2. Signals. (30%)

a. Let us obtain the self-convolution $y(t)$ of the signal $x(t)$ in Figure 3 where $y(t) = x(t) * x(t)$.

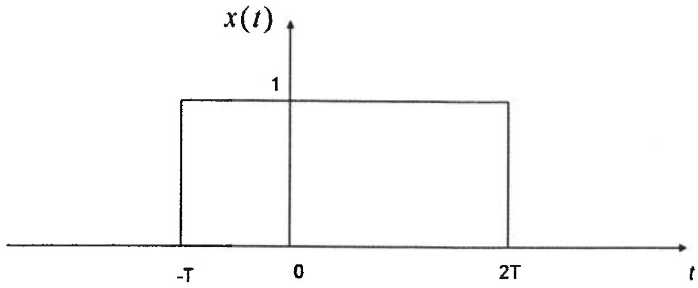


Figure 3. The signal $x(t)$.

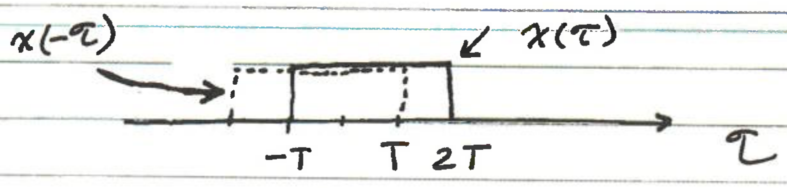
i. Express $y(t)$ as a convolution integral of $x(t)$. [3]

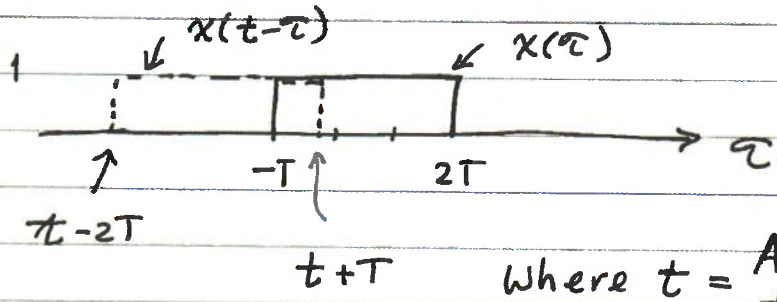
ii. Identify several time intervals of t and carry out the convolution integration for $y(t)$ for each of these time intervals. [10]

iii. Sketch a diagram for $y(t)$ as a function of t. [2]

2a i. $$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\,x(t-\tau)\,d\tau$$ 3

ii.





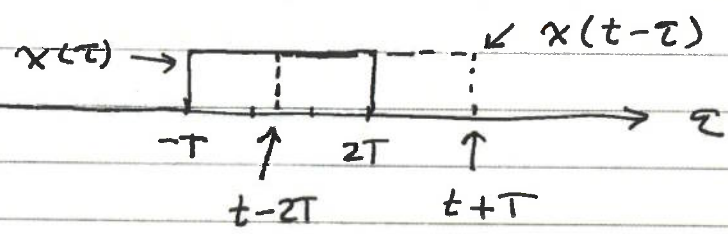
where t = ~~Amount of time shift~~

a) When $t+T < -T \Rightarrow t < -2T$, the two curves don't overlap. That is, $y(t)=0$ 2

b) when $-T \le t+T \le 2T \Rightarrow -2T \le t \le T$,

$$y(t) = \int_{\tau=-T}^{t+T} 1 \cdot d\tau = t+2T.$$ 3

c) When $-T \le t-2T \le 2T \Rightarrow T \le t \le 4T$

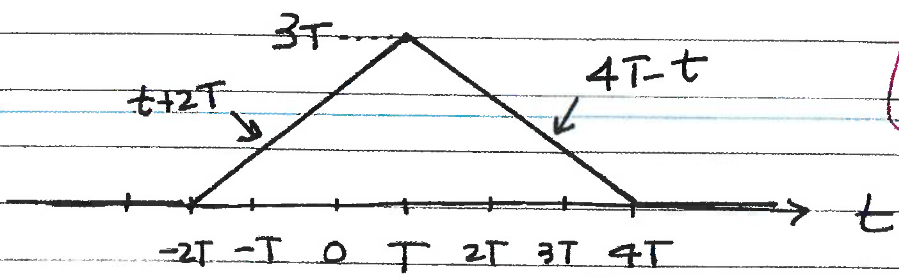


$$y(t) = \int_{\tau=t-2T}^{2T} 1 \cdot d\tau = 2T-(t-2T)$$ 3

$$y(t) = 4T - t$$

d) when $t>4T$, no overlap, $y(t)=0$. 2

2 a. iii.



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