Introduction to Signals and Communications

Problem Sheet One

1. Find the mean value (time average) and the power for the following signals

   (a) $v(t) = 3\cos(2\pi 10^3 t)$,

   [0, 9/2]

   (b) $v(t) = 3$,

   [3, 9]

   (c) $v(t) = 3\sin(40t) + 4\cos(\pi 10^4 t)$.

   [0, 25/2]

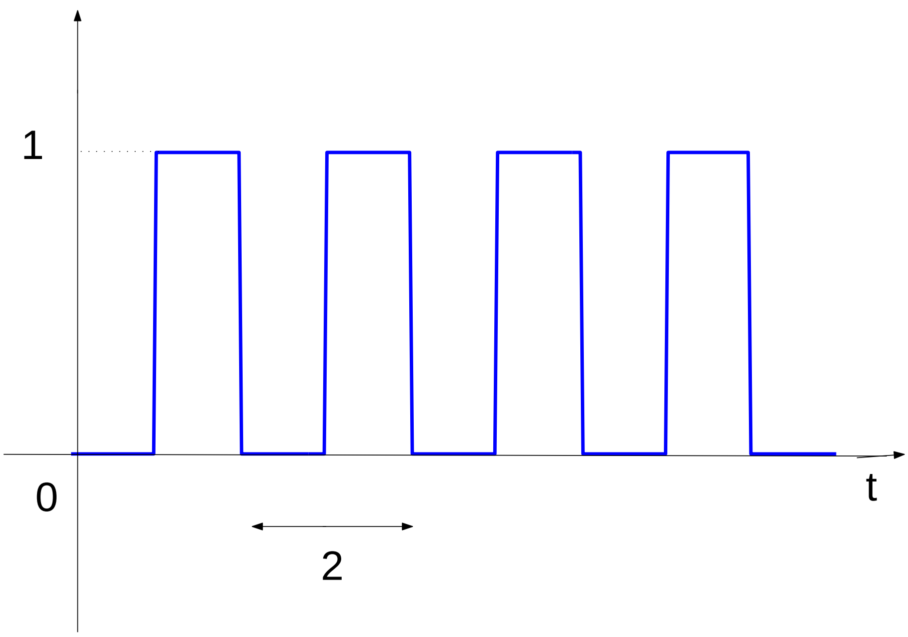
2. (a) Find the energy of the signal

$$g(t) = \begin{cases} \sin t & \text{for } 0 \le t \le 2\pi \\ 0 & \text{otherwise} \end{cases}$$

   [π]

   (b) Comment on the effect on energy of sign change or time shifting. What is the effect on the enegy if the signal is multiplied by k?

3. Find the power for the following periodic signal.



[1/2]

Problem Sheet Two

1. Find the energy E_x and E_y of the signals $x(t)$ and $y(t)$ shown in Figure 1. Find the correlation coefficient between $x(t)$ and $y(t)$. Sketch the signal $x(t) + y(t)$ and show that the energy of this signal is equal to $E_x + E_y$. (Why?)

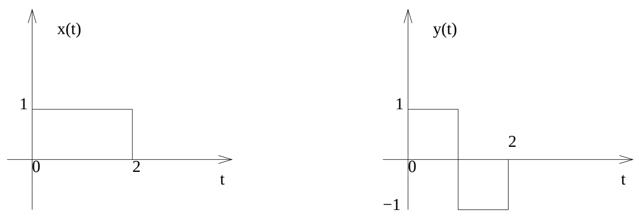


Figure 1:

2. Repeat the procedure for the signal pair shown in Figure 2. That is, compute:

   - Signal energies.
   - Correlation coefficient.
   - Energy of the signal $z(t) = x(t) + y(t)$. (Is $E_z \neq E_x + E_y$? Why?)

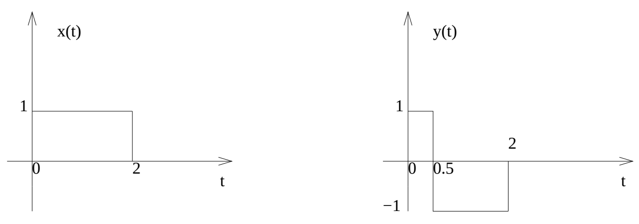


Figure 2:

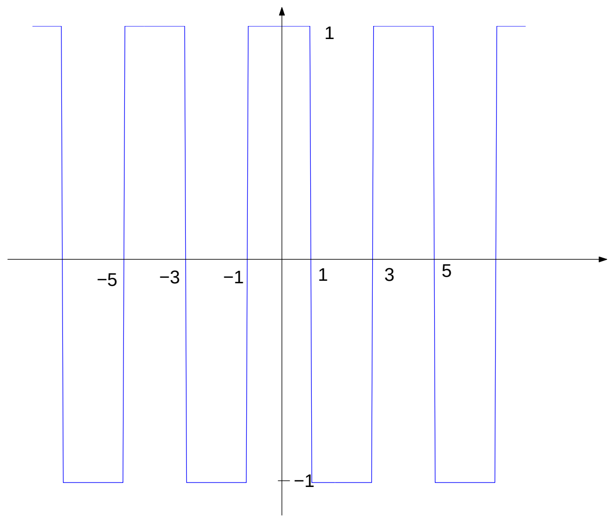
3. If a periodic signal satisfies certain symmetry conditions, the evaluation of the Fourier series component is somewhat simplified. Show that:

(a) If $g(t) = g(-t)$ (even symmetry), then all the sine terms in the trigonometric Fourier series vanish ($b_n = 0$).

(b) If $g(t) = -g(-t)$ (odd symmetry), then the dc and all the cosine terms in the Fourier series vanish ($a_0 = a_n = 0$).

4. Show that the trigonometric Fourier series of the signal shown below is

$$x(t) = \frac{4}{\pi}\left(\cos\frac{\pi t}{2} - \frac{1}{3}\cos\frac{3\pi t}{2} + \frac{1}{5}\cos\frac{5\pi t}{2} - \frac{1}{7}\cos\frac{7\pi t}{2} + \cdots\right)$$



Problem Sheet Three

1. Show that the Fourier transform of $g(t)$ may be expressed as

$$G(\omega) = \int_{-\infty}^{\infty} g(t)\cos\omega t dt - j\int_{-\infty}^{\infty} g(t)\sin\omega t dt$$

   Hence, show that

   - if $g(t)$ is a real and even function of t, then $G(\omega)$ is real and even.
   - if $g(t)$ is a real and odd function of t, then $G(\omega)$ is imaginary and odd.

2. Using the definition of the Fourier transform:

$$G(\omega) = \int_{-\infty}^{\infty} g(t)e^{-j\omega t}dt$$

   show that if $g(t)$ is real then $G^*(\omega) = G(-\omega)$.
   (An important consequence of the above property is that if $g(t)$ is real then $|G(\omega)|$ is an even function of ω and $\angle G(\omega)$ is an odd function of ω).

3. From the definition of the Fourier transform, find the Fourier transform of $\text{rect}(t-5)$.

4. Using time shift property, compute again the Fourier transform of $\text{rect}(t-5)$ and compare the two results.

Problem Sheet Four **(No need to try this problem set)**

1. Find the Power Spectral Density $S_g(\omega)$ of the power signal $g(t) = \cos\omega_0 t$. (Hint: Compute the autocorrelation function first, and then use the property $\mathcal{R}_g(\tau) \Longleftrightarrow S_g(\omega)$).

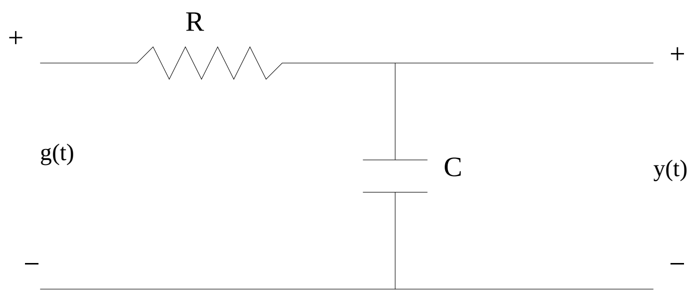
$[\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]]$

2. Find the power of the output signal $y(t)$ of the RC circuit shown below with $RC = 1$ if the input PSD $S_g(\omega)$ is given by:

   (a) $\text{rect}(\omega/2)$

   (b) $\delta(\omega+1)+\delta(\omega-1)$

   (Hint: recall that $S_y(\omega) = |H(\omega)|^2 S_g(\omega)$ and that $P_y = 1/2\pi \int_{-\infty}^{\infty} S_y(\omega)d\omega$)



[1/4, 1/2π]

Problem Sheet Five

1. Consider the two baseband signals $x_1(t) = \cos 2000t$ and $x_2(t) = \cos 1900t$. Plot the magnitude spectrum of the signal $s(t) = x_1(t)\cos 10000t + x_2(t)\cos 20000t$.

   (a) The signal $s(t)$ is multiplied by $\cos 10000t$ and fed to the filter with frequency response

   $$H(\omega) = \begin{cases} 1 & for \;\; |\omega| \leq 2050 \text{ rad/s} \\ 0 & otherwise \end{cases}$$

   Plot the magnitude spectrum of the output time waveform $y(t)$ (see Figure 3)

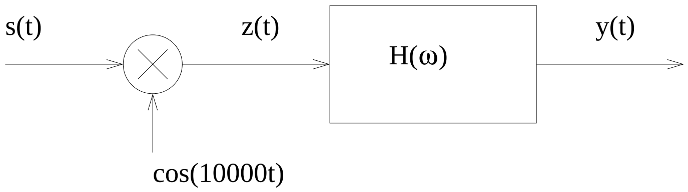


Figure 3: DSB-SC receiver

   (b) What happens if $s(t)$ is multiplied by $\cos 20000t$ and fed to the same filter? Plot the amplitude spectrum of this new output.

2. The signals $m_1(t) = \frac{2}{\pi} sinc(2t)$ and $m_2(t) = \frac{4}{\pi^2} sinc^2(2t)$ are to be transmitted simultaneously over a channel. Call $y(t) = (m_1(t) + x(t))\cos(1000t)$ the signal transmitted over the channel, where $x(t) = m_2(t)\cos 6t$.

   (a) Sketch the spectra of $x(t)$ and $y(t)$ [Hint: recall that a product in the time domain is equivalent to a convolution in the frequency domain).

   (b) What is the bandwidth of $m_1(t) + x(t)$? [$5/\pi$ Hz]

   (c) Can $m_1(t)$ and $m_2(t)$ be recovered from $y(t)$?

   (d) Design (only the block diagram) a sycronous receiver to recover $m_2(t)$.

Problem Sheet Six

1. Consider the amplitude modulated signal $s(t) = (A + m(t))\cos\omega_c t$, where $A = 2$, $\omega_c = 10000$rad/s and $m(t) = \cos 100t + \sin 100t$. Compute:

   (a) the peak amplitude m_p of $m(t)$.
   (Hint: use the identity $a\cos\omega_0 t + b\sin\omega_0 t = c\cos(\omega_0 t + \theta)$ with $c = \sqrt{a^2 + b^2}$ and $\theta = tan^{-1}(-b/a)$.

   (b) Compute the modulation index $\mu = \frac{m_p}{A}$.

   (c) Compute the power efficiency $\eta = \frac{P_s}{P_c + P_s}$

2. The output signal from an AM modulator is

$$u(t) = 5\cos(1800\pi t) + 20\cos(2000\pi t) + 5\cos(2200\pi t).$$

   Determine:

   (a) the modulating signal $m(t)$ and the carrier $c(t)$,

   (b) the modulation index,

   (c) the ratio of the power in the sidebands to the power in the carrier.

3. Consider the modulating signal $m(t) = \cos 100t$.

   (a) Sketch the spectrum of $m(t)$

   (b) Find and sketch the spectrum of the DSB-SC signal $\phi(t) = 2m(t)\cos 1000t$

   (c) From the spectrum of $\phi(t)$, suppress the LSB spectrum to obtain the USB spectrum.

   (d) From the USB spectrum, write the expression of $\phi_{USB}(t)$.

Problem Sheet Seven

1. Over an interval $0 \leq t \leq 1$, an angle modulated signal is given by

$$\phi(t) = 10 \cos 13000t$$

The carrier frequency is $\omega_c = 10000$.

   (a) If this were a PM signal with $k_p = 1000$, determine $m(t)$ over $0 \leq t \leq 1$.

   (b) If this were an FM signal with $k_f = 1000$, determine $m(t)$ over $0 \leq t \leq 1$.

2. An angle modulated signal has the form $u(t) = 100 \cos[2\pi f_c t + 4 \sin(2000\pi t)]$ where $f_c = 10$ MHz.

   (a) Determine the average transmitted power.

   (b) Is this an FM or PM signal? Expain.

   (c) Determine the frequency deviation Δf. [4000Hz]

   (d) Using Carson's rule, find the bandwidth of the modulated signal. [10kHz]

3. The message signal $m(t) = 10 sinc(400\pi t)$ frequency modulates the carrier $c(t) = 100 \cos(2\pi f_c t)$. The modulation index is $\beta = 6$.

   (a) Write an expression for the modulated signal $u(t)$. [Hint: you need to find the value of k_f]

   (b) What is the maximum frequency deviation of the modulation signal? [1200Hz]

   (c) Using Carson's rule, find the bandwidth of the modulated signal. [2800Hz]

Problem Sheet Eight

1. Consider the FM signal

$$\varphi(t) = 10\cos[2\pi f_0 t + k_f \int_{-\infty}^{t} x(\alpha)d\alpha]$$

where $k_f = 10\pi$. The message $x(t)$ is given by

$$x(t) = \sum_{n=0}^{2} m_n(t)$$

with

$$m_n(t) = \frac{2^n}{\pi}\text{sinc}(t)\cos(2nt).$$

(a) Sketch and dimension the Fourier transform of $m_1(t)$.

(b) Sketch and dimension the Fourier transform of $x(t)$.

(c) Using Carson's rule, determine the bandwidth of $\varphi(t)$.

[$75/\pi$Hz]

(d) Assume now that $x(t) = Ae^{-10t}u(t)$. Using Carson's rule, the bandwidth of $\varphi(t)$ is 50.4 Hz. Find the amplitude A of $x(t)$. Select the bandwidth, B, of the baseband message $x(t)$ so that it contains 95% of the signal energy.

[$A = 1$]