

# Adaptive Plenoptic Sampling

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# Overview

- Image Based Rendering and Motivation
- Plenoptic Sampling Theory
- Spectral Analysis of a Slanted Plane
- Adaptive Sampling Algorithm
- Conclusions and Future Work

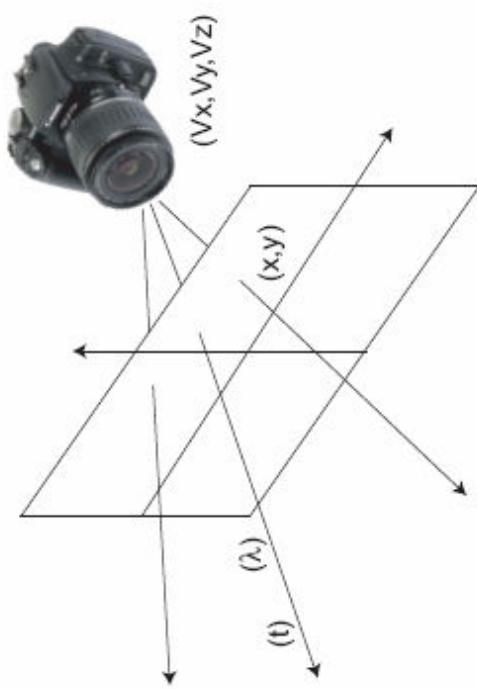
# Image Based Rendering

Image Based Rendering (IBR)  $\rightarrow$  Rendering New Viewpoints of a Scene from a Multi-View Image Set

In more detail:

- Images sample a set of lights rays from the scene to the camera
- New rendering interpolated from captured light rays
- Lights ray modelled using the 7D *Plenoptic Function* [1]

$\rightarrow$  IBR viewed as the Sampling and Reconstruction of the Plenoptic Function



# Motivation

Perform IBR on an Unknown Scene using a Camera Mounted to a Robot.

Unknown Scene

Important Questions:

- How should the camera travel relative to the scene?
- Where to sample along the path?
- Whether to zoom or not?



Current Work:

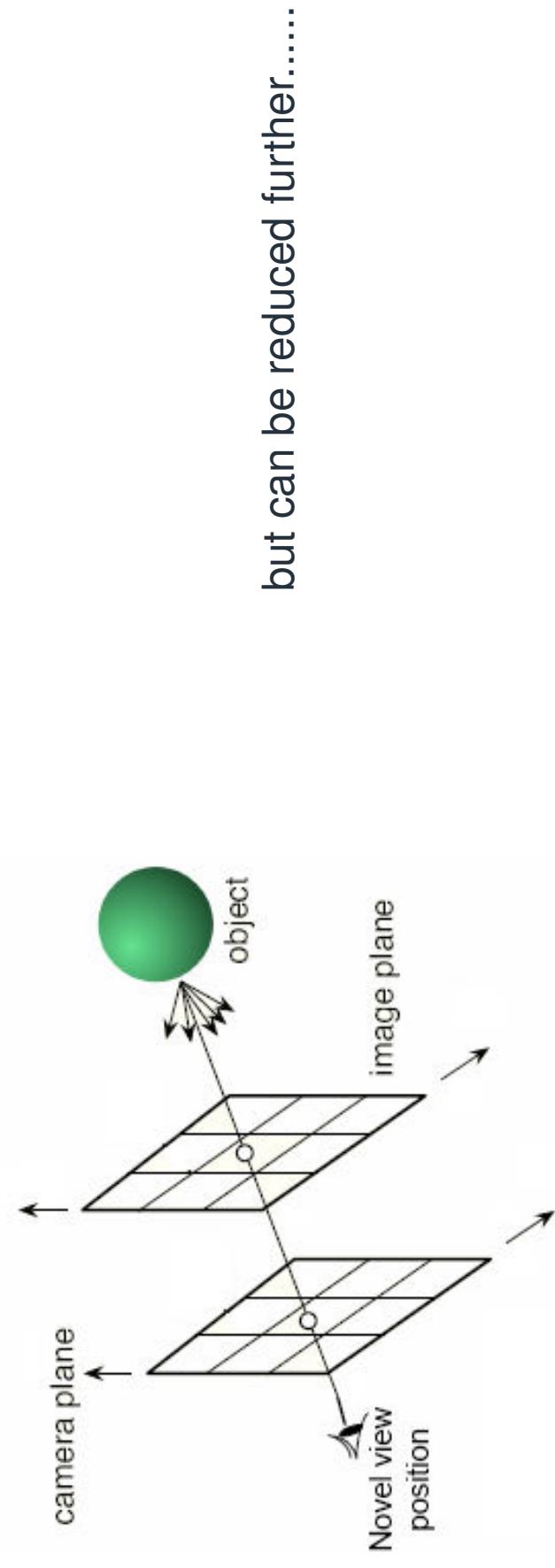
- Model the scene with a smoothly varying surface and bandlimited texture
- Non-uniform sampling based on approximating the surface with slanted planes

# Reducing Plenoptic Dimensionality

Three Assumptions:

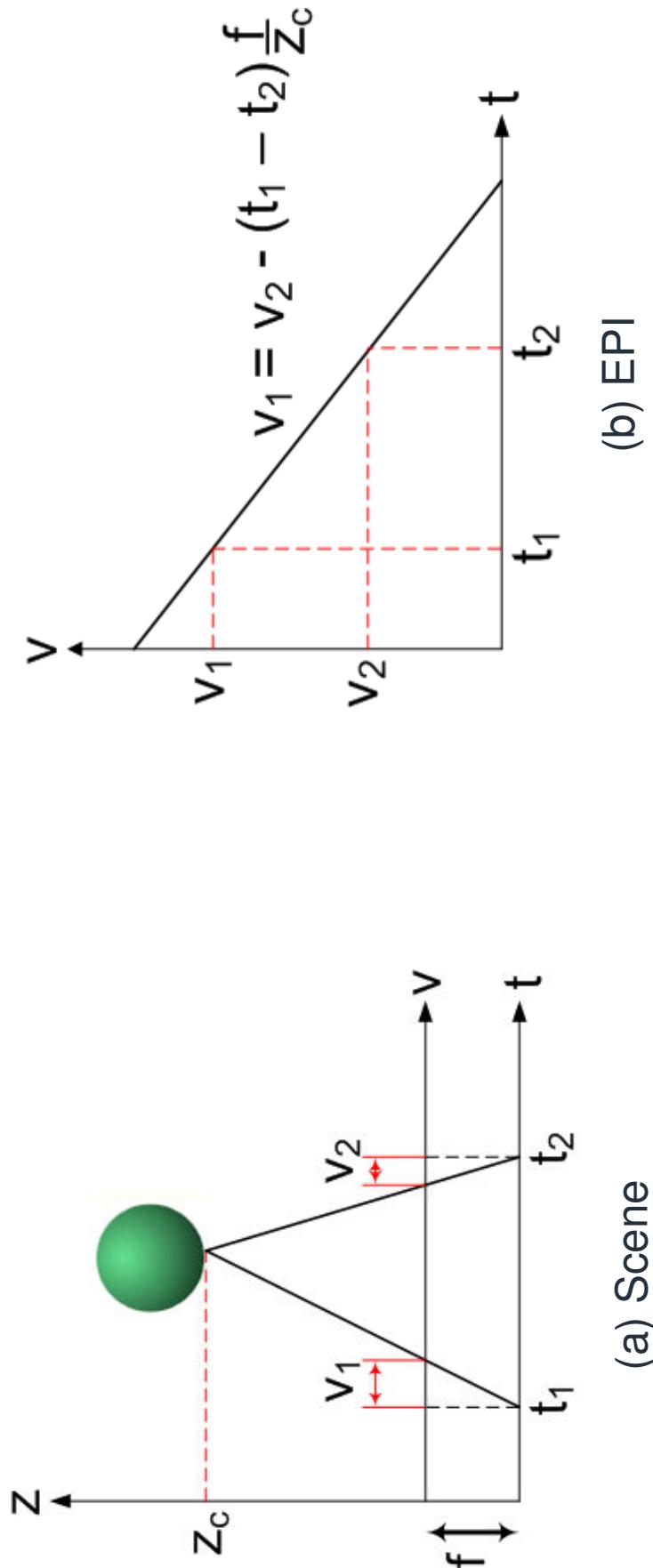
1. Static Scene  $\Rightarrow$  remove  $\tau$
2. Three constant wavelength bands  $\Rightarrow$  remove  $\nu$
3. Radiance of a light ray is constant along its path  $\Rightarrow$  Light ray define by the intersection with two planes

$\Rightarrow$  4D Light Field Parameterisation



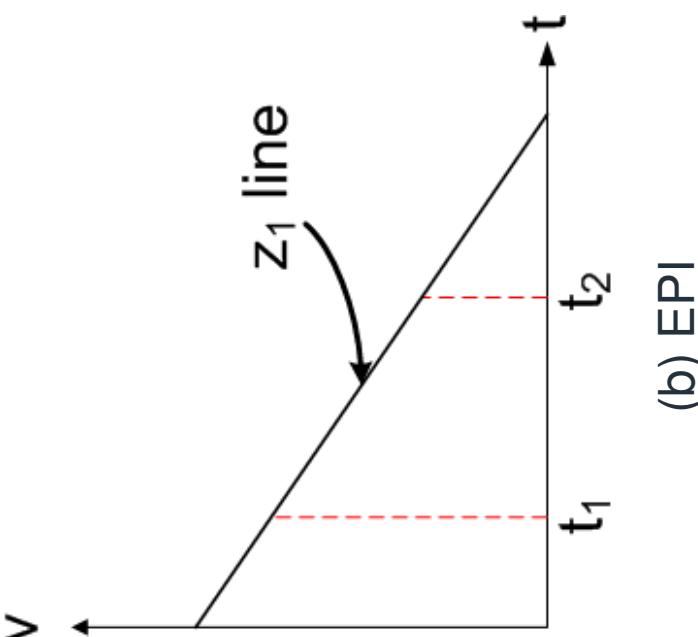
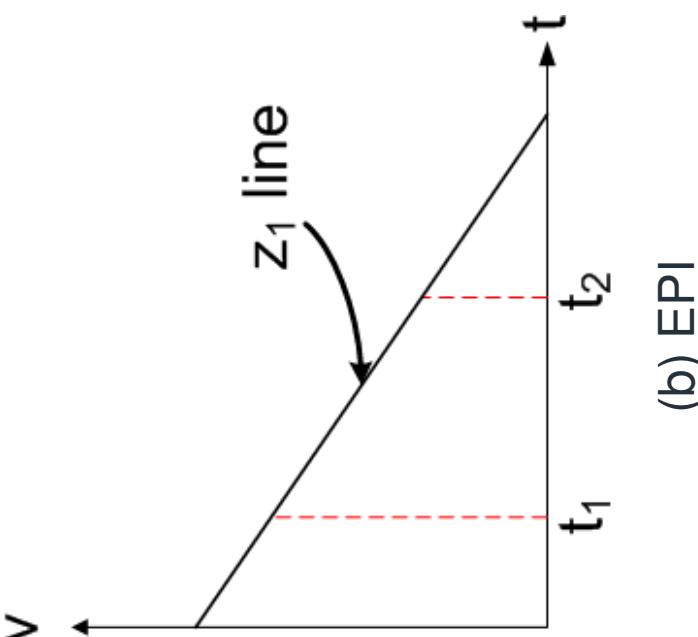
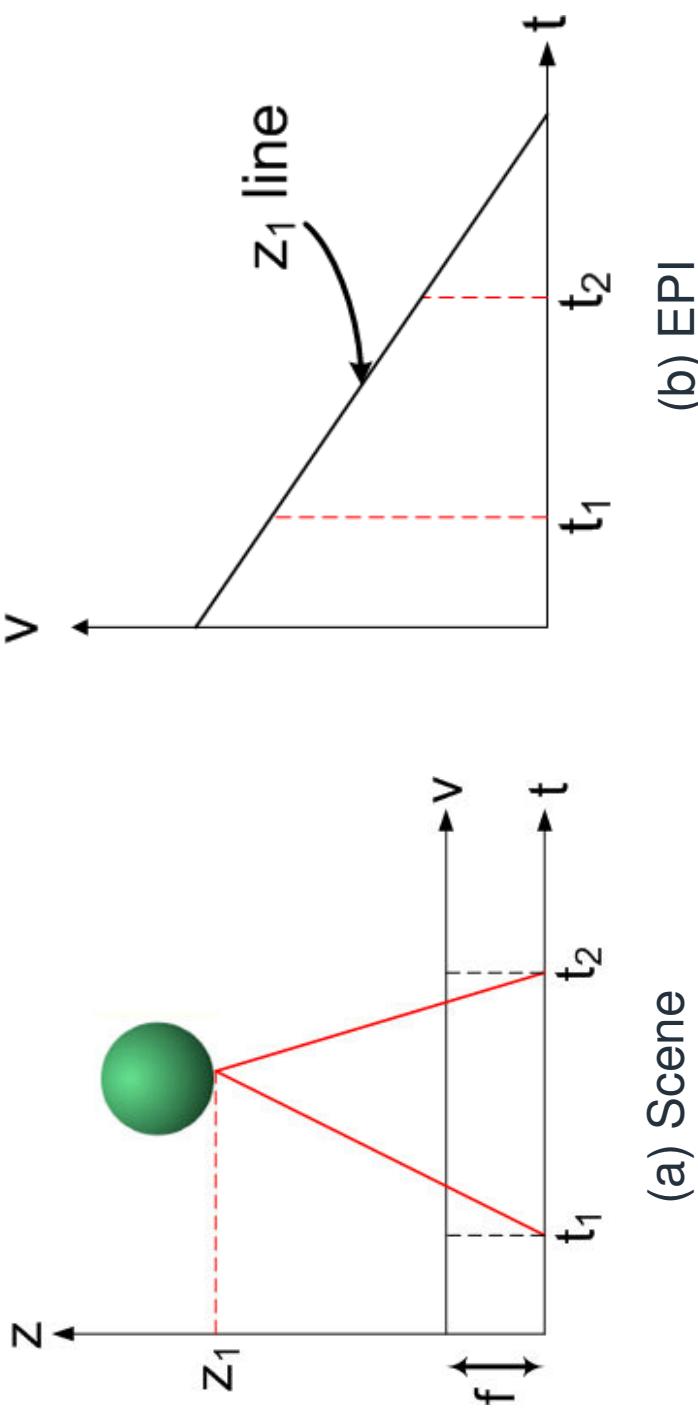
# Plenoptic Function and the Epipolar Plane Image

Consider the 2D Plenoptic Function,  $p(t, v)$ , known as the Epipolar Plane Image (EPI) [2]

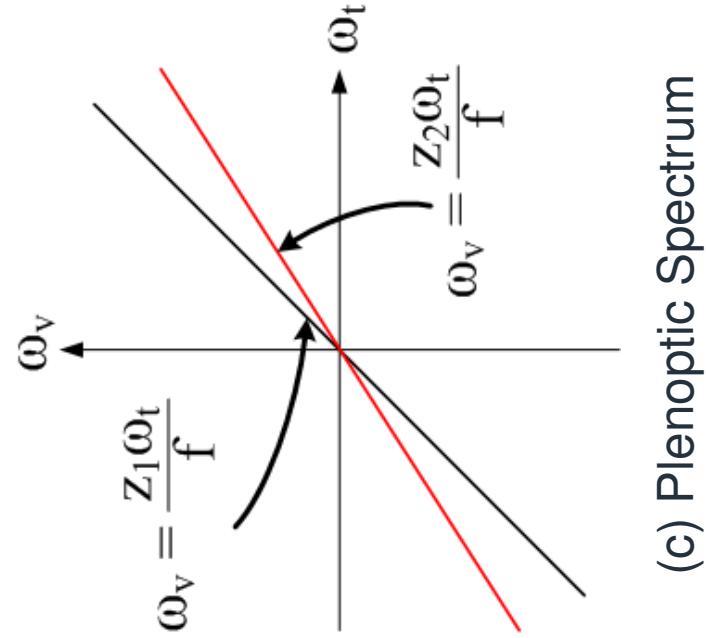
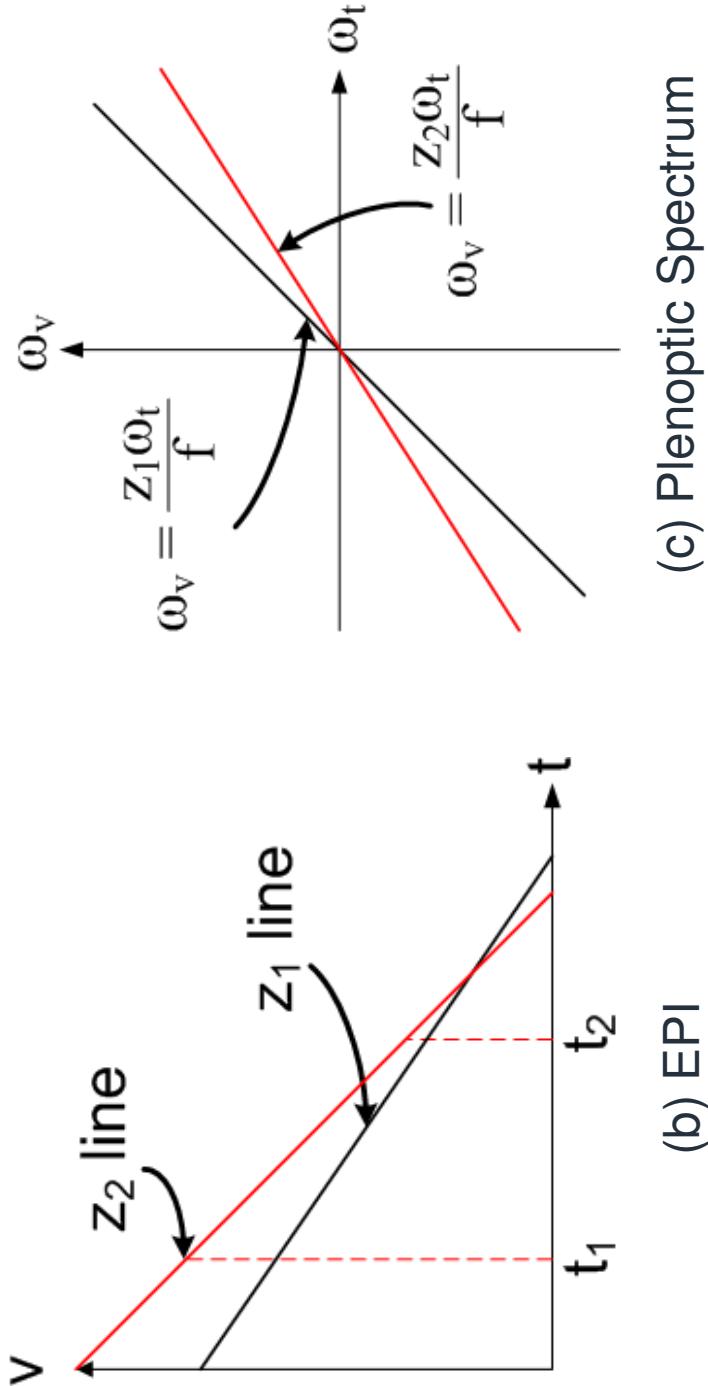
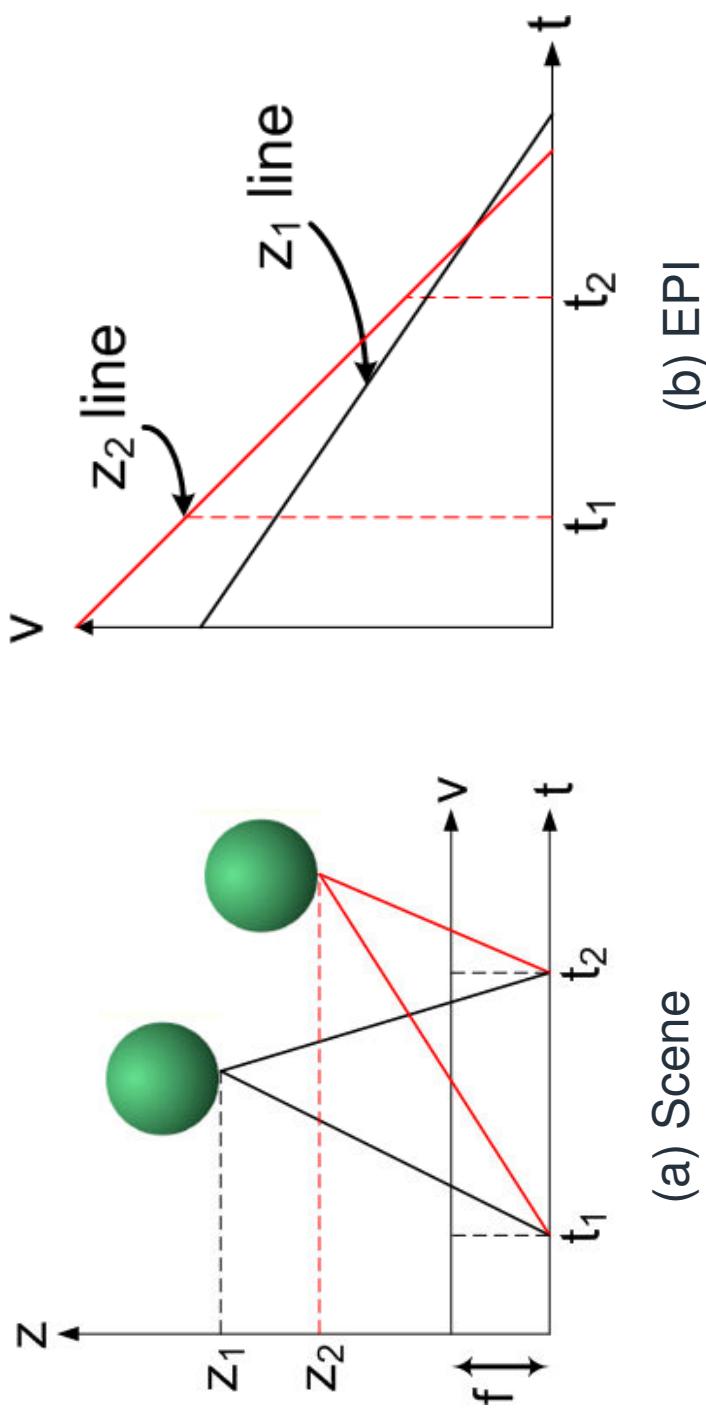


- Point in the scene  $\Rightarrow$  Line in the EPI plane where the slope depends on the depth
- Fixing a camera position  $t_1 \Rightarrow$  1D image signal
- Fixing a pixel  $v_1 \Rightarrow$  1D signal of the pixel captured by all cameras

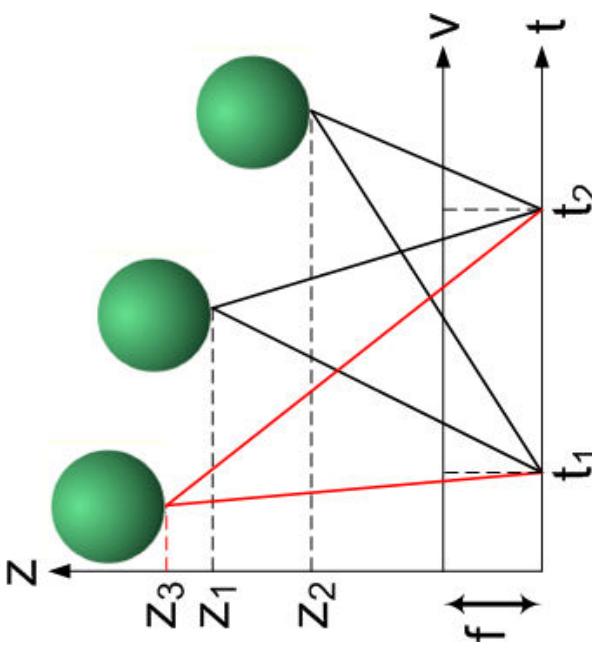
# Plenoptic Spectral Analysis



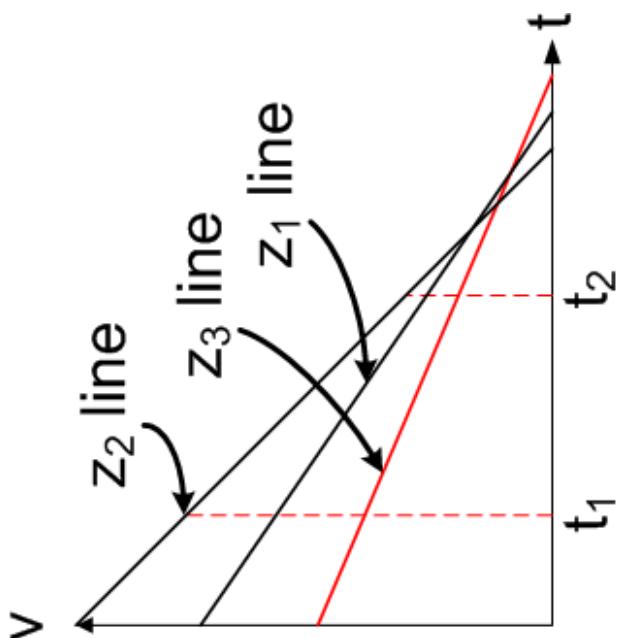
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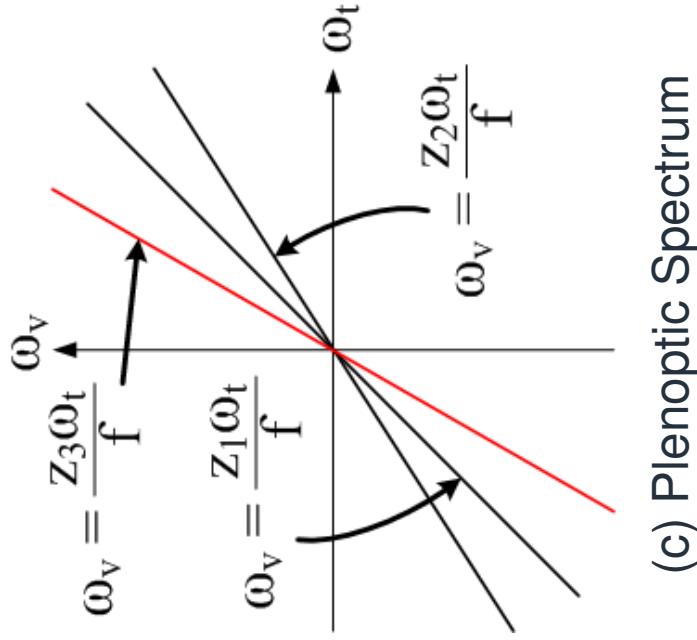
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(a) Scene



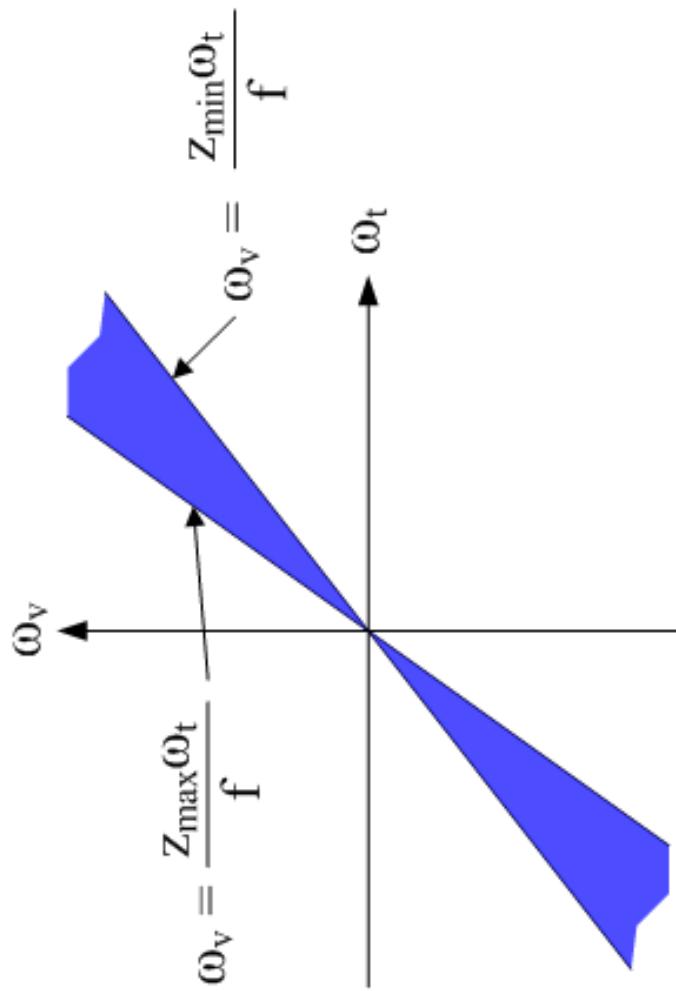
(b) EPI



(c) Plenoptic Spectrum

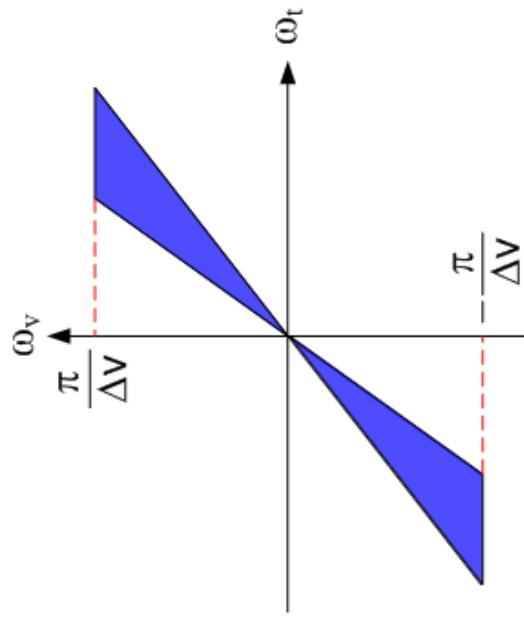
# Plenoptic Spectral Analysis

Plenoptic Spectrum exactly bound within two lines relating to the minimum and maximum depths of the scene [2,3]

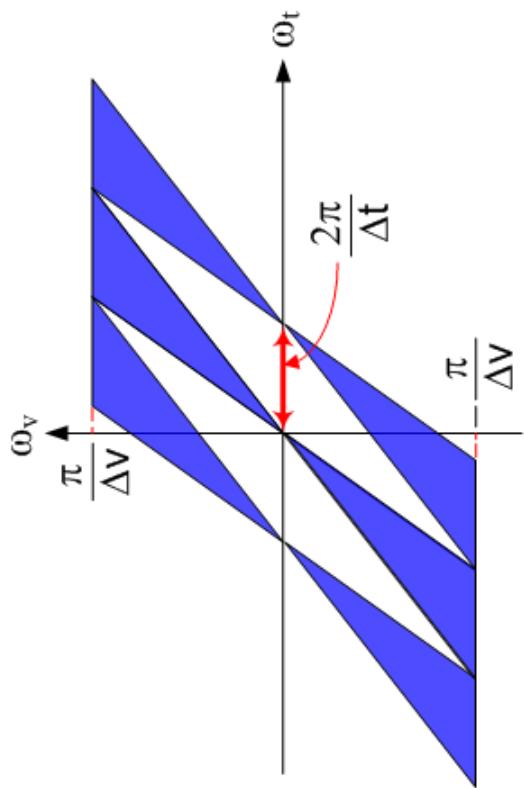


# Sampled Plenoptic Spectrum

- Finite Camera Resolution  $\Delta v \implies$  Enforced Lowpass Filtering in  $\omega_v$
- Sampling in  $t$  of period  $\Delta t \implies$  Replicated Plenoptic Spectra
- Undersampling  $\implies$  Replicated Spectra Overlap  $\implies$  Aliasing



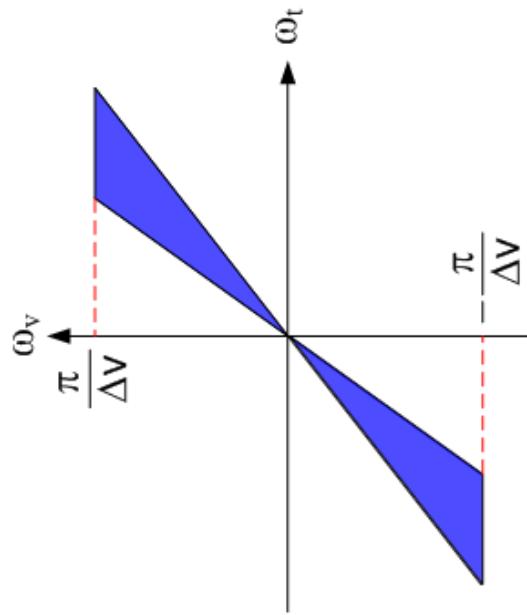
(a) Plenoptic Spectrum Sampled in  $v$



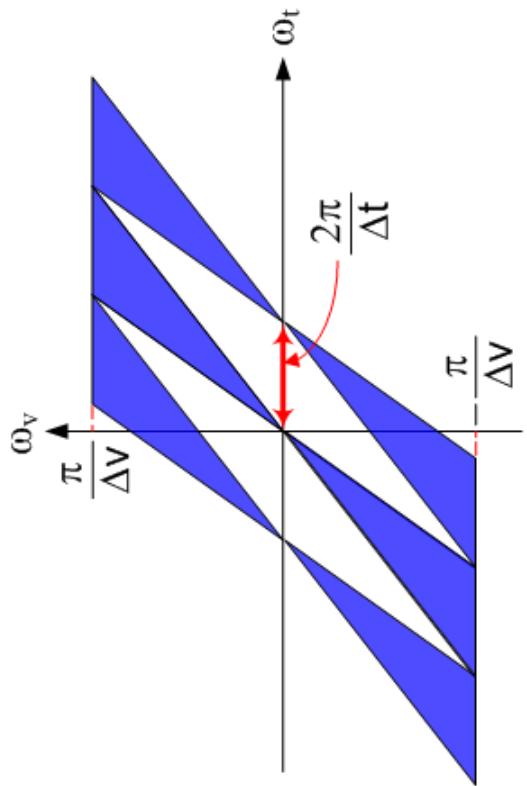
(b) Plenoptic Spectrum Sampled in  $t$

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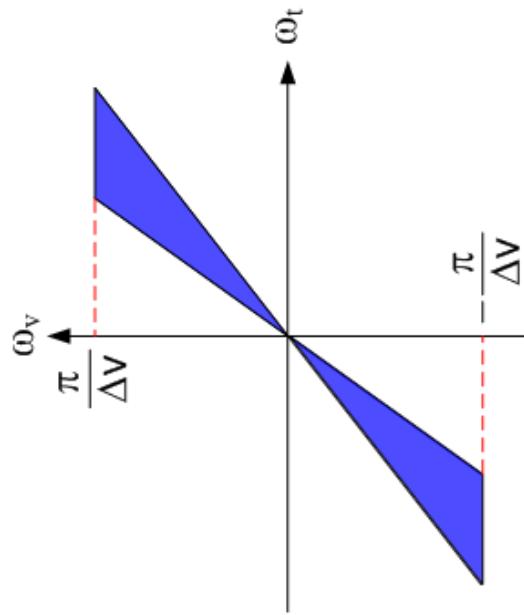


(b) Plenoptic Spectrum Sampled in  $t$

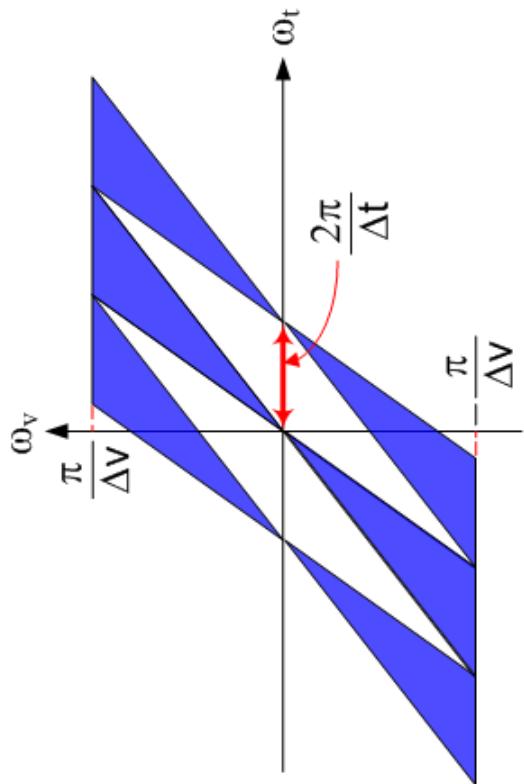
Assumes  $\implies$  Infinite Scene Width and Infinite Field of View (FoV),

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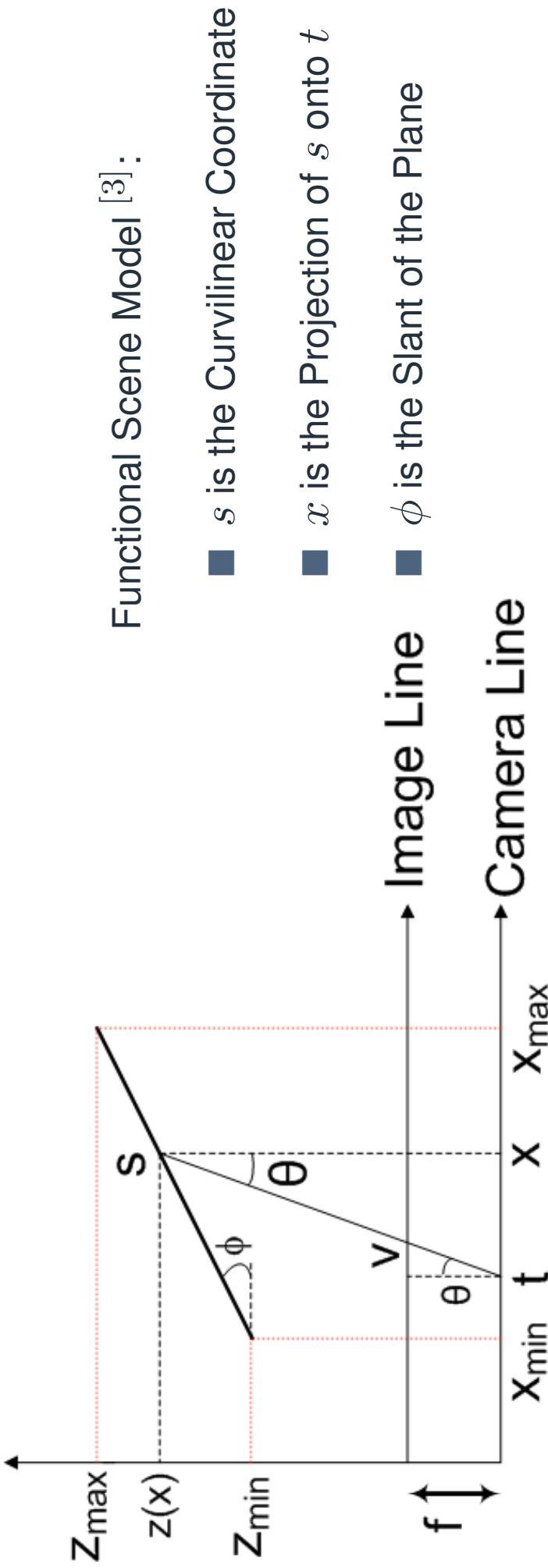
(a) Plenoptic Spectrum Sampled in  $v$



(b) Plenoptic Spectrum Sampled in  $t$

Assumes  $\implies$  Infinite Scene Width and Infinite Field of View (FoV),  
 $\implies$  Uniform Sampling in  $t$

# Slanted Plane Geometry

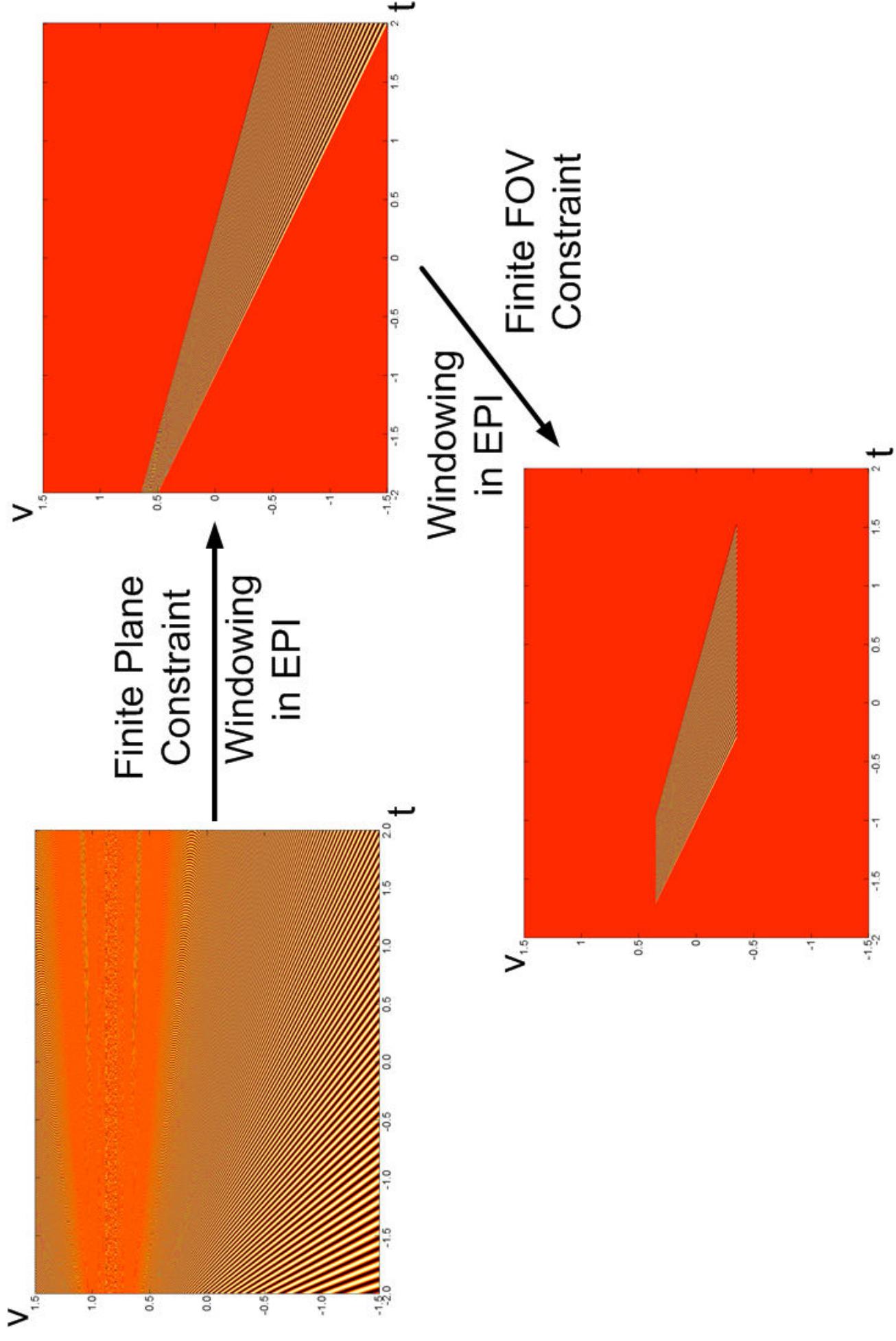


Bandlimited Texture Signal Pasted to Scene Surface

Constraints:

- Finite Field of View (FoV) for the Cameras  $\implies v \in [-v_m, v_m]$
- Finite Plane Width  $\implies s \in [0, T]$
- Lambertian Scene

# Effect of the Constraints on the EPI



# Evaluating Plenoptic Spectrum for Slanted Plane

Fourier Transform of Plenoptic Function:

$$\int_{t=-\infty}^{t=\infty} \int_{v=-\infty}^{v=\infty} p(t, v) e^{-j(\omega_t t + \omega_v v)} dv dt$$

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Changing from  $t$  to  $x$ :

$$\Rightarrow \int_{x=-\infty}^{x=\infty} \int_{v=-\infty}^{v=\infty} l(x) \left[ 1 - v \frac{z'(x)}{f} \right] e^{-j(\omega_v - z(x) \frac{\omega_t}{f}) v} e^{-j\omega_t x} dv dx$$

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Finite FoV Constraint:

$$\Rightarrow \int_{x=-\infty}^{x=\infty} l(x) \int_{v=-v_m}^{v=v_m} \left[ 1 - v \frac{z'(x)}{f} \right] e^{-j(\omega_v - z(x) \frac{\omega_t}{f}) v} e^{-j\omega_t x} dv dx$$

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Finite Plane Width Constraint:

$$\Rightarrow \int_{s=0}^{s=T} g(s) \cos(\phi) \int_{v=-v_m}^{v=v_m} \left[ 1 - v \frac{\tan(\phi)}{f} \right] e^{-j(\omega_v - s \frac{\sin(\phi) \omega_t}{f}) v} e^{-j\omega_t \cos(\phi) s} dv ds$$

# Plenoptic Spectrum when $g(s) = e^{j\omega_s s}$

## The Plenoptic Spectrum

$$|P| = \left| \frac{\omega_s f}{\sin(\phi)\omega_t^2} [\zeta(jb(c-1)) - \zeta(ja(c-1)) - \zeta(jb(c+1)) + \zeta(ja(c+1))] \right. \\ \left. + \frac{2v_m}{\omega_t} \left[ \text{sinc}(a) e^{-jca} - \text{sinc}(b) e^{-jcb} \right] \right|$$

Where  $\zeta(jx)$ , for  $x \in \mathbb{R}$ , is defined as

$$\zeta(jx) = \begin{cases} \text{E}_1(jx) + \ln(jx) + \gamma & , \text{ if } x > 0 \\ \text{E}_1(-jx) - 2j\text{Si}(-x) + j\pi + \ln(jx) + \gamma & , \text{ if } x < 0 \\ 0 & , \text{ if } x = 0 \end{cases}$$

$\text{E}_1(w)$  is the Exponential Integral,  $\text{Si}(w)$  is the Sine Integral and  $\gamma$  is Euler's Constant [1]

$$\text{and } a = \omega_v v_m - \omega_t \frac{z_{max} v_m}{f} , b = \omega_v v_m - \omega_t \frac{z_{min} v_m}{f} , c = \frac{-f(\omega_t \cos(\phi) - \omega_s)}{\sin(\phi) \omega_t v_m}$$

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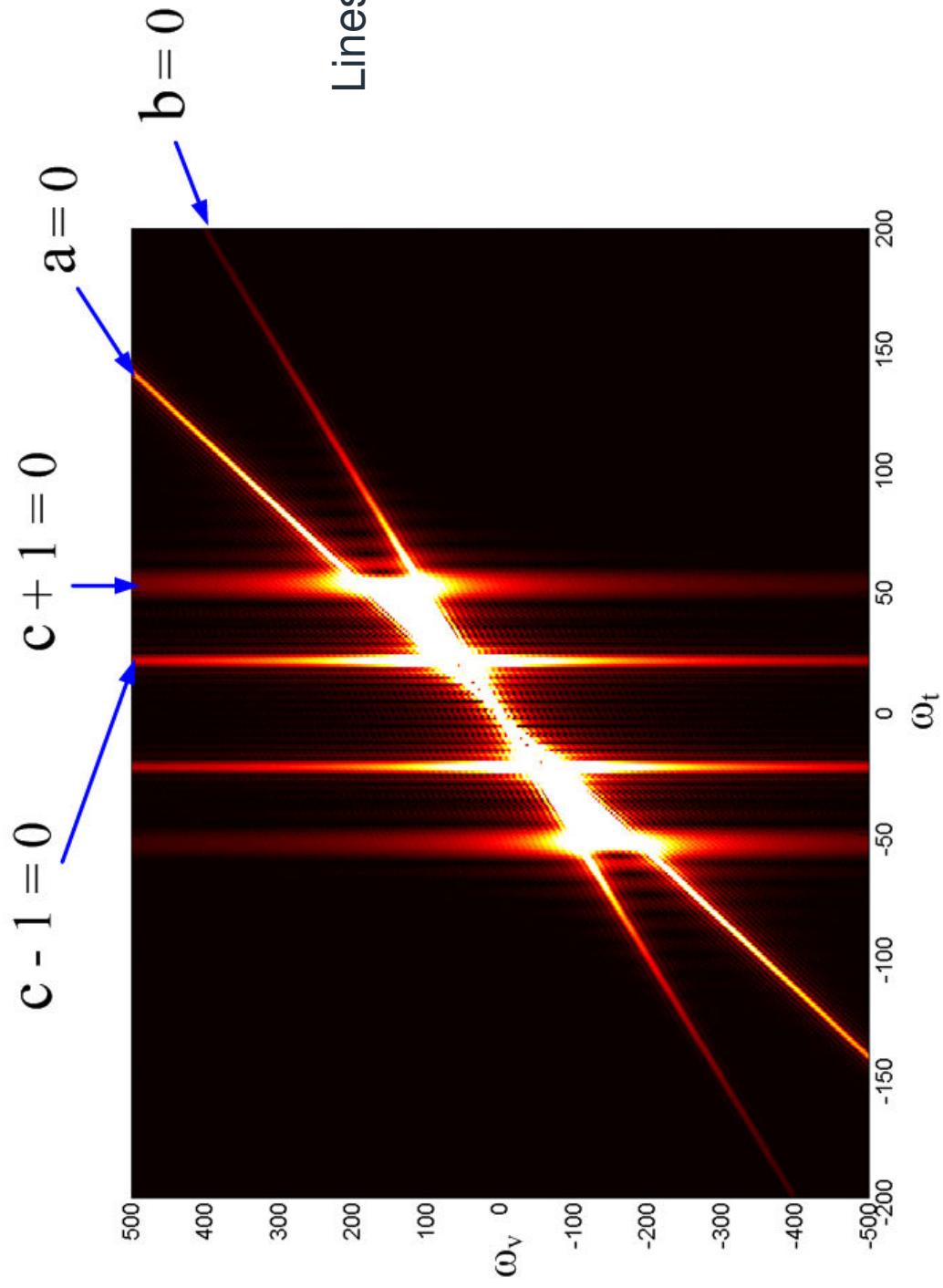
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# Slanted Plane Plenoptic Spectrum



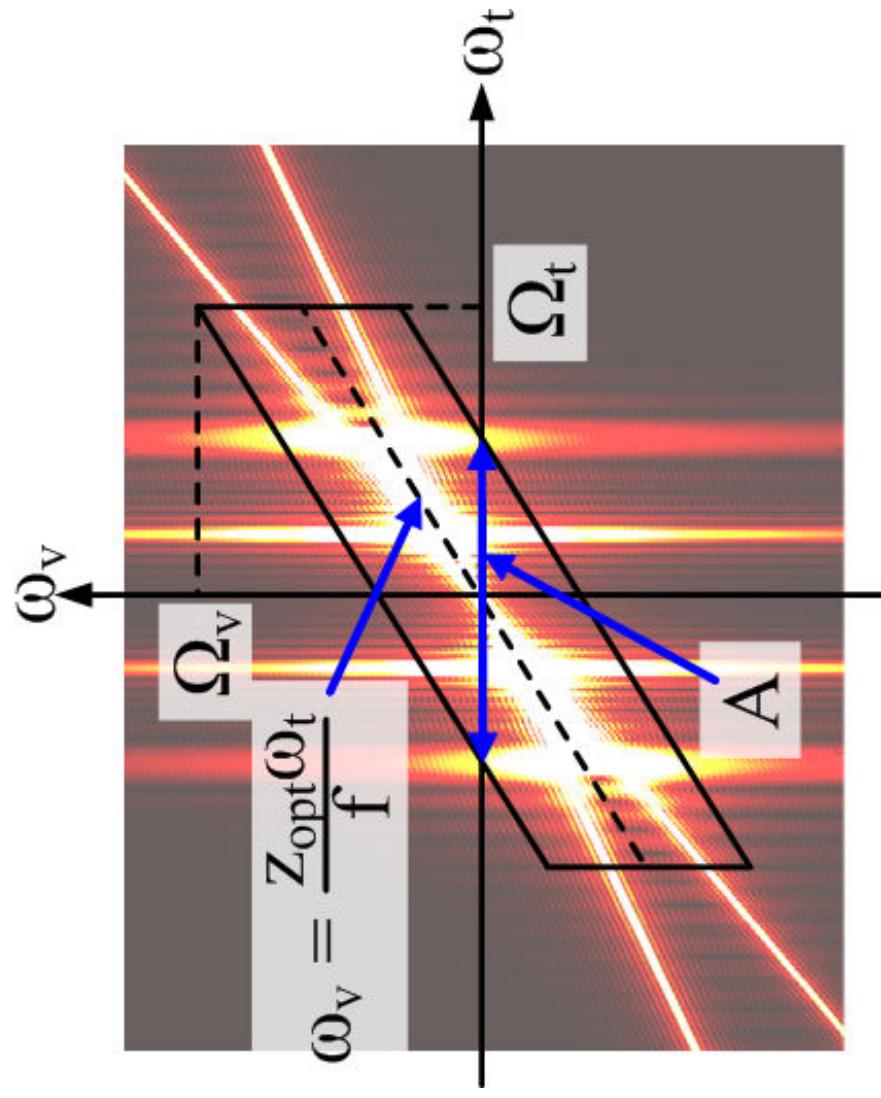
Lines Relating to Finite FoV:

$(c - 1) = 0 \implies$  Minimum Modulated Frequency Projected onto the Image

$(c + 1) = 0 \implies$  Maximum Modulated Frequency Projected onto the Image

# Essential Bandwidth for a Slanted Plane

Parametric model of the essential bandwidth, consisting of 4 parameters:



$$\Omega_t = \frac{\omega_s f}{f \cos(\phi) - v_m |\sin(\phi)|} + \frac{2\pi}{T},$$

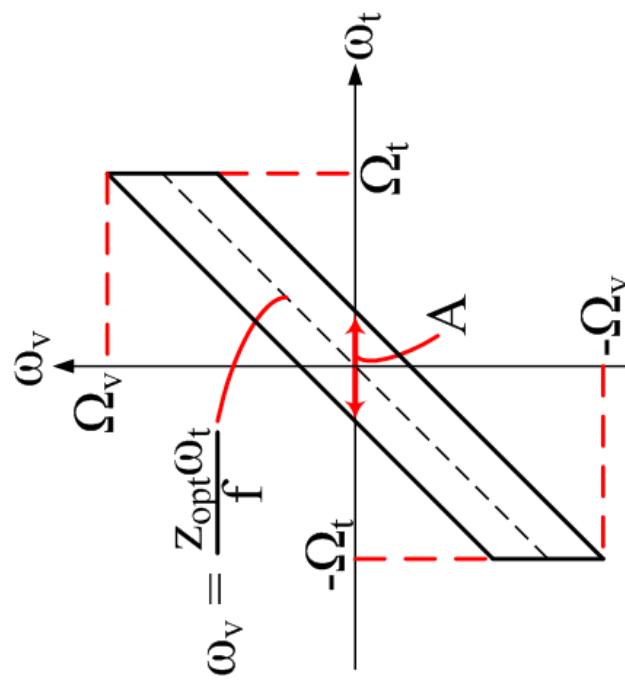
$$\Omega_v = \Omega_t \frac{z_{max}}{f} + \frac{\pi}{v_m},$$

$$z_{opt} = \frac{z_{max} + z_{min}}{2},$$

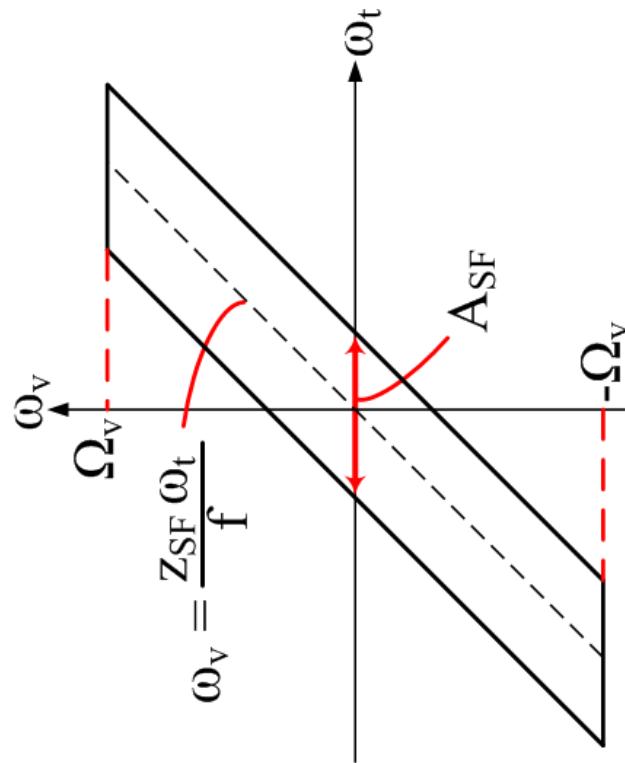
$$A = \frac{T |\sin(\phi)| \Omega_t}{z_{opt}} + \frac{2\pi f}{z_{opt} v_m}$$

# Plenoptic Sampling of a Slanted Plane

Comparison between the essential bandwidth and the standard filter:



(a) Essential Bandwidth



(b) Standard Filter [2]

The differences:

$$z_{opt} = \frac{z_{max} + z_{min}}{2}$$

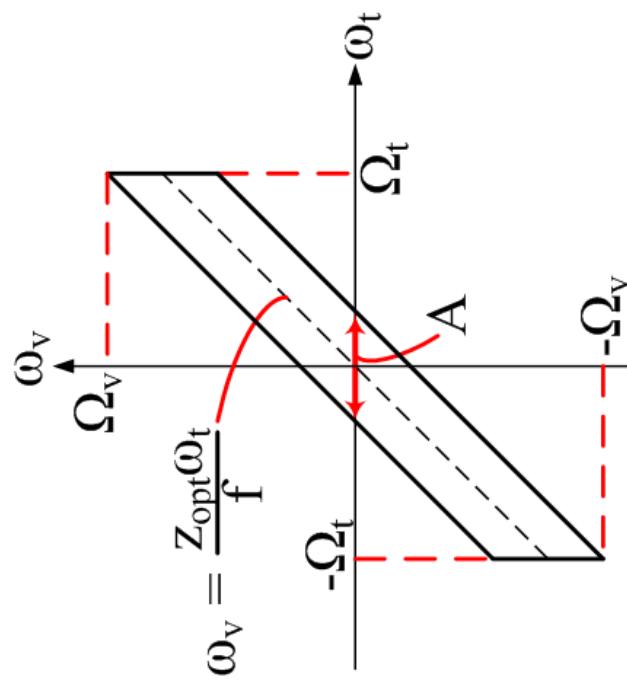
$$z_{SF} = \frac{2}{z_{max}^{-1} + z_{min}^{-1}}$$

$$A = \frac{T |\sin(\phi)| \Omega_t}{z_{opt} v_m} + \frac{2\pi f}{z_{opt} v_m}$$

$$A_{SF} = \Omega_v f \left( \frac{1}{z_{min}} - \frac{1}{z_{max}} \right)$$

# Plenoptic Sampling of a Slanted Plane

Comparison between the essential bandwidth and the standard filter:

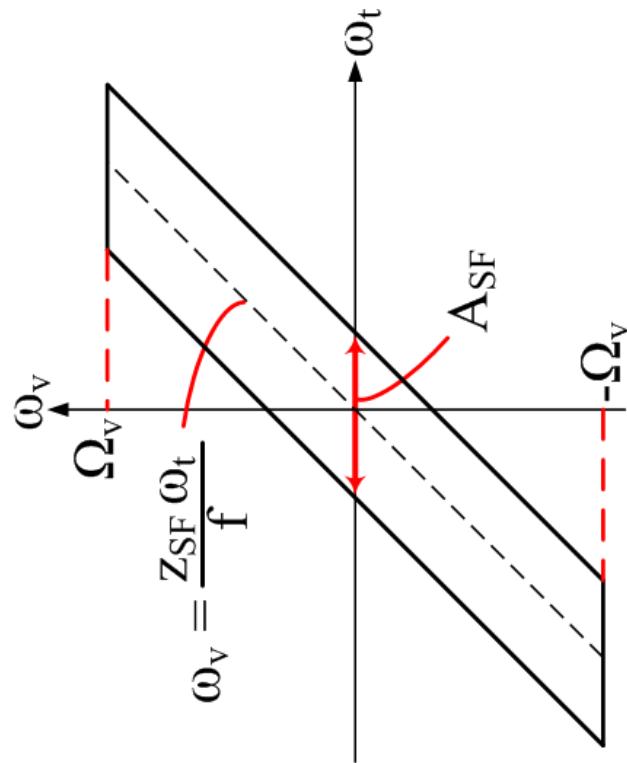


(a) Essential Bandwidth

At Critical Sampling:

$$\Delta t_c = \frac{2\pi}{A},$$

$$= \frac{2\pi z_{opt} v_m}{v_m \Omega_t T |\sin(\phi)| + 2\pi f}$$



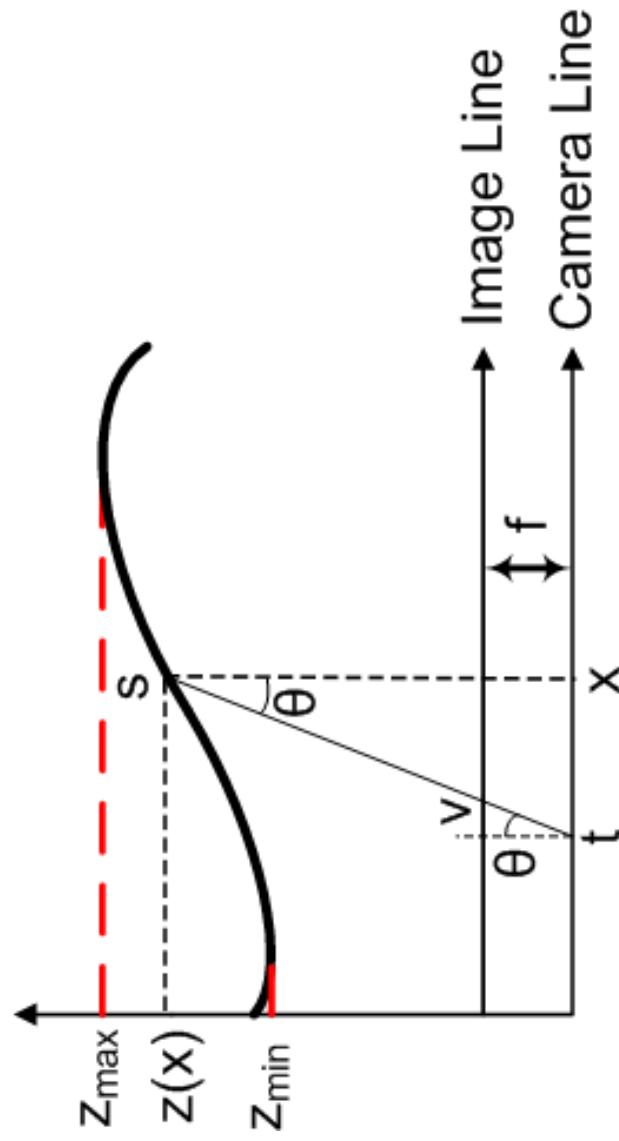
(b) Standard Filter [2]

$$\Delta t_c = \frac{2\pi}{A_{SF}},$$

$$= \frac{2\pi z_{max} z_{min}}{f \Omega_v (z_{max} - z_{min})}$$

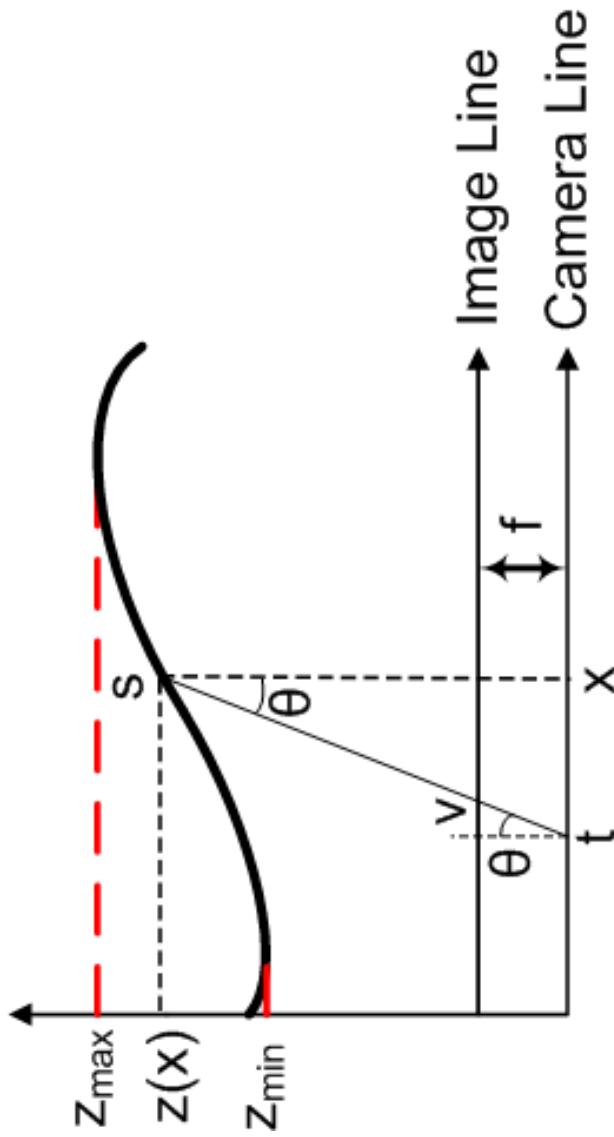
# Sampling More Realistic Scenes

Smoothly varying scene surface with bandlimited texture:



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Smoothly varying scene surface with bandlimited texture:



Our approach:

- Approximate the scene surface using a set of  $L$  slanted planes.
- Use the previous theory to determine non-uniformly sampling and reconstructing the scene.

Important Questions:

→ How best to approximate the scene and with how many planes?

→ How should we allocate the samples?

# Evaluating the Surface Approximation

Define the distortion function,  $D$ , as the measure of the distortion in the plenoptic domain due to approximating the surface.

Consists of two Parts:

$$D(N_i) = \underbrace{\gamma_i}_{\text{Geometric Error}} + \underbrace{\alpha_i(N_i)}_{\text{Aliasing Error}}$$

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The error between the actual plenoptic function,  $p(t, v)$  and the plenoptic function generated by the approximate surface,  $\hat{p}(t, v)$ .

However

Need to consider a limited number of samples,  $N_T$ .

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$\alpha_i(N_i)$  is the error caused by undersampling the  $i$ th plane with  $N_i$  samples.

Therefore:

Small number of samples  $\implies$  Aliasing error dictates the surface approximation.

Large number of samples  $\implies$  Geometric error dictates the surface approximation.

# Sample Allocation per Plane

The sample allocation problem is defined in terms minimising the distortion function given  $N_T$  samples:

The problem:

$$\min \left\{ \sum_{i=1}^L D_i(N_i) \right\} \text{ s.t. } N_T = \sum_{i=1}^L N_i,$$

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Solved using a Lagrange multiple  $\lambda$ , thus the cost function:

$$\sum_{i=1}^L (D_i(N_i) + \lambda N_i)$$

where

$$\lambda = -\frac{d}{dN_1} \{D_1(N_1)\} = -\frac{d}{dN_2} \{D_2(N_2)\} = \dots = -\frac{d}{dN_L} \{D_L(N_L)\}$$

# Sample Allocation per Plane

Approximating  $\alpha_i(N_i)$  as twice the amount of energy outside the reconstruction filter, the cost function becomes:

$$\sum_{i=1}^L \left( \gamma_i + \frac{16K_i\Delta v}{A_i\pi} + \frac{4A_iK_i\Delta v}{\pi} + \frac{8\pi K_i}{\Delta v} \tan^{-1} \left( \frac{r_i W_i}{\pi N_i} \right) + \lambda N_i \right).$$

where

- $A_i$  is the width of the essential bandwidth of the  $i$ th plane (in  $\omega_t$ ).
- $W_i$  is the width in  $t$  that the  $i$ th plane is visible.
- $K_i$  and  $r_i$  are coefficients of the approximation for the  $i$  plane.

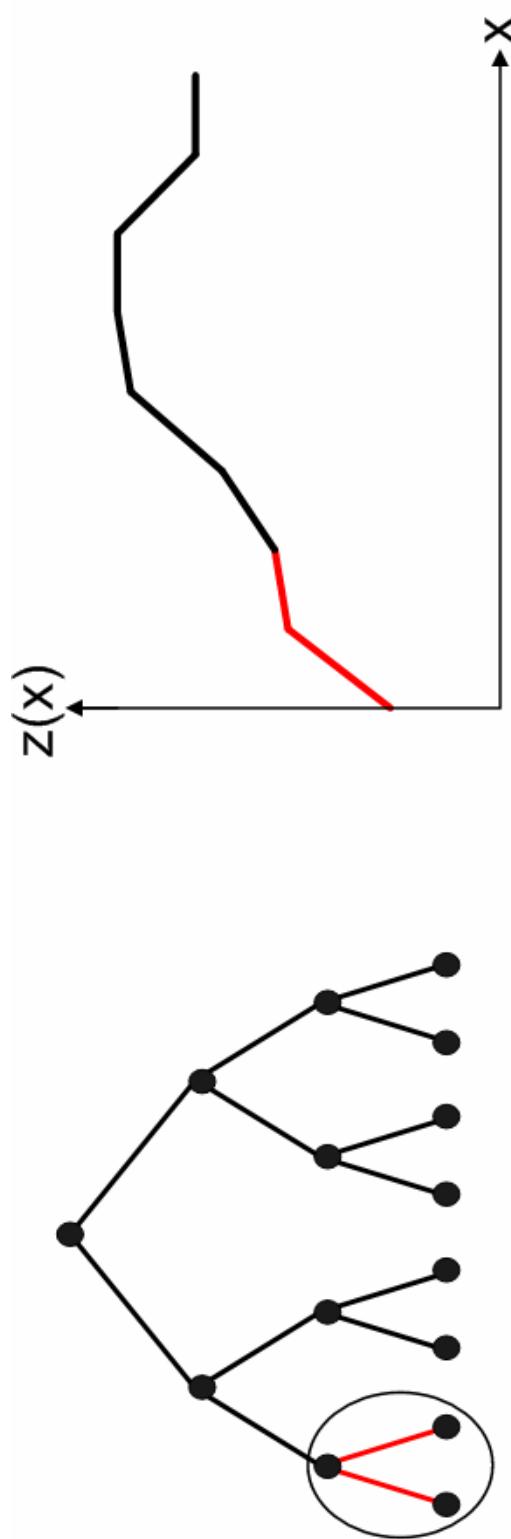
As a result,  $\lambda$  is the solution to

$$N_T - 1 = \sum_{i=1}^L \sqrt{\frac{8K_i W_i r_i}{\Delta v \lambda} - \left( \frac{r_i W_i}{\pi} \right)^2}$$

# Optimising the Surface Approximation

Determining the optimum surface approximation:

- Binary-tree approach  $\implies$  Start with an initial ‘fine-grain’ approximation.
- Initially split the surface into  $2^k$  equal pieces resulting in  $L$  planes.
- Determine the initial  $\lambda$  and sample allocation between the  $L$  planes by solving the minimisation problem.
- Merge the leaves of the tree to reduce the overall distortion.



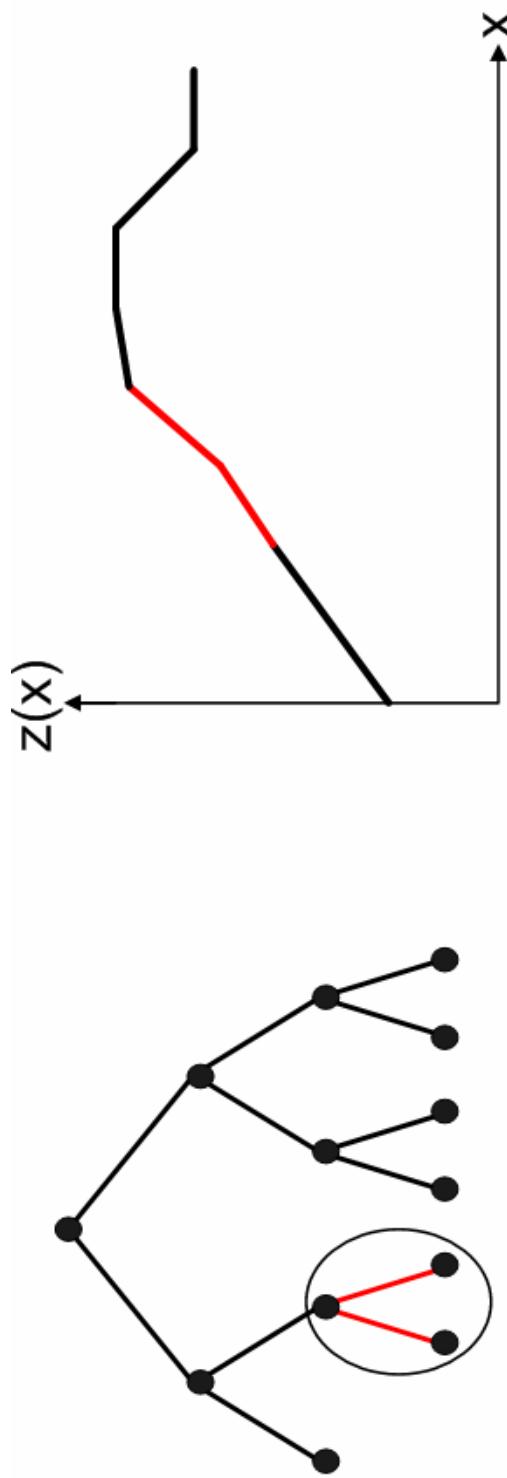
(a) Quad-tree

(b) Surface Approximation

# Optimising the Surface Approximation

Determining the optimum surface approximation:

- Binary-tree approach  $\implies$  Start with an initial ‘fine-grain’ approximation.
- Initially split the surface into  $2^k$  equal pieces resulting in  $L$  planes.
- Determine the initial  $\lambda$  and sample allocation between the  $L$  planes by solving the minimisation problem.
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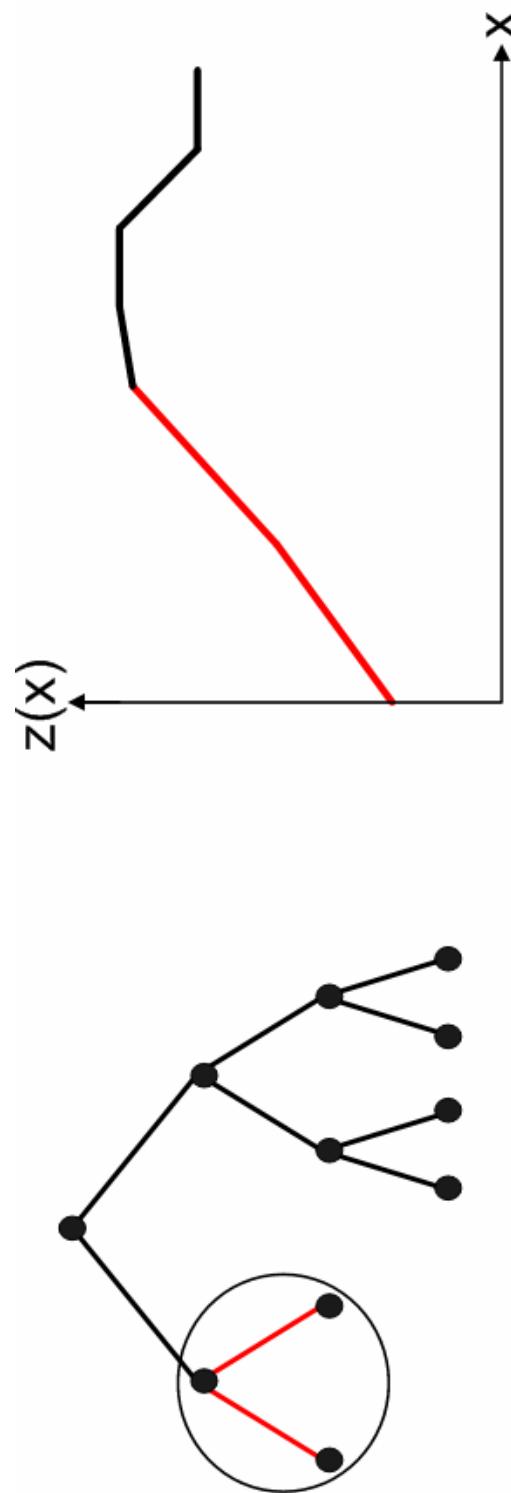
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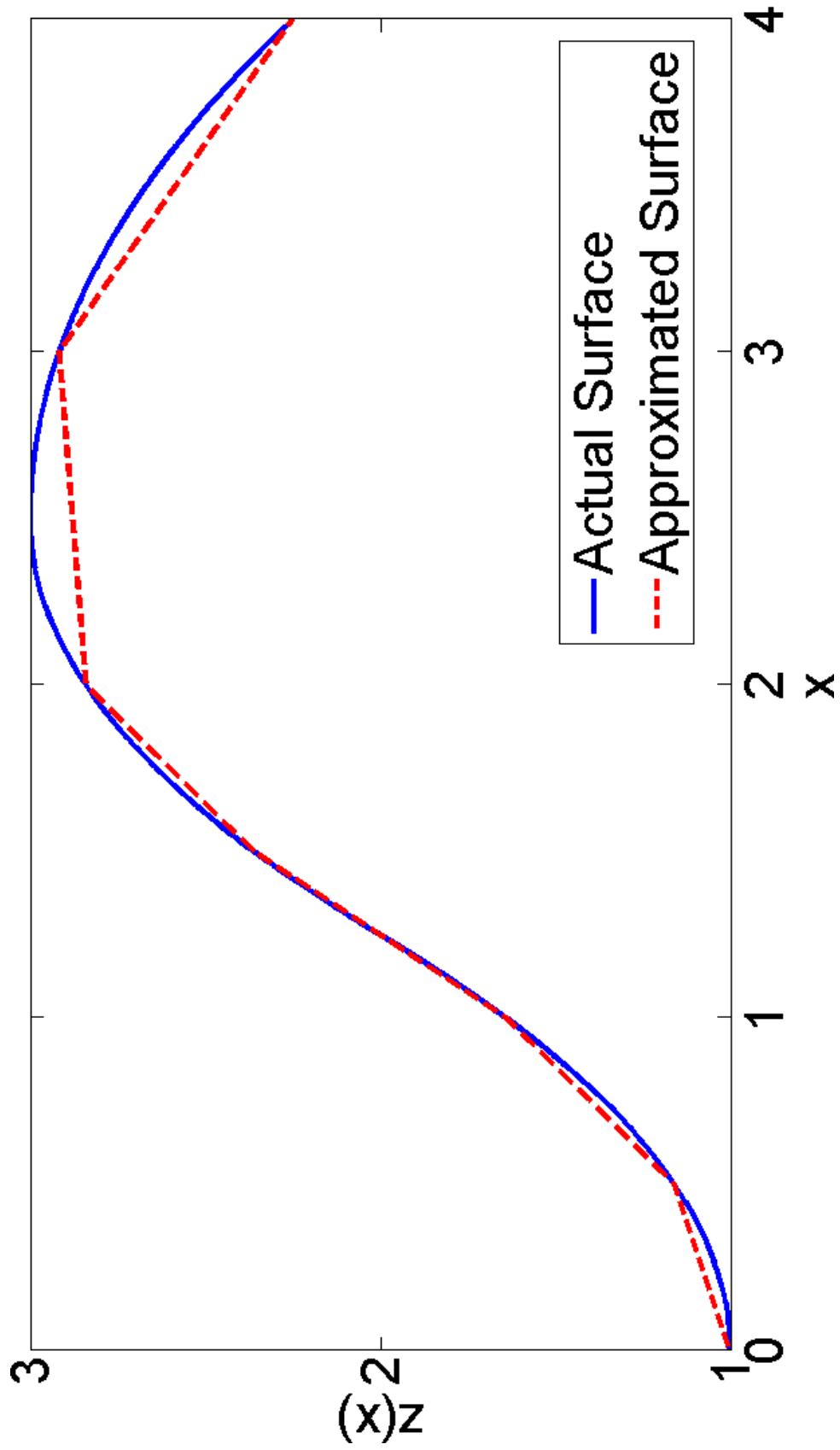


(a) Quad-tree

(b) Surface Approximation

# Surface Approximation Simulations

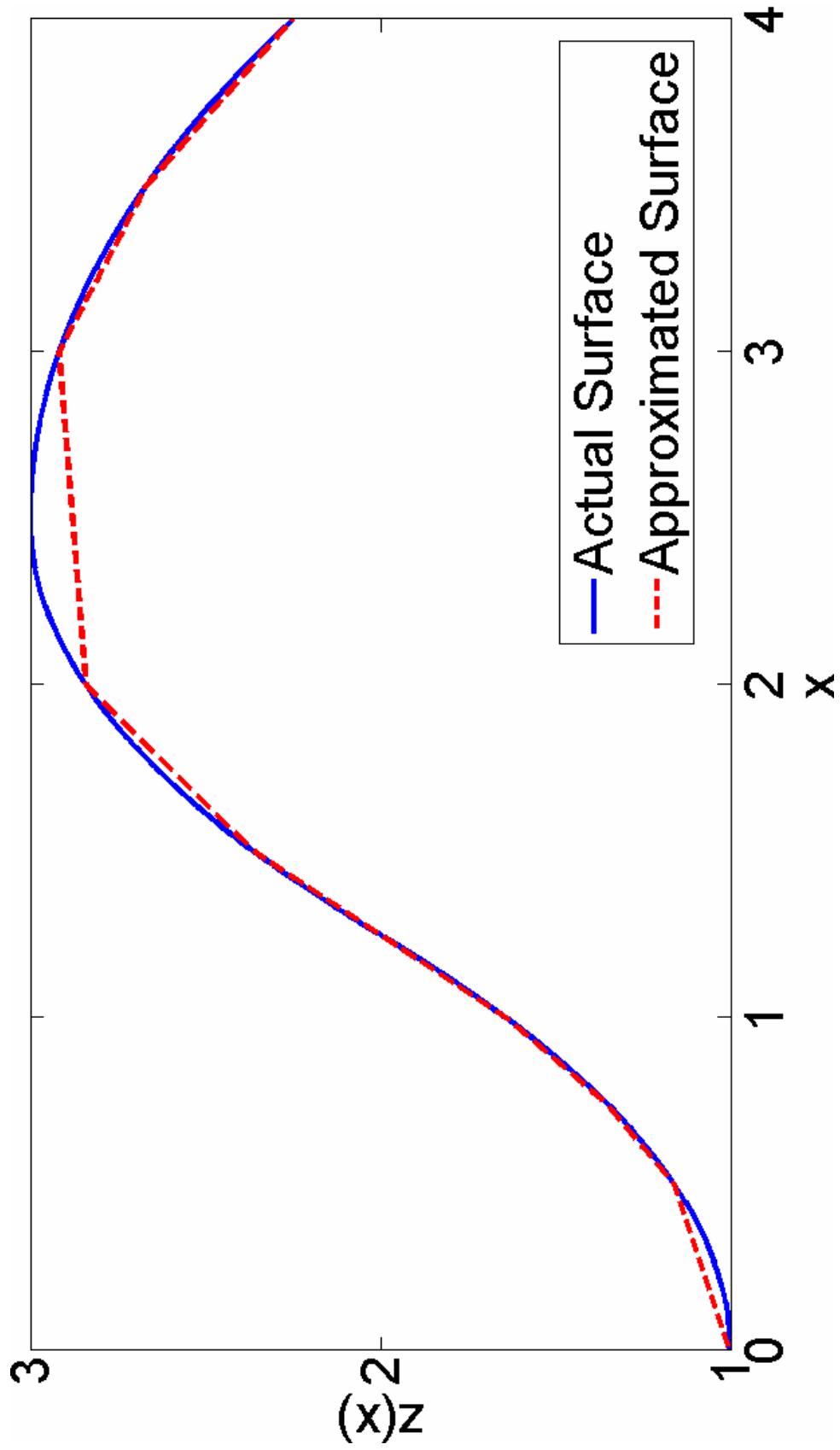
Approximation of the Piecewise Quadratic Surface using  $N_T = 20$ :



Initial Number of Planes = 16, Final Number of Planes = 6

# Surface Approximation Simulations

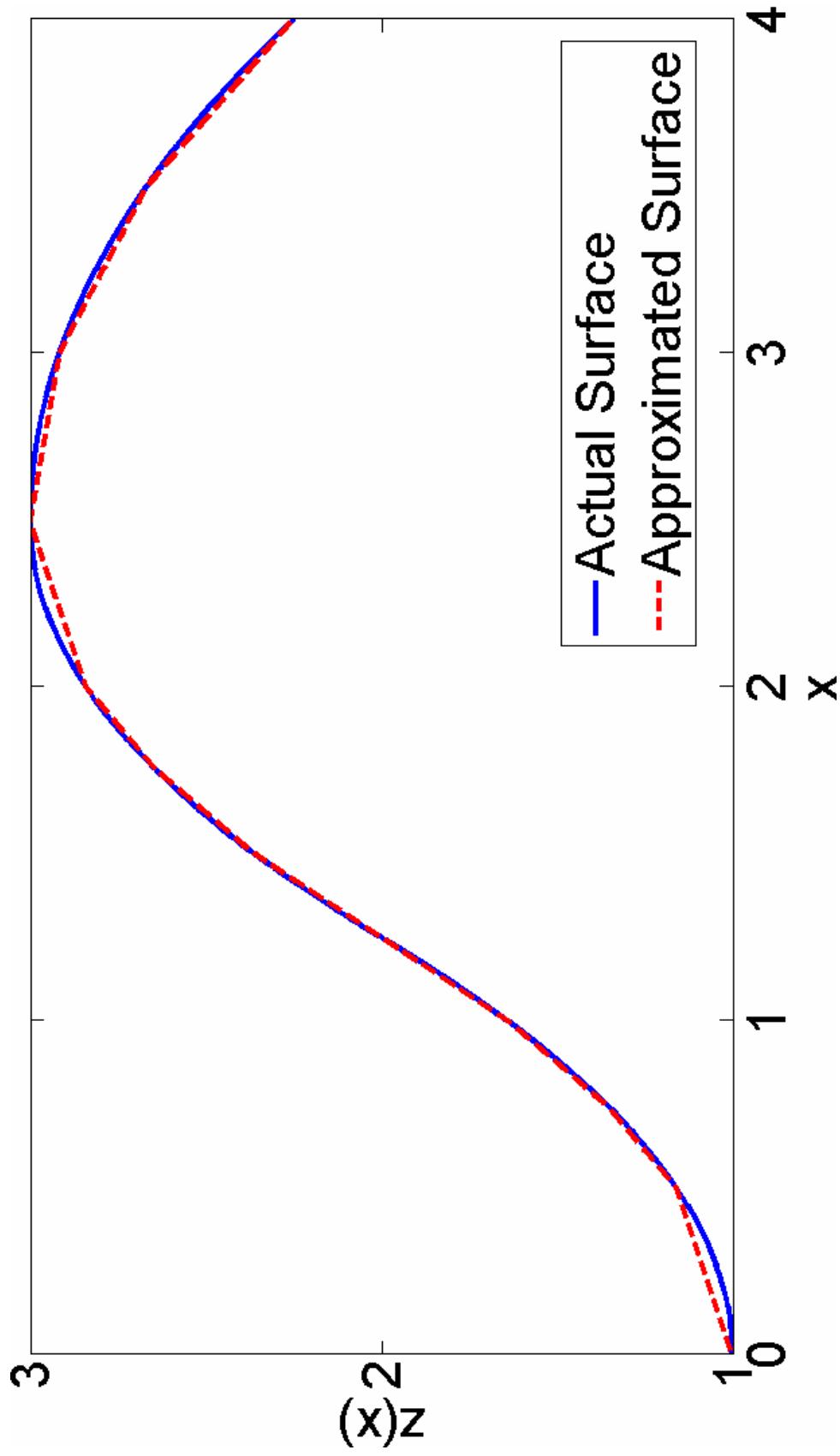
Approximation of the Piecewise Quadratic Surface using  $N_T = 150$ :



Initial Number of Planes = 16, Final Number of Planes = 8

# Surface Approximation Simulations

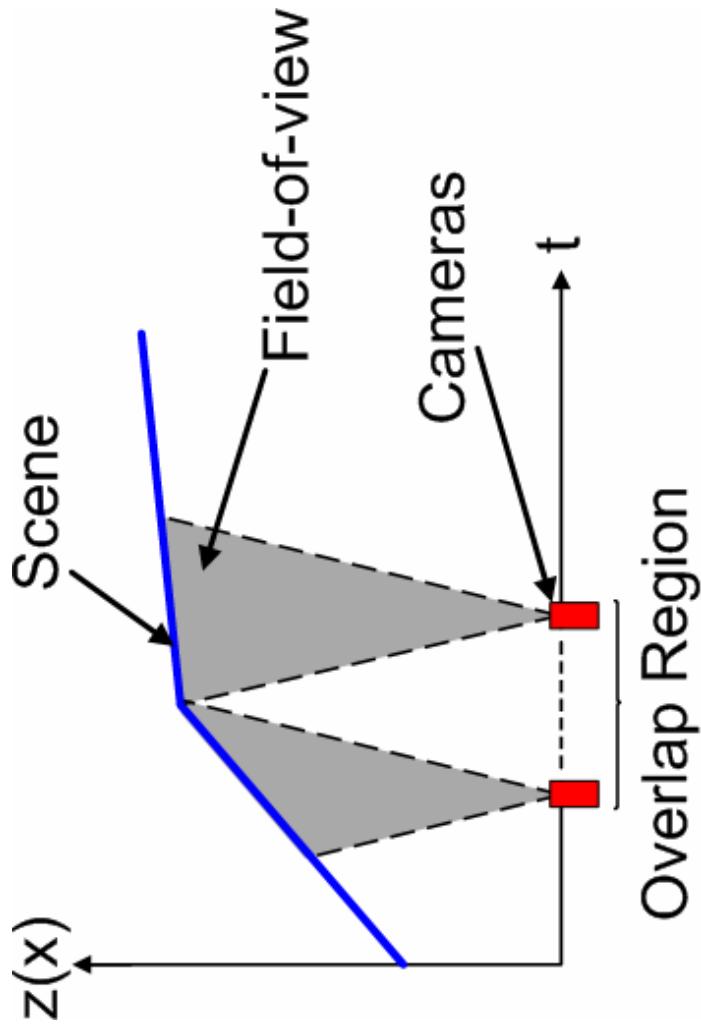
Approximation of the Piecewise Quadratic Surface using  $N_T = 300$ :



Initial Number of Planes = 16, Final Number of Planes = 10

# Sample Allocation to Sample Positions

Remember that multiple planes are visible in a single image

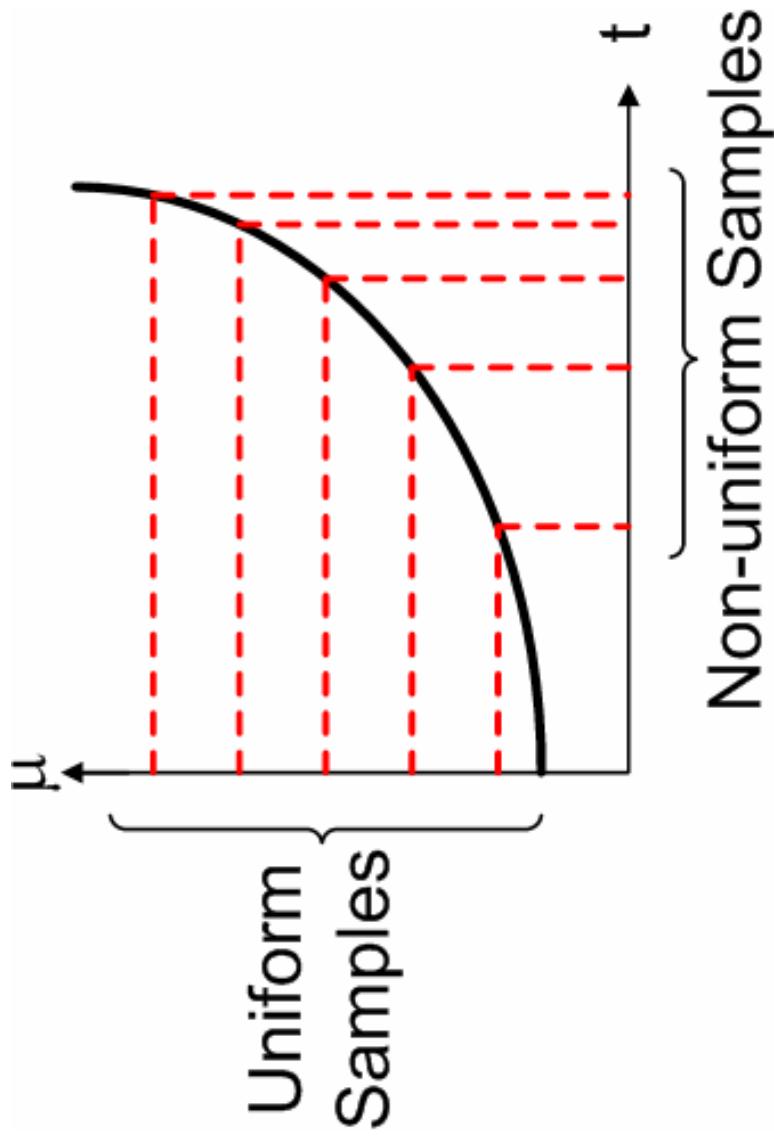


⇒ Choose the highest sample rate in overlap region.

End result ⇒ A piecewise constant sample rate profile in  $t$ .

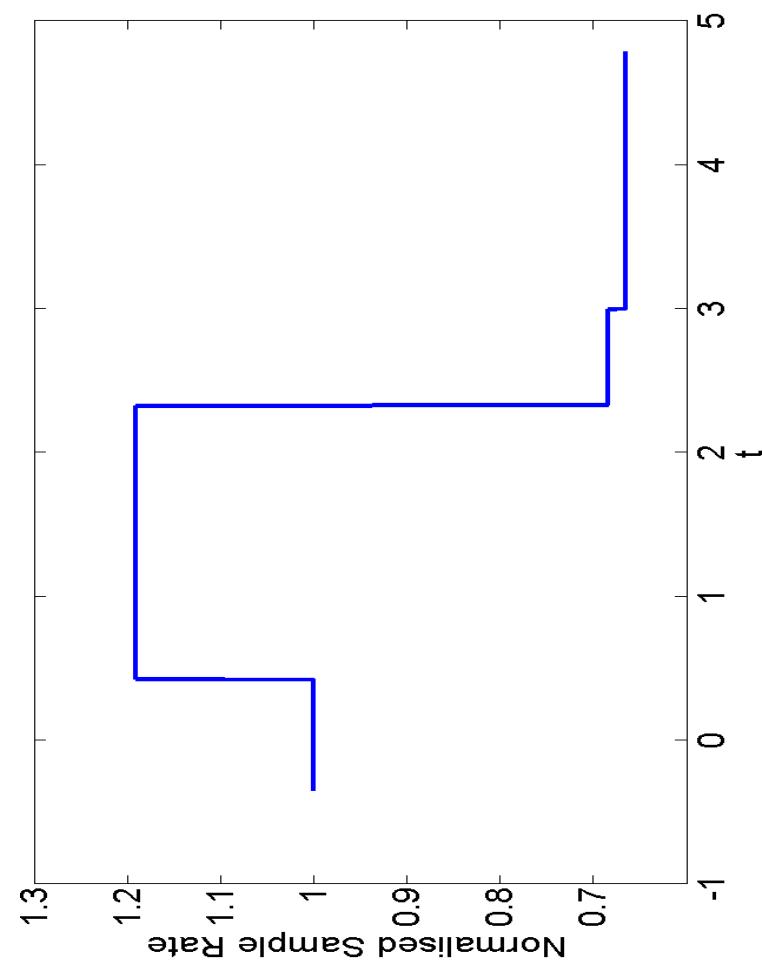
# Sample Allocation to Sample Positions

Integrate sample rate profile to give a warping function  $\Rightarrow$  allowing coherent non-uniform samples

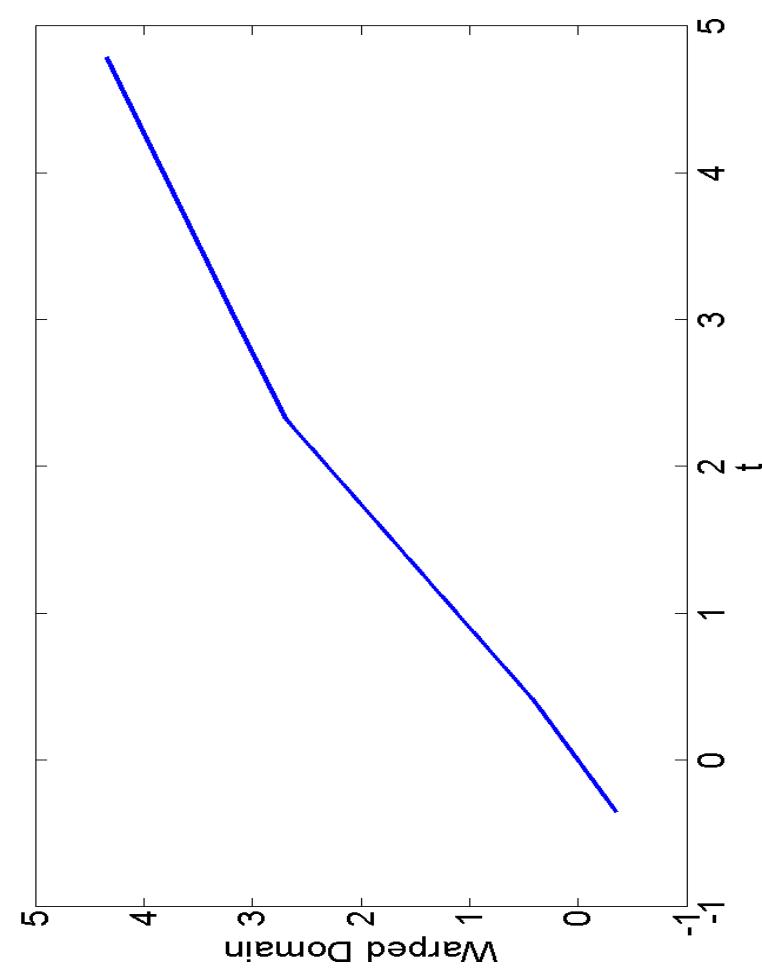


# Sample Allocation to Sample Positions Example

An example when  $N_T = 150$ ,



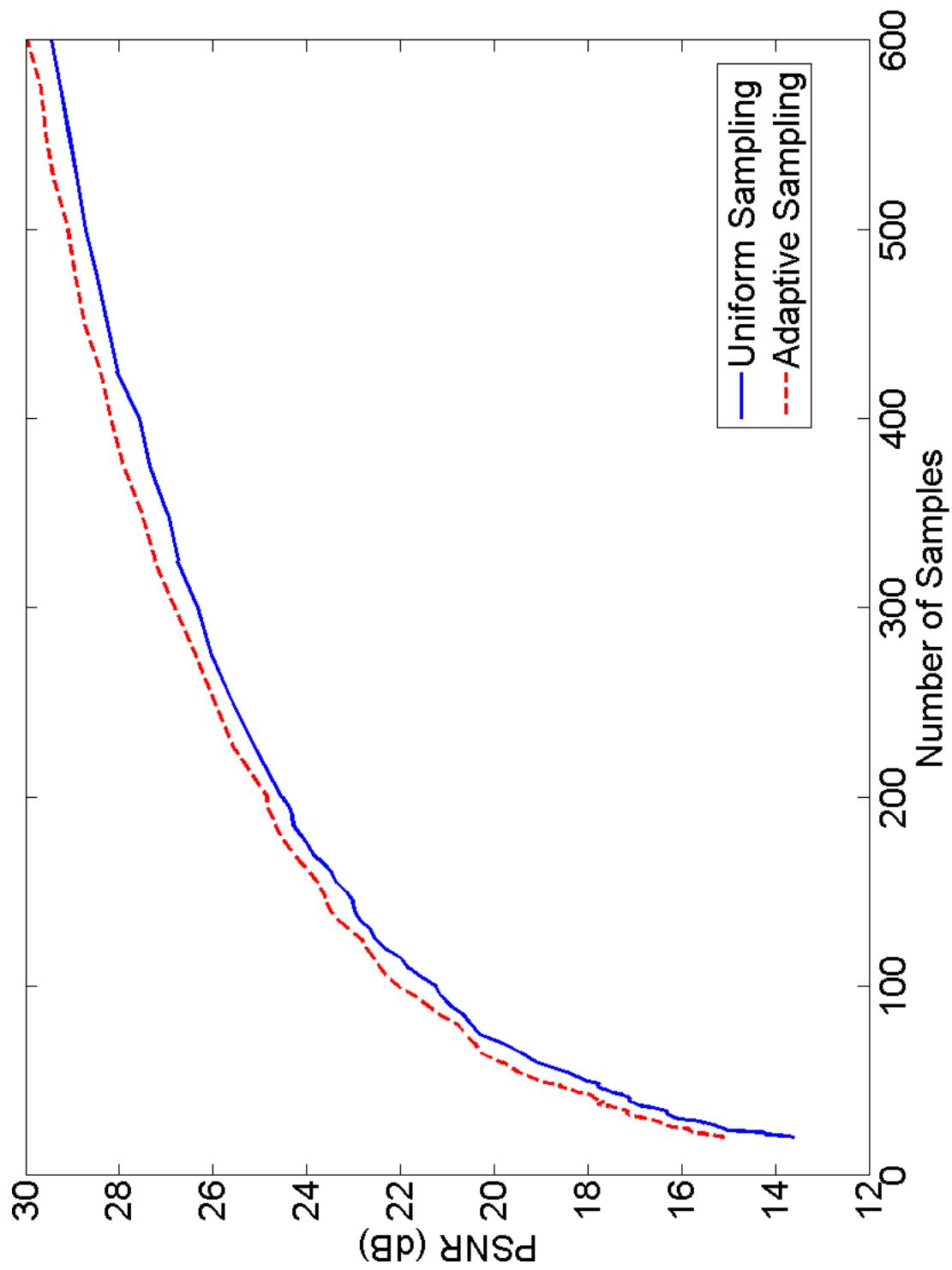
(a) Sample Rate Profile



(b) Warping Function

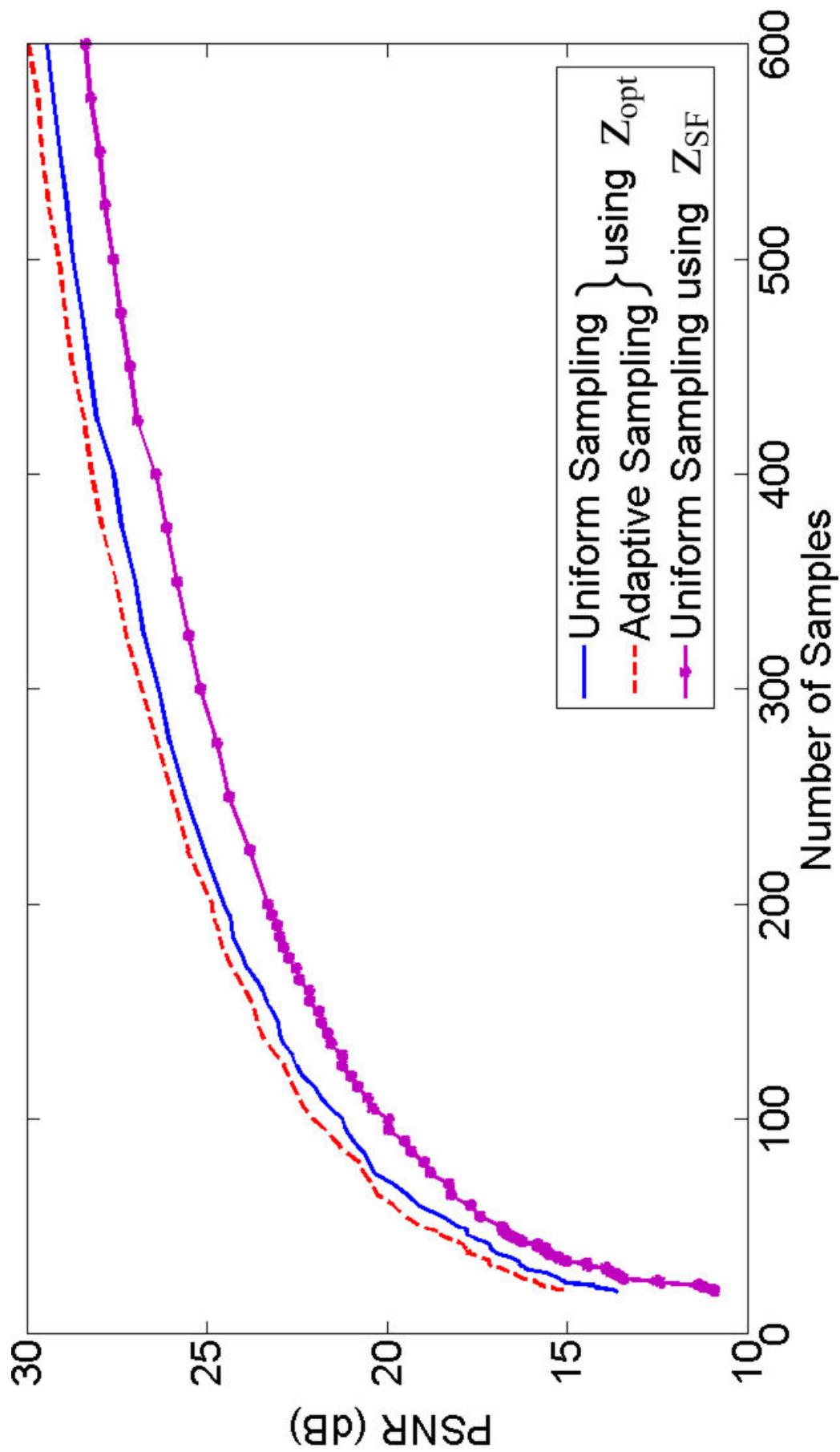
# Simulations Results

Comparison between uniform and adaptive reconstruction for the piecewise quadratic surface.



# Comparison with Standard Filter

Comparison between the reconstruction of the piecewise quadratic surface when using the standard filter and the new filter.

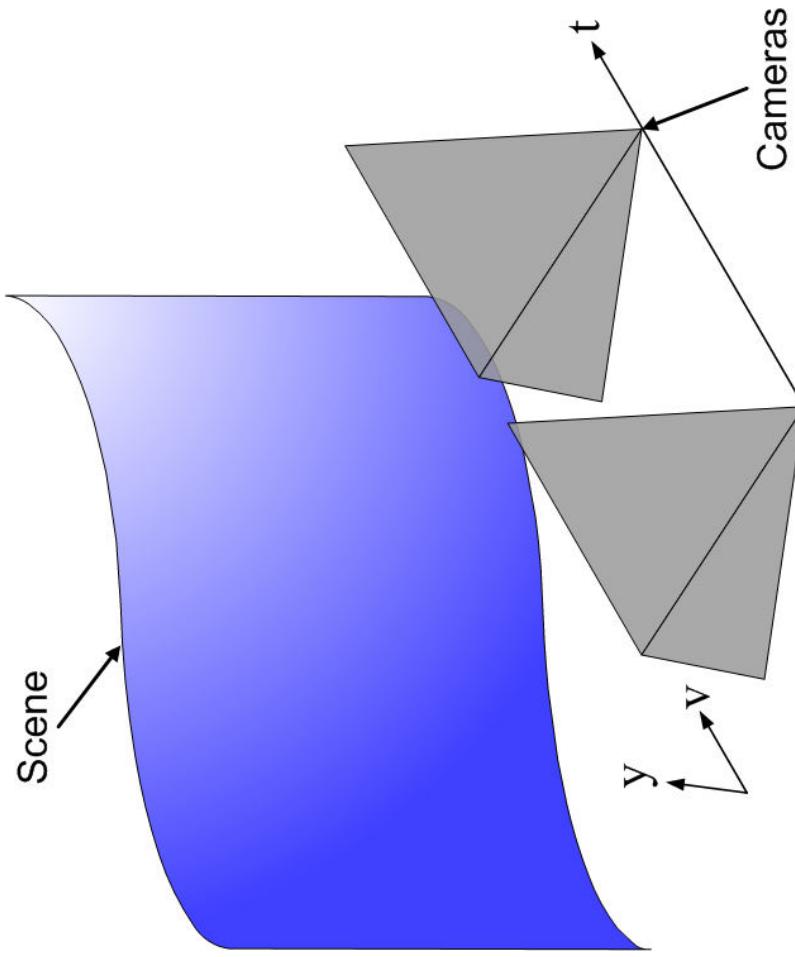


# Conclusions

- We present a method for adaptively sampling a scene with a smoothly varying surface by approximate the surface with a set of slanted planes.
- For a given approximation the samples are adaptive allocated to minimise the distortion in the plenoptic function, using a Lagrange multiplier.
- The surface approximation is optimised in a binary-tree framework, going from fine to coarse, depending on the number of samples.
- The plenoptic sampling theory for a single slanted plane is used to determine a piecewise constant sample rate  $\Rightarrow$  Non-uniform samples.
- Non-uniform sampling scheme outperforms normal uniform sampling.

# Future Work

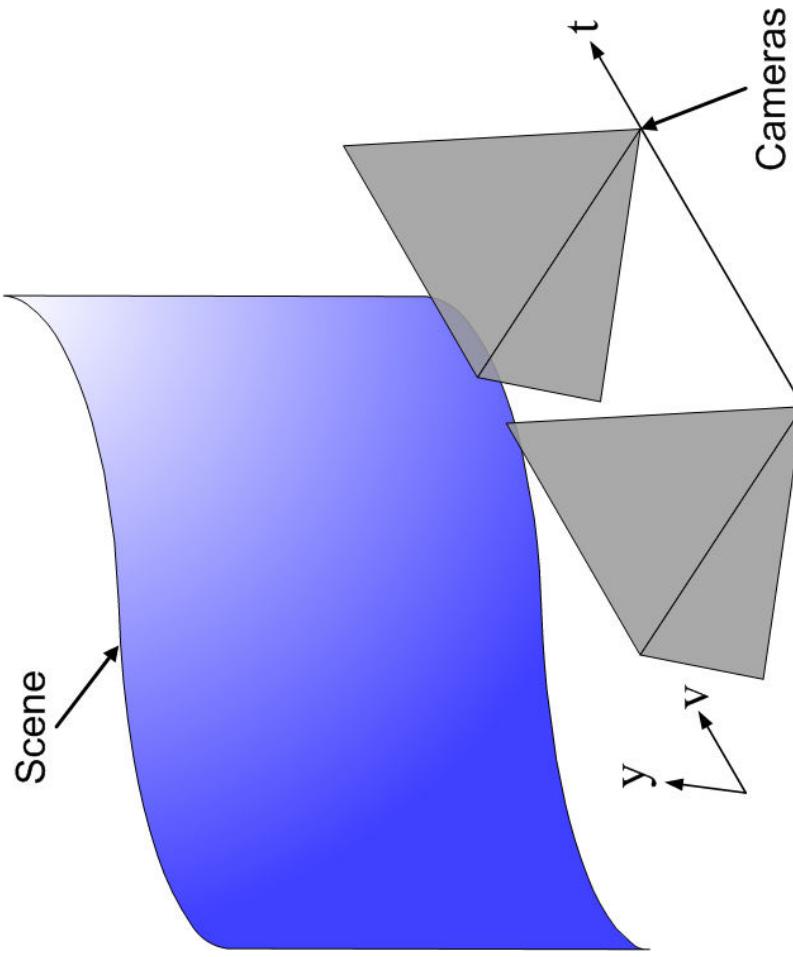
Extending the adaptive algorithm to higher dimensions:



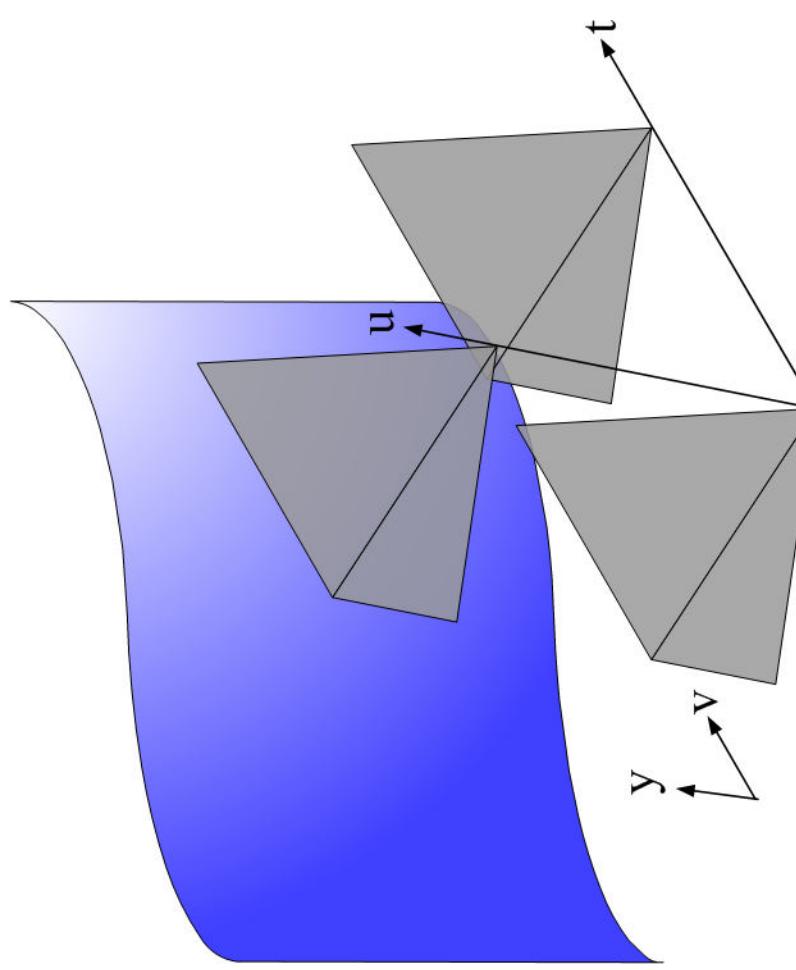
(a) 3D Lightfield

# Future Work

Extending the adaptive algorithm to higher dimensions:



(a) 3D Lightfield



(b) 4D Lightfield

# References

1. E.H. Adelson and J.R. Bergen. The plenoptic function and the elements of early vision. In *Computational Models of Visual Processing*, pages 3-20. MIT Press, Cambridge, MA, 1991.
2. J.X. Chai, S.C. Chan, H.Y. Shum, and X. Tong. Plenoptic sampling. In *Computer graphics (SIGGRAPH'00)*, pages 307-318, 2000.
3. M.N. Do, D Marchand-Maillet, and M. Vetterli. On the bandwidth of the plenoptic function. *IEEE Transactions on Image Processing*, 2009. Preprint.
4. C. Gilliam, P.L. Dragotti, and M. Brookes. A closed-form expression for the bandwidth of the plenoptic function under finite field of view constraints. *IEEE ICIP 2010*, pages 3965-3968.
5. B.P Lathi. *Modern Digital and Analog Communication Systems*. Oxford University Press, 1998.