

# Sequential local FRI sampling of infinite streams of Diracs

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## Outline

### Sampling Finite Rate of Innovation Signals

Signals with Finite Rate of Innovation

Sampling process

### Sequential algorithm

Sampling an infinite sequence of Diracs

The noisy scenario

Application: neural activity detection

# Signals with Finite Rate of Innovation (FRI)

- ▶ Signals that have a finite number of free parameters

$$x(t) = \sum_{k \in \mathbb{Z}} \sum_{r=0}^{R-1} a_{k,r} g_r(t - t_k).$$

If the set of functions  $\{g_r(t)\}_{r=0,1,\dots,R-1}$  is known, the signal  $x(t)$  is perfectly determined by the coefficients  $(a_{k,r}, t_k)$ .

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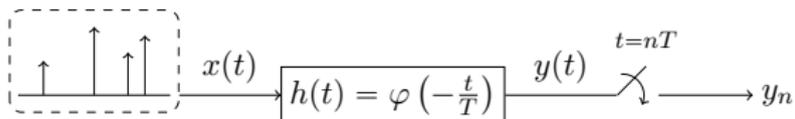
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- ▶ This signal has  $2K$  degrees of freedom in a temporal interval  $\tau$
- ▶ Local rate of innovation:  $\rho = \frac{2K}{\tau}$

- ▶ We acquire the signal with a sampling device at regular intervals of time  $t = nT$



- ▶ The output samples can be expressed as  $y_n = \langle x(t), \varphi(t/T - n) \rangle$ .

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- ▶ State of the art FRI algorithms do not deal well with infinite streams:
  - ▶ Based on isolating bursts of Diracs
  - ▶ Require high sampling rates
- ▶ We present a novel sequential algorithm that is able to reconstruct these type of signals:
  - ▶ Able to recover 1k Diracs from 10k samples
  - ▶ Robust under high noise conditions
  - ▶ Works in real time
  - ▶ Successfully applied in neuroscience to infer spiking activity of individual neurons from calcium fluorescence imaging

# Sampling process

- ▶ We sample  $x(t)$  with a very specific kernel:  $\varphi(t)$  together with its shifted versions can reproduce exponentials of the form  $e^{\alpha_m t}$

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- ▶ If coefficients  $\alpha_m$  are real, or complex but appear in complex conjugate pairs, the kernel is real valued.
- ▶ E-splines present the advantage of being of compact support  $P + 1$ .

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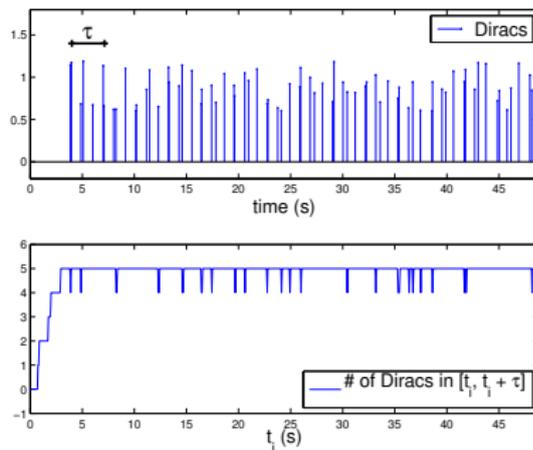
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- ▶ If we have an infinite stream we face some problems:
  - ▶ This approach requires knowledge of all samples  $y_n$  in order to compute  $s_m$
  - ▶ The number of Diracs is infinite so the order of the E-spline must be infinite as well

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- ▶ There are an infinite number of Diracs, but with a limited rate of at most  $K$  Diracs per  $\tau$  interval.



**Figure:** Infinite stream. Local maximum rate of innovation  $\rho = 2K/\tau$  ( $K = 5$ ,  $\tau = 3.125$  s).

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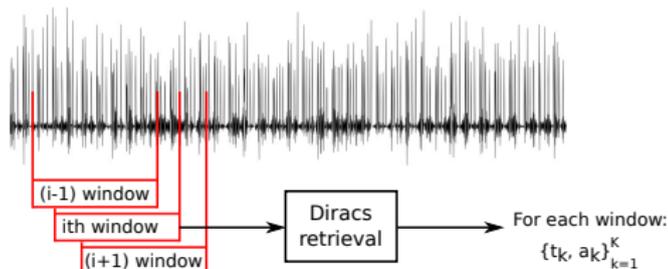


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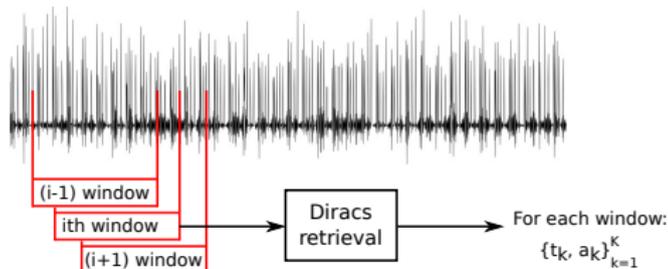


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- ▶ Problem  $\Rightarrow$  if we only process  $N$  samples at a time there are border effects when Diracs are located near the borders of the sliding window

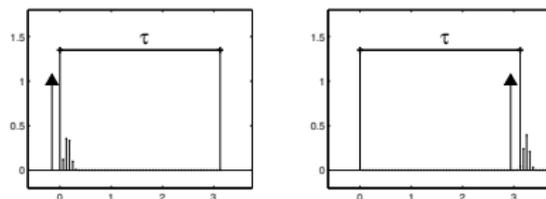


Figure: Border effects.

- ▶ The border effect in the left side is due to Diracs before the  $\tau$  interval that leak into the  $N$  samples  $y_n$  of the current window.

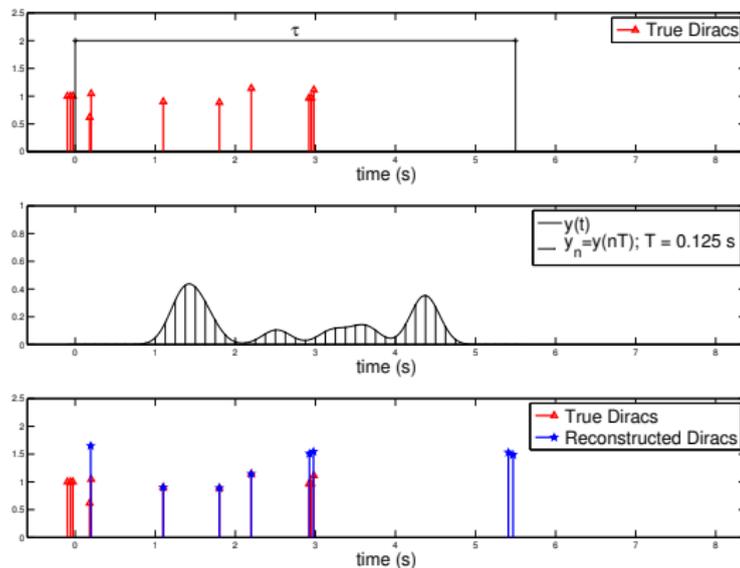


Figure: Diracs are not perfectly recovered because past Diracs corrupt current samples.

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- ▶ If we assume that we have already recovered Diracs up to the current position of the sliding window we can remove the contribution to  $y_n$  of nearby Diracs that happened before.

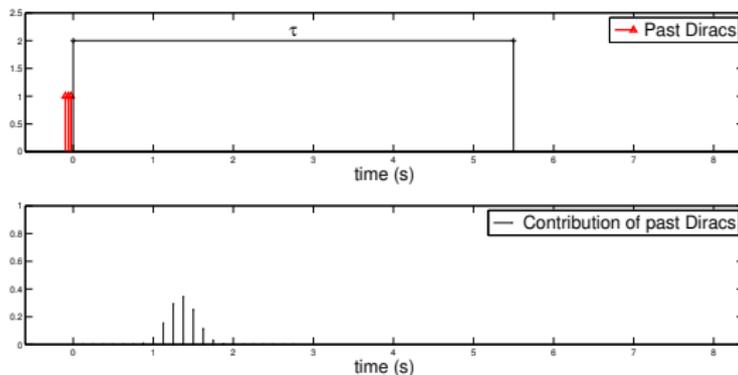


Figure: Contribution of past Diracs to samples  $y_n$ .

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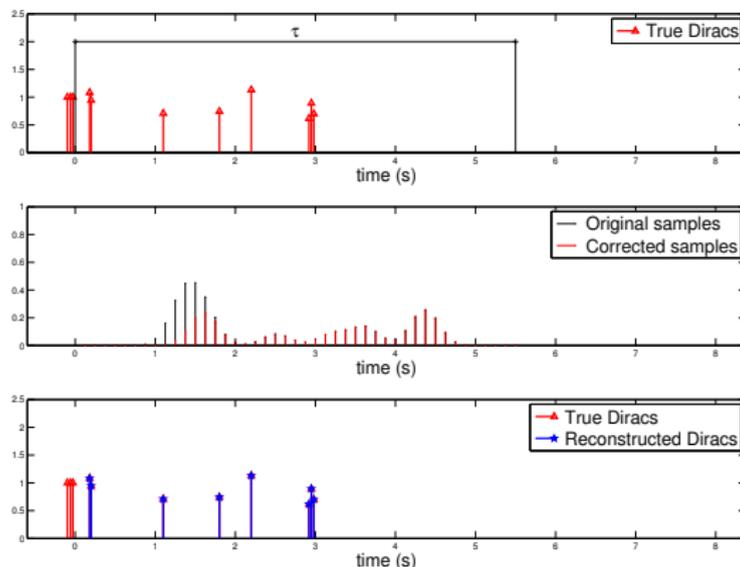


Figure: Perfect reconstruction after correcting past Diracs effect.

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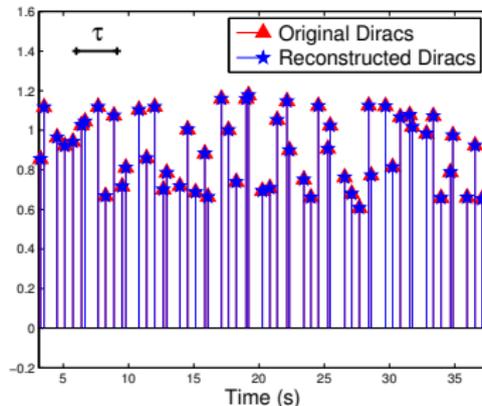
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**Figure:** Sequential perfect reconstruction of a noiseless stream of Diracs. Section of a stream of 1000 Diracs and 10220 samples  $y_n$ . Rate  $K = 5$  Diracs per  $\tau = 3.125$  s,  $N = 50$  samples,  $T = 1/16$  s and order of the E-spline  $P = 9$ .

# The noisy scenario

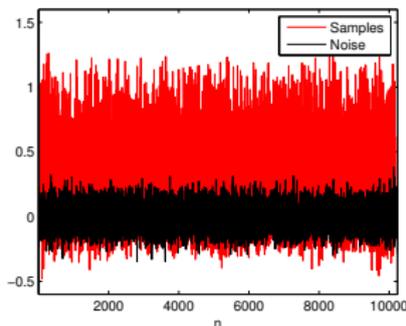
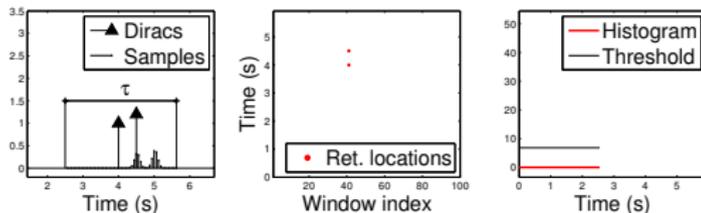


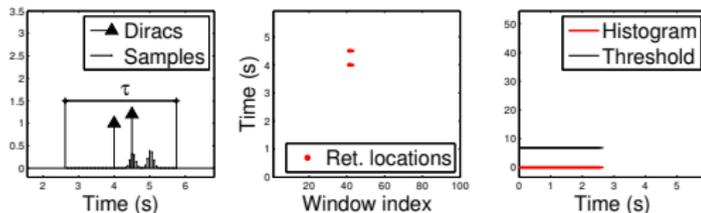
Figure: 1k Diracs, 10k samples, SNR = 10 dB.

- ▶ Perfect reconstruction conditions do not hold anymore.
- ▶ We can relax conditions on  $T$  and  $P$ 
  - ▶ We allow the sampling kernel to be of higher order in order to be more robust against noise.
- ▶ The idea is to estimate Diracs by analysing the consistency of the retrieved locations among different positions of the sliding window.

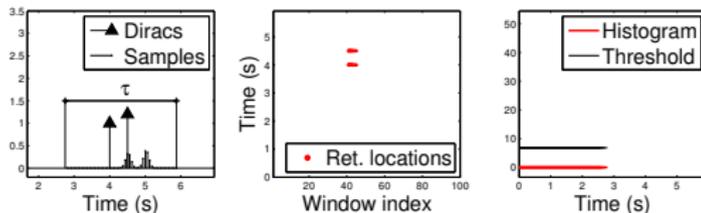
- ▶ A Dirac is captured among different positions of the sliding window:
  - ▶ If a retrieved location corresponds to a true Dirac this location will be consistent among different positions of the sliding window.



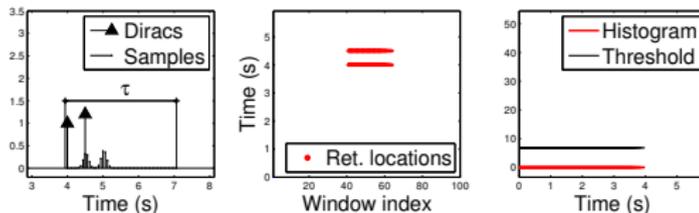
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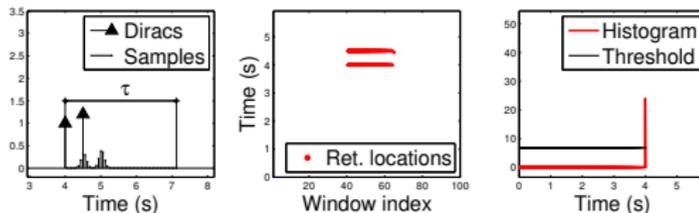
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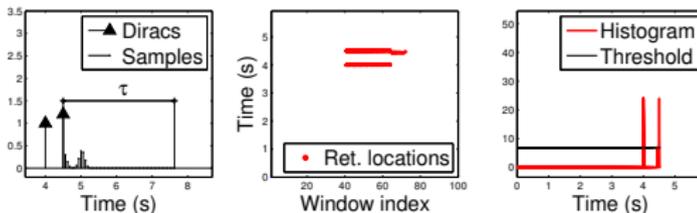
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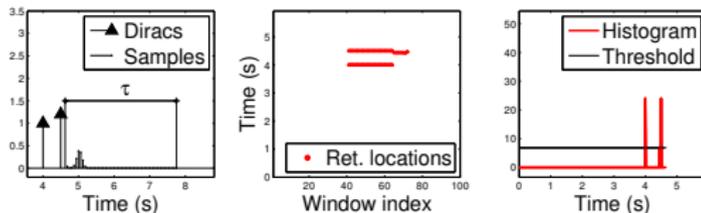
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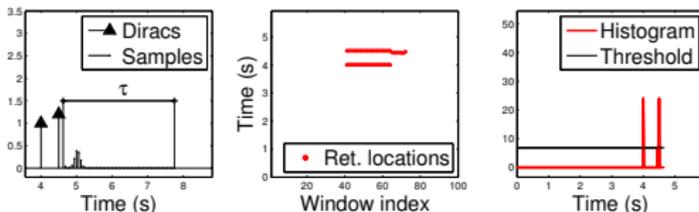
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- ▶ If we analyse the consistency of the retrieved locations we can estimate the Diracs from the peaks of the histogram of the locations:

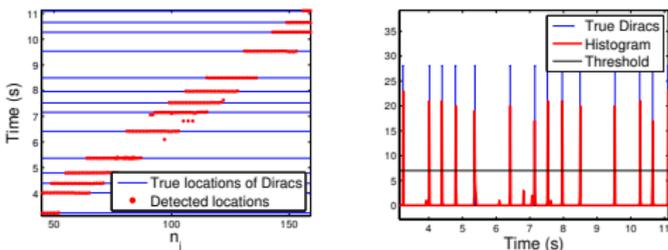
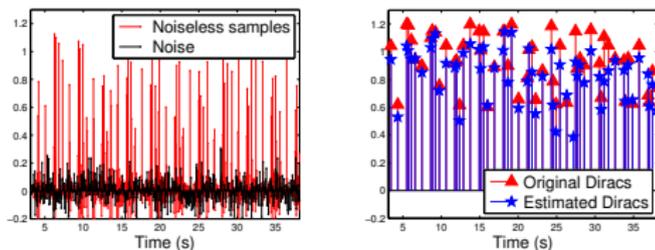


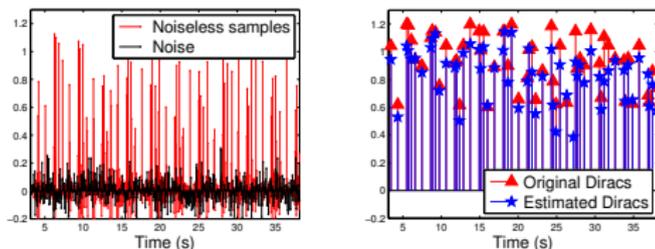
Figure: Retrieved locations among different positions of the sliding window and histogram of locations.

- ▶ The consistency analysis makes the retrieval algorithm robust against noise.



**Figure:** Sequential reconstruction of a noisy stream of Diracs (SNR = 10 dB). Section of a stream of 1000 Diracs and 10220 samples  $y_n$ . Rate  $K = 5$  Diracs per  $\tau = 3.125$  s,  $N = 50$  samples,  $T = 1/16$  s and order of the E-spline  $P = 22$ .

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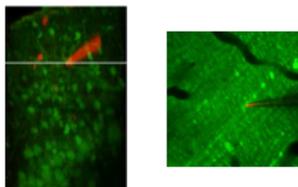
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- ▶ Some results for different levels of noise (experiment repeated 100 times for each level of noise):

SNR (dB)	5	10	15	20
Detection rate	97.69 %	99.97 %	100.00 %	100.00 %
False positives	351.7	37.8	0.5	0.3
Precision (s)	0.0086	0.0049	0.0028	0.0018

## Application: neural activity detection

- ▶ This framework has been successfully applied to the detection of neural activity in calcium concentration movies <sup>1</sup>.



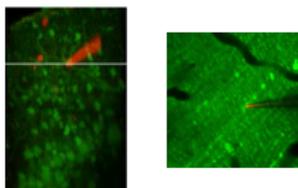
**Figure:** Simultaneous multiphoton calcium imaging of a region of the cortex and electrophysiological recording of a targeted cell with a micropipette.

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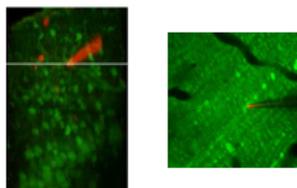
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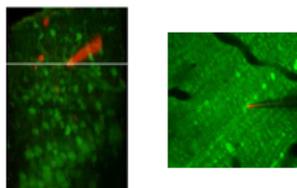
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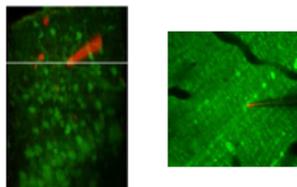
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- ▶ This is a Finite Rate of Innovation signal and with a correct processing of the fluorescence samples we can apply our sequential algorithm.

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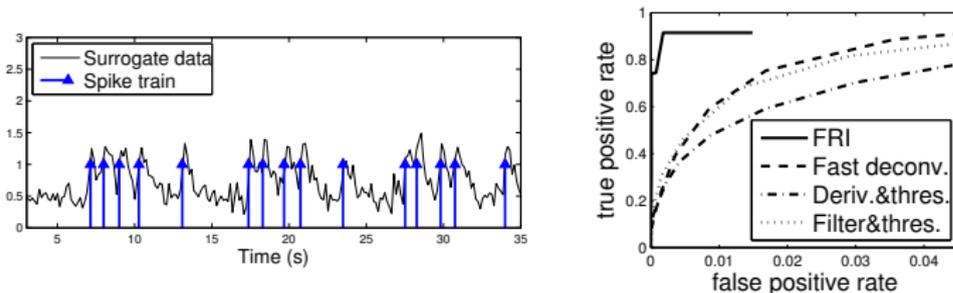


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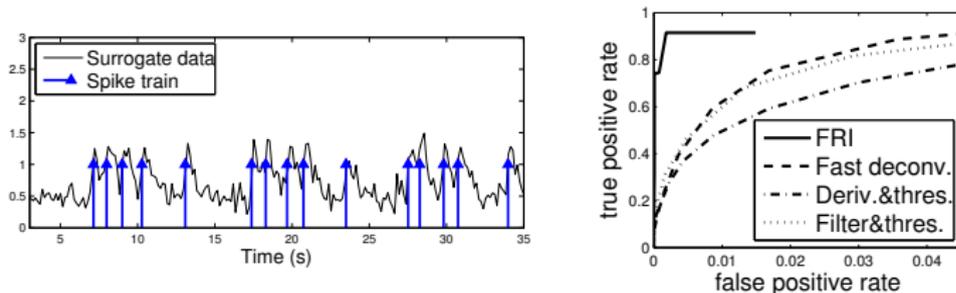


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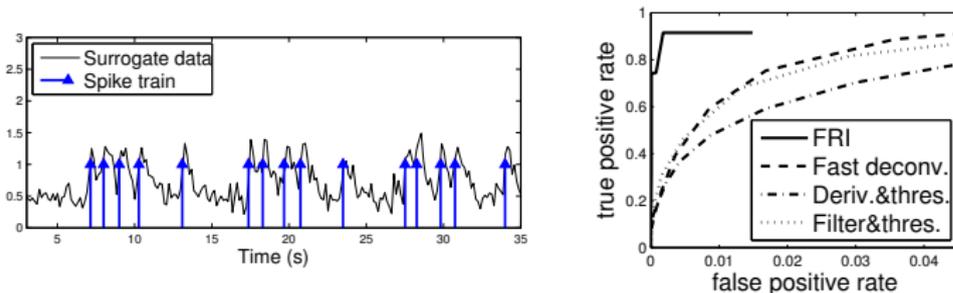


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- ▶ This technique can be used to monitor tens of neurons simultaneously since the fluorescence movie captures a volume that contains many neurons.
- ▶ The algorithm is fast enough to perform real-time spike inference:
  - ▶ The current MATLAB implementation can process more than 80 datastreams in parallel on a commercial laptop (2.5 GHz Intel Core i5 CPU).

Questions?