tfrac{A^2}{4\omega_0 T} \sin(\omega_0 T + 2\theta) + \lim_{T\to\infty} \tfrac{A^2}{4\omega_0 T} \sin(2\theta - \omega_0 T) \\
&= \tfrac{A^2}{2},
\end{aligned}$$

where the last identity follows from the fact that

$$\lim_{x\to\infty} \frac{\sin x}{x} = 0.$$

Therefore

$$P_g = \frac{A^2}{2}.$$

   (a) $v_{dc} = 0, \quad P_{v(t)} = \frac{9}{2}$.
   (b) $v_{dc} = 3, \quad P_{v(t)} = 9$.
   (c) $v_{dc} = 0, \quad P_{v(t)} = \frac{25}{2}$.

2. (a)

$$E_g = \int_0^{2\pi} \sin^2 t dt = \frac{1}{2}\int_0^{2\pi} dt - \frac{1}{2}\int_0^{2\pi} \cos 2t dt = \pi + 0 = \pi.$$

   (b) Sign change and time shift do not affect the signal energy. The energy of $kg(t)$ is $k^2 E_g$.

3.

$$P_v = \frac{1}{2}\int_1^2 (1)^2 dt = \frac{1}{2}$$