Distributed Network Utility Optimization in Wireless Sensor Networks Using Power Control*

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***Abstract*—We extend the existing network utility maximization (NUM) framework for wired networks to wireless sensor networks by formulating it in order to take into account interference among radio links. We study the conditions under which the formulated problem is a feasible convex optimization problem. Under such conditions, a distributed algorithm is proposed to solve the problem optimally. Finally, we provide numerical results, based on computer simulations, to show the performance of the proposed algorithm and the rate of convergence of its solution.**

***Index Terms*—Network Utility Maximization, Power Control, Rate Allocation, Wireless Sensor.**

I. Introduction

Wireless Sensor Networks are considered as a new class of distributed systems that are being extensively used to sense critical information that require us to take some actions. One of the most important challenges in wireless sensor networks is the development of a self-optimization technique that would enable sensors to adjust their transmissions for optimal network performance.

This paper makes the following contributions to the problem of optimizing wireless sensor networks with respect to network resource allocation:

1. Extend the network utility maximization (NUM) framework used in wired networks with the main characteristic of the wireless medium, namely, *interference* among links;
2. Propose a new formulation for the resource allocation;
3. Propose a distributed algorithm that is shown to solve the resource allocation problem optimally.

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The remainder of this paper is organized as follows: Section 2 is a brief literature review of the most relevant work in the area of NUM. Section 3 includes a presentation of the reformulated problem, based on *Power Control*, and a solution of the problem with a distributed algorithm that is shown to solve it optimally. In addition, we state the necessary and sufficient conditions needed for feasibility of the Power Control mechanism. Section 4 provides some numerical results concerning the performance of the algorithm and, finally, Section 5 concludes our results and outlines of our future work.

II. RELATED WORK

A. Wired networks

Evidently, the first work on *Network Utility Maximization* was published in [1], which proposes the following formulation:

$$\max \sum_{r \in R} U_r(x_r) \tag{1}$$

$$s.t. \quad Ax \le C$$
$$x \ge 0$$

where r, x_r and U_r denote the source sensor, the data rate of source sensor r and the utility of sensor r when transmitting at rate x_r respectively. Then, element A_{jr} is 1 when resource j lies on route r, and 0 otherwise. The authors propose an algorithm that enables the network nodes to determine the optimal way to share the link bandwidths among different traffic flows using the *max-min fairness criterion*. The solution is a set of differential equations, proved to be stable. The same problem was also solved in [2] following a different methodology based on Lagrange theory and two algorithms, both synchronous and asynchronous, are proposed. In [3] and [4], the authors use multi-level decomposition techniques to provide different optimization algorithms, each with a different trade-off among convergence speed, message overhead and distributed computation architecture. A systematic framework is also presented to decompose the optimization problems.

B. Wireless Networks

The first attempt for NUM in wireless networks was made in [5]. Its most important contribution is the proof that the logarithmic relation between capacity and transmission power is neither convex nor concave and the proposal of a centralised algorithm converging to the optimal. In [6] and [7], the NUM framework was converted into *mission-centric*, where utility functions depend on more than one flows and flows may have more than one sinks. Moreover, the notion of *maximal clique* is introduced, which is a set of links that mutually interfere with each other. In addition, a distributed algorithm is proposed for solving the problem using a scheduling mechanism to avoid the difficulties of considering interference between links.

To the best of our knowledge, existing work on the NUM has not actually taken into account the major characteristic of wireless networks, the *interference* among links. The key contribution of this work is to propose a NUM framework that indeed takes interference into account and makes use of *power control* to cope with interference.

III. Network Utility Optimization Using Power Control

A. Problem Formulation

Consider a set of M randomly deployed sensors that act as data sources and send traffic to other nodes in a multihop wireless network. Assume that $\Omega = \{1, 2, \ldots, S\}$ is the set of all nodes in the network and that each traffic flow is generated by one of the M sensors with only one node as its destination. A rate vector $\underline{r} = [r_1, r_2, \ldots, r_M]^T$ denotes the transmission rates for all the individual nodes. Moreover, assume that there exists a set $T = \{1, 2, \ldots, L\}$ of links in the network. There is a path loss matrix G of size $L \times L$ which depends on the physical characteristics of the link and whose element G_{ij} is the path loss gain from the transmitter of link i to the receiver of link j. Vector $\underline{p} = [p_1, p_2, \ldots, p_L]^T$ consists of all the individual transmission powers of the links. Furthermore, we use a vector $\underline{c} = [c_1, c_2, \ldots, c_L]^T$ to denote link capacities. Each traffic flow i is characterized by a utility function, $U_i(r_i)$, a function of the transmission rate of the flow. Moreover, there is a cost function $V_j(p_j)$ for each link j, which depends on its transmission power, p_j, and represents the cost of using the limited power resources in the wireless sensor network.

We propose the following NUM formulation:

$$\textit{Problem } P: \qquad \max_{\underline{r}, \underline{p}} \; \sum_{i=1}^{M} U_i(r_i) - \sum_{j=1}^{L} V_j(p_j) \tag{2}$$

$$\text{s.t.} \quad \sum_{i \in Z(j)} r_i \le c_j \qquad \forall \text{link j}$$

$$\frac{G_{jj} \cdot p_j}{\sum_{\substack{k=1,\\ k \ne j}}^{L} G_{jk} \cdot p_k + n_j} \ge \gamma_j \qquad \forall \text{link j}$$

where γ_j is the target signal-to-interference-plus-noise ratio (SINR) for link j and $Z(j)$ represents the set of traffic flows passing through each link j.

According to its first constraint, the total traffic flow passing from each link j should not exceed the link capacity, c_j. This is actually the same constraint as in the NUM framework for wired networks [1]. Even though it is difficult to know the exact capacity of a wireless link, it is possible to estimate it under some specific conditions based on the SINR at each node of the network. This is the contribution of the second constraint which is actually an expression of the power control problem. According to these constraints, we maximize the objective function so that the SINR is at least equal to a target value, which is a measure of the quality offered to the users of the network. In that way, we can capture the *interference* between links. As a result, if we know that the SINR will always be higher than a certain limit, one can be sure that the capacity of a link will always be above a specific value (c_j in our case). Hence, the combination of the two constraints can adequately capture the necessary characteristics of wireless links.

B. Solution of the Problem

Readers not familiar with *Convex Optimization Theory* are advised to refer to [8]. To solve problem P using local search algorithms, we need to make some assumptions about the concavity of the objective function. Both constraints of *Problem P* are linear and, therefore, convex. So, according to [8], three conditions must hold:

- functions $U_i(r_i)$, $i = 1,\ldots,M$, must be strictly concave, increasing and twice differentiable functions of r_i for $i = 1,\ldots,M$,
- functions $V_j(p_j)$, $j = 1,\ldots,L$, must be strictly convex, increasing and twice differentiable functions of p_j for $j = 1,\ldots,L$,
- variables r_i take values in the range $I_r = [m_r, M_r]$, where $m_r \geq 0$ and $M_r < \infty$, and p_j take values in the range $I_p = [m_p, M_p]$, where $m_p \geq 0$ and $M_p < \infty$.

We regard *Problem P* as our primal problem. As we can see, both source rates r_i and transmission powers p_j are coupled by constraints 1 and 2, respectively. So, in order to solve the problem in a distributed way, we need to look at the problem from a different point of view in order to reveal its distributed nature. Hence, we solve the *Dual Problem*. We define the *Lagrangian* function L of an optimization problem:

$$\min f_0(x) \tag{3}$$

$$\begin{aligned} s.t. \quad & f_i(x) \leq 0, \quad & i = 1,\ldots,m \\ & \mathrm{h}_i(x) = 0, \quad & i = 1,\ldots,p \end{aligned}$$

as a function $L : R^n \times R^p \to R$ of the form:

$$L(x,\mu,\lambda) = f_0(x) + \sum_{i=1}^{m} \mu_i f_i(x_i) + \sum_{i=1}^{p} \lambda_i h_i(x_i) \tag{4}$$

Note that the variables μ_i and λ_i are called *Lagrange Multipliers* and are also the optimization variables of the *Dual Problem*. Please, refer to [8] for a complete description of the *Lagrangian* function and its properties. The objective function of the *Dual Problem* is

$$l(\underline{\mu},\underline{\lambda}) = \max_{r_i \in I_r, p_j \in I_p} L(\underline{x},\underline{p},\underline{\mu},\underline{\lambda}) \iff$$

$$l(\underline{\mu},\underline{\lambda}) = \sum_{i=1}^{M} \max_{r_i \in I_r} \left[U_i(r_i) - r_i \cdot \lambda^i \right] -$$

$$-\sum_{j=1}^{L} \min_{p_j \in I_p} \left[V_j(p_j) - \mu_j \left(G_{jj} p_j - \gamma_j \sum_{\substack{k=1, \\ k \neq j}}^{L} G_{jk} p_k - \gamma_j n_j \right) \right] + \sum_{i=1}^{L} \lambda_j c_j \tag{5}$$

Based on (5) we could define two optimization problems. The first one is:

$$\textit{Problem } P_1^i : \qquad \max_{r_i \in I_r} \; U_i(r_i) - r_i \cdot \lambda^i \tag{6}$$

where
$$\lambda^i = \sum_{j \in S(i)} \lambda_j . \tag{7}$$

Each source sensor i can solve the above problem in order to determine the optimal traffic rate that it should transmit its data. The only piece of information that each source sensor needs is the aggregate cost of the links it is using, which does not produce significant message overhead. Also note that problem P_1^i is actually the maximum benefit that sensor i can achieve for that given aggregate price of links.

The second distributed problem is:

Problem P_2^j *:*

$$\min_{p_j \in I_p} \left[V_j(p_j) - \mu_j \cdot \left(G_{jj} p_j - \gamma_j \sum_{\substack{k=1, \\ k \neq j}}^{L} G_{jk} p_k - \gamma_j n_j \right) \right] \tag{8}$$

It is clear that *Problem* P_2^j can be solved by each link j independently. At each iteration, link j should be aware of the value of the dual variable μ_j. Note that the optimal value of problem P_2^j is actually the minimum transmission cost that a link can have for the given price μ_j. So, the objective function of the *Dual Problem* can be written in the form:

$$l(\underline{\mu},\underline{\lambda}) = \sum_{i=1}^{M} P_1^i - \sum_{j=1}^{L} P_2^j + \sum_{j=1}^{L} \lambda_j c_j \tag{9}$$

Then, the *Dual Problem* of the network is given by:

$$\text{Dual Problem } D : \quad \min_{\underline{\mu},\underline{\lambda}} \; l(\underline{\mu},\underline{\lambda}) \tag{10}$$

$$\begin{aligned} s.t. \quad & \underline{\lambda} \geq \underline{0} \\ & \underline{\mu} \geq \underline{0} \end{aligned}$$

Under the assumptions made earlier, the *Duality Gap* is zero and the optimal prices of the dual values, which are also the Lagrange multipliers of the *Primal Problem*, exist and hence the solution of the *Dual* problem is also the solution of the *Primal* one. In order to solve it, we will use an iterative algorithm, the *Gradient Projection Method* described in [9]. Recall that the recursive formula for determining the value of a variable x at time $t+1$ is:

$$x(t+1) = \left[x(t) - \alpha \cdot \nabla F\left(x(t)\right)\right]^{+} \qquad (11)$$

where α is a positive constant. In our case, we obtain the update equations for the shadow prices μ_j and λ_j as follows:

$$\mu_j(t+1) = \left[\mu_j(t) - \alpha_1\left(-G_{jj}p_j(\mu_j) + \gamma \sum_{k=1,k\neq j}^{L} G_{jk}p_k + \gamma n_j\right)\right]^{+} \qquad (12)$$

and

$$\lambda_j(t+1) = \left[\lambda_j(t) - \alpha_2\left(c_j - \sum_{i\in Z(j)} r_i(\underline{\lambda})\right)\right]^{+} \qquad (13)$$

where $[x]^{+} = \max(x,0)$.

Using the same procedure, we also obtain the optimal solutions for problems P_1^i and P_2^j as

$$r_i^* = \left[U_i^{'}\left(\sum_{j\in S(i)} \lambda_j\right)^{-1}\right]_{m_r}^{M_r} \qquad (14)$$

and

$$p_j^* = \left[V_j^{'}\left(\mu_j G_{jj}\right)^{-1}\right]_{m_p}^{M_p} \qquad (15)$$

respectively, where $[x]_a^b = \min\left(\max(x,a),b\right)$.

C. Distributed Optimization Algorithm

We propose the following distributed algorithm that can be used by each node in the network to reach the optimal performance. The algorithm consists of two sub-algorithms, one carried out by each source sensor i and another one implemented by each link j, as shown in tables 1 and 2.

TABLE I: SOURCE SENSOR'S i DISTRIBUTED ALGORITHM

Each time slot $t = 1,\ 2,\ldots$ do:

1. Receive the aggregate cost λ^i , given by (5), for all the links that sensor *i* is using for sending its traffic.
2. Calculate the new data rate for time $t+1$ using (14)
3. Send the new rate $r_i(t+1)$ to all the links that sensor *i* is using

The message overhead of the algorithm does not seem to be significant. The aggregate price of each flow can be calculated by sending a packet from sink to source and each time it reaches an intermediate node, the link increases the cost header by its shadow price. Then, when the packet reaches the source the value of this header variable will be equal to the aggregate cost λ^i.

TABLE II: LINK'S j DISTRIBUTED ALGORITHM

Each time slot $t = 1,\ 2,\ldots$ do:

1. Receive the aggregate traffic rate that is passing through it using formula $r^j = \sum_{i\in Z(j)} r_i$.
2. Calculate the value of the transmission power price μ_j for time $t+1$ using (12).
3. Calculate the value of the rate price λ_j for time $t+1$ using (13).
4. Use the new value $\mu_j(t+1)$ in order to calculate the optimal transmission power p_j^*, using (15).
4. Send the new price $\lambda_j(t+1)$ to all the sources that are using link j.

D. Feasibility of the Algorithm

Even though we have proposed an algorithm for solving the NUM problem, this does not mean that the problem has always a feasible solution and hence we need to provide some necessary and sufficient conditions for feasibility. Observe that the problem P can be viewed as two separable subproblems that are connected by the objective function. The first one is the rate allocation problem and the second is the power control problem. The rate allocation sub-problem is actually presented and solved in [1] and [2], and is proved to be feasible since the zero vector $\underline{0}$ is a feasible point of the problem. On the contrary, the power control sub-problem is not always feasible. However, Lemma 1 in [10] represents a necessary and sufficient condition that guarantees the existence of a feasible solution. Accordingly, the SINR targets γ_j, $j = 1,\ldots,L$, are feasible if and only if $\rho < 1$ where ρ is the Perron-Frobenius eigenvalue of the matrix $Y \cdot H^{-1} \cdot E$, where

$$Y = \begin{bmatrix} \gamma_j & 0 & \cdots & & 0 \\ 0 & \gamma_j & \cdots & & 0 \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ \vdots & \vdots & & \gamma_j & 0 \\ 0 & 0 & \cdots & 0 & \gamma_j \end{bmatrix} \qquad (16)$$

Matrix H is defined as $H_{ij} = \begin{cases} G_{ii}, & i = j \\ 0, & i \neq j \end{cases}$ and matrix E is

$$E_{ij} = \begin{cases} 0, & i = j \\ G_{ij}, & i \neq j \end{cases}.$$

IV. Numerical Results

In this section, we present numerical results to show the performance of the proposed distributed algorithm. In particular, we have simulated the algorithm in a network consisting of 10 sensor nodes and a total of 9 links, as shown in Figure 1. Assume that three sensors, sensors 1, 2 and 3, in the network generate and send data flows to three other sensors, sensors 7, 9 and 10, and that the routing matrix of these flows is known a priori and fixed. For illustration purposes, we have set the utility functions $U_i(r_i) = \log(r_i)$ and the cost functions $V_j(p_j) = (p_j)^2$. Note that $U_i(r_i)$ is a concave function of r_i and $V_j(p_j)$ is a convex function of p_j, as required by the conditions mentioned earlier. Finally, the SINR target at each link is equal to $\gamma_{\text{target}} = 10\ dB$ and the capacity of each link is equal to $c^* = 10\ Mbps$.

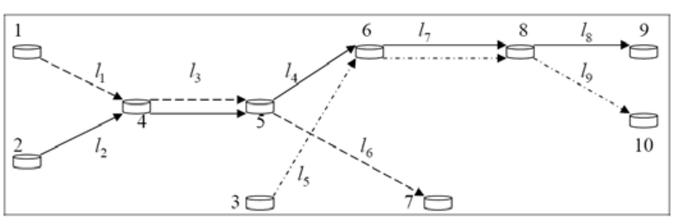

Figure 1 – The network topology

To study the convergence rate and the deviation from the optimality for the proposed algorithm, we have simulated our network for two different sets of update coefficients α_1 and α_2 and compared it with the actual optimal solution. The performance of our distributed algorithm is shown in Figures 2 to 6. According to [9], the values of α_1 and α_2 can be any positive and sufficiently small number, so that each iteration of the *Gradient Projection* method will decrease the value of the cost function unless we reach the optimal solution.

We show the convergence of the rate allocation for two flows (Figures 2 and 3) and transmission power of three links (Figures 4 and 5). Figure 6 depicts the convergence of the aggregate SINR error with respect to the SINR target. The convergence metric is a normalized summation of the differences between the actual and the target SINR values as follows:

$$SINR_{perf} = \frac{\sum_{i=1}^{M} \left| SINR_i - SINR_{\text{target}} \right|}{SINR_{\text{target}}} \qquad (17)$$

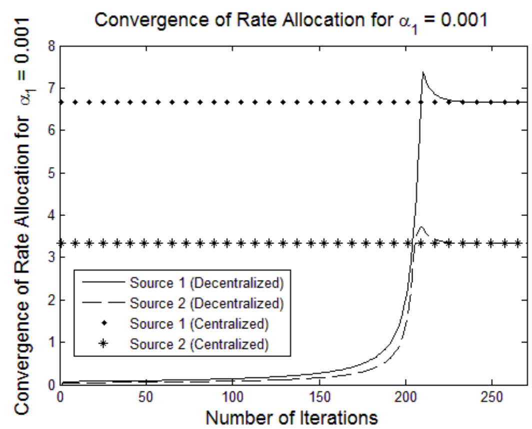

Figure 2 – Allocated Data Rate for $\alpha_1 = 0.001$

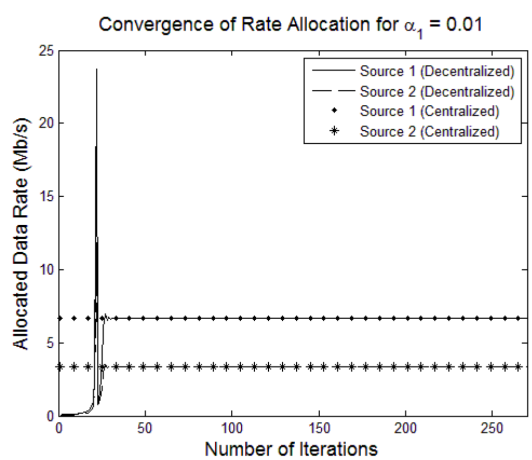

Figure 3 – Allocated Data Rate for $\alpha_1 = 0.01$

According to these graphs, it can be seen that for greater values of α_1 and α_2 the algorithm is converging quicker. However, the smaller the values of these parameters of the gradient projection method, the smoother the convergence to the optimal value will be. As shown in Figures 2 and 3, the allocated data rate may exceed the link capacity temporarily before it reaches the optimal solution at steady state. We further observe that the greater the coefficient α_1, the greater the data rate may exceed the capacity temporarily. Therefore, it may be worthwhile to choose small coefficient values to limit such excessive data rate.

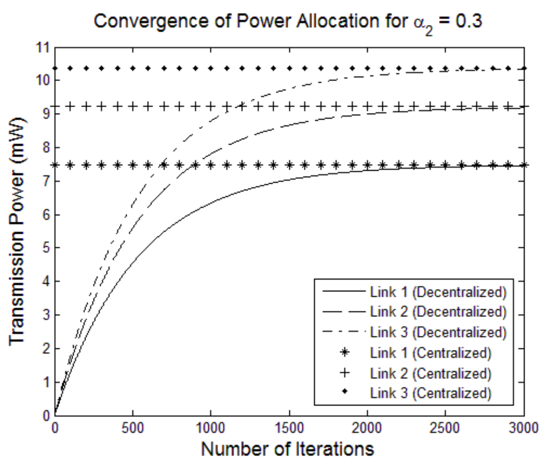

Figure 4 - Transmission Power for $\alpha_2 = 0.3$

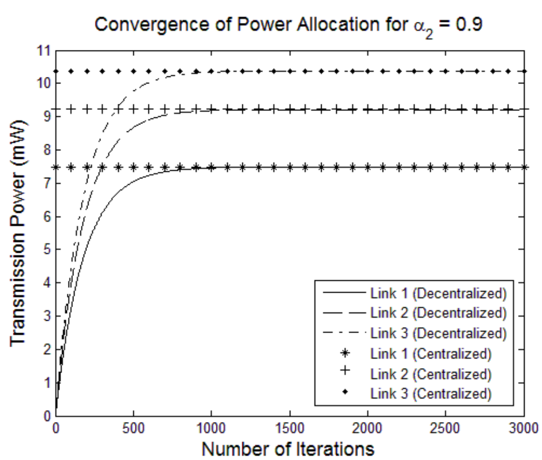

Figure 5 - Transmission Power for $\alpha_2 = 0.9$

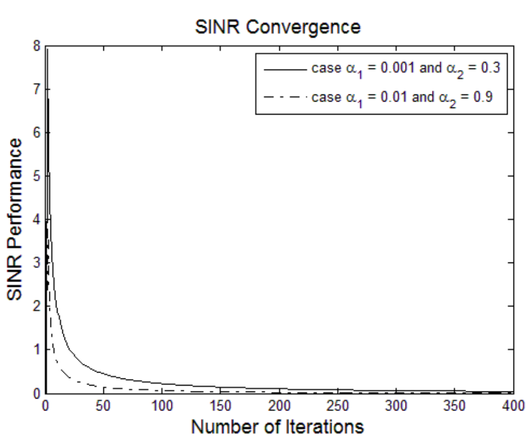

Figure 6 - SINR Performance

Hence, we see that the optimal decision for these parameters actually represents a trade-off between convergence rate and convergence smoothness in practice. Nevertheless, these simulation results reveal that the proposed algorithm always converges at the actual optimal value which means that it can indeed solve the maximization problem optimally by selecting appropriate data rates and transmission power levels.

V. CONCLUSIONS AND FUTURE WORK

In this paper, we have extended the existing *Network Utility Maximization* framework for wireless networks so that it takes into account the interference between links and the path loss effect by proposing a problem formulation based on *power control*. In addition, we have proposed a distributed algorithm for the optimization problem and obtained the conditions under which the problem is solvable and a feasible solution exists. Finally, we have verified the performance of the proposed algorithm in terms of convergence of rate allocation and transmission power by simulation.

We plan to extend the proposed framework further so that it takes into account multi-sink and multi-source flows. In addition, we also prefer to enhance the formulation to capture the coupling between SINR and link capacity.

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