Time signal

$$f(t) = e^{-a|t|}$$

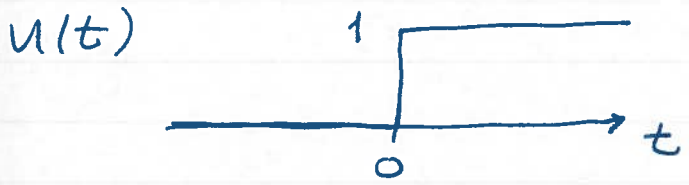
where $a > 0$

Fourier transform

$$\Longleftrightarrow \quad \frac{2a}{a^2+\omega^2}$$

$$\lim_{a\to 0} f(t) = \lim_{a\to 0} e^{-a|t|} = 1 \quad \Longleftrightarrow \quad \lim_{a\to 0} \frac{2a}{a^2+\omega^2} = 2\pi\delta(\omega)$$

From Fourier transform of 1.



$$u(t) = \lim_{a\to 0} e^{-at} \quad \text{for } t \geq 0$$

$$f(t) = \begin{cases} e^{-at} & t \geq 0 \\ 0 & t < 0 \end{cases} \quad \Longleftrightarrow \quad \frac{1}{a+j\omega}$$

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$$= \frac{a}{a^2+\omega^2} - \frac{j\omega}{a^2+\omega^2}$$

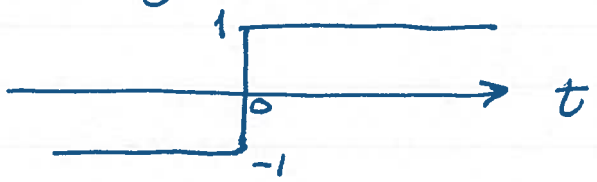
$$u(t) = \lim_{a\to 0} f(t) \quad \Longleftrightarrow \quad \lim_{a\to 0} \left[\frac{a}{a^2+\omega^2} - \frac{j\omega}{a^2+\omega^2} \right]$$

$$= \pi\delta(\omega) - \frac{j}{\omega}$$

$$\Rightarrow \quad u(t) \quad \Longleftrightarrow \quad \pi\delta(\omega) + \frac{1}{j\omega}$$

Time signal Fourier transform

sgn(t)



$$\frac{1+\text{sgn}(t)}{2} = u(t)$$

$$\Rightarrow \quad \text{sgn}(t) = 2u(t) - 1$$

$$2u(t) - 1 \quad \Longleftrightarrow \quad 2\left[\pi\delta(\omega) + \frac{1}{j\omega}\right]$$

$$- \underbrace{2\pi\delta(\omega)}$$

↑ Fourier transform of 1

$$\Rightarrow \quad \text{sgn}(t) \quad \Longleftrightarrow \quad \frac{2}{j\omega}$$