A Resource Allocation Strategy for Distributed MIMO Multi-Hop Communication Systems

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***Abstract*—An extended form of multi-hop communication systems is introduced which allows the application of multiple-input-multiple-output (MIMO) capacity enhancement techniques over spatially separated relaying mobile terminals to drastically increase end-to-end capacity. An explicit resource allocation strategy is deduced in terms of fractional bandwidth and power allocations to each relaying hop over ergodic Rayleigh flat fading channels employing orthogonal frequency-division multiple-access (FDMA)-based relaying.**

***Index Terms*—Distributed information systems, MIMO systems.**

I. Introduction

A MESH NETWORK topology allows a source mobile terminal (s-MT) to communicate with a target mobile terminal (t-MT) via a number of relaying mobile terminals (r-MTs) [1]. The end-to-end connection is accomplished through a multi-hop link, where the data routing path usually depends on channel conditions between the hops (and other factors) [1]. To avoid saturating the receiver power amplifier, a mobile terminal (MT) cannot relay information at the same time and in the same frequency band. Therefore, either frequency-division multiple access (FDMA) or time-division multiple access (TDMA) has to be utilized. From an information theoretical point of view, it can be shown that both access methodologies yield the same capacities [2]; however, analysis herein concentrates on FDMA-based relaying only.

FDMA-based regenerative relaying requires that the bandwidth W is divided between the relaying nodes. It is further assumed that the allocation process of the bandwidth is such that r-MTs are not interfering with each other at any time. The relaying process can therefore be characterized as orthogonal regenerative FDMA-based relaying.

If serial relaying is deployed then the end-to-end data throughput depends on the capacity of each relaying hop, which is mainly dictated by the predominating channel conditions. An optimum relaying system should allocate fractional bandwidth and transmission power to each hop such as to maximize the end-to-end capacity. Clearly, a link with very bad channel conditions will require additional resources, therefore diminishing the total throughput.

This letter introduces the concept of distributed MIMO multi-hop systems, a natural extension to the prior developed concept of virtual antenna arrays (VAAs) [3], [4]. VAAs allow a tractable approach to exploiting MIMO channels since the t-MT is provided with redundant information by adjacent r-MTs. The concept is equally applicable in the transmission mode, where adjacent r-MTs act as a transmit array. This allows the deployment of space-time capacity enhancement techniques to SISO MTs, e.g., space-time block codes (STBCs) or space-time trellis codes (STTCs). The deployment of a generalized form of VAA to mesh networks, as depicted in Fig. 1, has been proposed in [5].

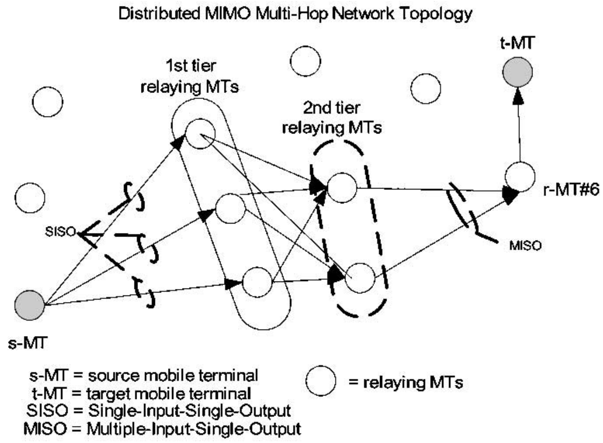

Fig. 1. Topology of a distributed MIMO multi-hop communication network.

The operation of distributed MIMO multi-hop systems is introduced in Section II. A fractional capacity allocation strategy is formulated and explicitly solved in Section III, and its performance is analyzed in Section IV. Finally, conclusions are drawn in Section V.

II. Distributed MIMO Multi-Hop Systems

Referring to Fig. 1, an s-MT desires to communicate with a t-MT in a network with a plurality of available r-MTs. Thus, the s-MT broadcasts the information intended for the t-MT from an antenna array with at least one element. This information is received by at least two r-MTs forming the first tier of r-MTs, each of which possesses an antenna array with at least one element. Knowledge of their mutual existence allows the first tier MTs to space-time re-encode the data stream; each r-MT transmits only a spatial fraction of the space-time code word such that the total output from the first tier r-MTs accomplishes a MIMO transmission. The second tier of r-MTs receives the data stream, de-codes it, re-encodes, and re-transmits it to the third tier r-MTs in the same manner as described above. This process is continued until the t-MT is reached.

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The encoding happens at spatially distributed r-MTs; the communication system is therefore referred to as a Distributed MIMO Multi-Hop Communication System [5]. For such a system to reach Shannon capacity, the codebook entries have to be of infinite size and known to all r-MTs of the same and consecutive tiers. Although such an encoding strategy would introduce infinite delay, the Shannon capacity forms an upper bound and is therefore a useful measure of the performance of the distributed MIMO multi-hop system.

III. Capacity of Distributed MIMO Multi-Hop Systems

The Shannon capacity C in bits per second of an ergodic zero-mean circular-symmetric wireless channel with t uncorrelated transmit antennas and r receive antenna is elegantly derived in [6]. After some elementary manipulations, it can be expressed as

$$C = W \cdot \mathrm{E}_{\lambda}\left\{ m \log_2\left(1 + \lambda \frac{\gamma S/t}{WN_0}\right)\right\} \tag{1}$$

with the probability density function (pdf) of an unordered eigenvalue λ [6]

$$\mathrm{pdf}_{\lambda}(\lambda) = \frac{1}{m}\sum_{k=0}^{m-1} \frac{k!\left[L_k^{n-m}(\lambda)\right]^2 \lambda^{n-m} e^{-\lambda}}{(k+n-m)!}. \tag{2}$$

Here, W is the available bandwidth, S is the transmitted signal power (normalized by the number of transmit antennas t), γ is the pathloss, and $N = WN_0$ is the received noise power with N_0 being the received noise spectral density. Furthermore, $\mathrm{E}_{\lambda}\{\cdot\}$ represents the statistical expectation with respect to λ, and $m = \min\{r,t\}, n = \max\{r,t\}$. Finally, $L_k^{n-m}(\lambda)$ is the associated Laguerre polynomial of order k, the Rodrigues representation of which is $L_k^{n-m}(\lambda) = \sum_{l=0}^{k}(-1)^l((k+n-m)!)/((k-l)!(n-m+l)!l!)\lambda^l$ [7, §8.970.1].

The logarithmic expression of the Shannon capacity in (1) does not allow optimization problems related to capacity to be solved in a closed form. To this end, the approximation

$$\log_2(1+x) \approx \sqrt{x} \tag{3}$$

will be useful for subsequent analysis.

In a multi-hop distributed communication scenario with K hops, fractional bandwidth $\alpha_{k=(1,\ldots,K)}$ and power $\beta_{k=(1,\ldots,K)}$ are allocated to each hop, respectively. The total bandwidth and power utilized to deliver the information is normalized to W and S, respectively. For an FDMA-based system, the constraints can be expressed as $\sum_{k=1}^{K}\alpha_k = 1$ and $\sum_{k=1}^{K}\beta_k = 1$. The capacity C_k of the kth-hop is then

$$C_k = \alpha_k \cdot W \cdot \mathrm{E}_{\lambda_k}\left\{\log_2\left(1 + \lambda_k \frac{\beta_k}{\alpha_k}\frac{\gamma_k S/t_k}{WN_0}\right)\right\} \tag{4}$$

where t_k is the number of transmit antennas and γ_k the pathloss of the kth-hop. It is assumed that the r-MTs belonging to the same relaying tier are spatially sufficiently close as to justify a common link attenuation γ_k; however, sufficiently far apart as to justify uncorrelated fading. Note further that shadowing has not been considered here. Because regenerative relaying is deployed, the end-to-end capacity C is dictated by the smallest capacity C_k [8], i.e., $C = \min(C_1,\ldots,C_K)$. The aim is to find the fractional bandwidth α_k and power β_k allocations for given channel conditions γ_k, such as to maximize the minimum capacity C. With the given constraints, increasing any one capacity inevitably reduces the others. The minimum is therefore maximized if all capacities are equated and then maximized. Equating (4) for $k = 1,\ldots,K$ enables the set of equations to be solved for any α_k, where

$$\alpha_k = \left[\sum_{i=1}^{K} \frac{\mathrm{E}_{\lambda_k}\left\{\log_2\left(1+\lambda_k\frac{\beta_k}{\alpha_k}\frac{\gamma_k S/t_k}{WN_0}\right)\right\}}{\mathrm{E}_{\lambda_i}\left\{\log_2\left(1+\lambda_i\frac{\beta_i}{\alpha_i}\frac{\gamma_i S/t_i}{WN_0}\right)\right\}}\right]^{-1}. \tag{5}$$

Furthermore, approximating (4) with (3) yields $C_k \propto W\sqrt{\alpha_k}\sqrt{\beta_k}\sqrt{(\gamma_k S/t_k)/(WN_0)}\mathrm{E}_{\lambda_k}\{\sqrt{\lambda_k}\}$. The optimization problem is therefore approximately symmetric with respect to α_k and β_k; therefore, $\alpha_k \approx \beta_k$, which leads to $\sum_{k=1}^{K}(\beta_k)/(\alpha_k) \approx K$. With this in mind, inserting (5) into (4), yields for the maximum end-to-end capacity

$$C \approx W \cdot \left[\mathrm{E}_{\lambda_1}^{-1}\left\{\log_2\left(1+\lambda_1\left[K - \sum_{i=2}^{K}\frac{\beta_i}{\alpha_i}\right]\frac{\gamma_1 S/t_1}{WN_0}\right)\right\} + \sum_{i=2}^{K}\mathrm{E}_{\lambda_i}^{-1}\left\{\log_2\left(1+\lambda_i\frac{\beta_i}{\alpha_i}\frac{\gamma_i S/t_i}{WN_0}\right)\right\}\right]^{-1}. \tag{6}$$

The logarithmic capacity expression is now approximated by (3); further, the maximum is obtained by equating the first derivative along (β_i/α_i) for $i = (2,\ldots,K)$ to zero. Solving the $K-1$ equations for any (β_k/α_k), simply yields

$$\frac{\beta_k}{\alpha_k} \approx K \cdot \sum_{i=1}^{K} \frac{\sqrt[3]{\frac{\gamma_k}{t_k}\cdot \mathrm{E}_{\lambda_k}^2\left\{\sqrt{\lambda_k}\right\}}}{\sqrt[3]{\frac{\gamma_i}{t_i}\cdot \mathrm{E}_{\lambda_i}^2\left\{\sqrt{\lambda_i}\right\}}}. \tag{7}$$

Finally, the fractional frame bandwidth allocations α_k are obtained by substituting (7) into (5). The fractional power allocations are obtained by inserting the values of α_k into (7) and solving for β_k.

If each terminal involved possesses only one antenna element, then the communication from the kth-tier toward the $(k+1)$th-tier of r-MTs forms t_{k+1} channels with t_k transmit antennas and one receive antenna. For these cases, $\mathrm{E}_{\lambda_k}\{\sqrt{\lambda_k}\}$ has a closed solution that leads to an explicit expression for (7). From [6, eq. (10)] and [7, Sect. 3.381.4], one obtains $\mathrm{E}_{\lambda_k}\{\sqrt{\lambda_k}\} = \frac{\Gamma(t_k+1/2)}{\Gamma(t_k)}$ where $\Gamma(\cdot)$ is the complete Gamma function.

IV. Performance of the Algorithm

The performance of a distributed MIMO multi-hop network and the precision of the derived capacity allocation strategy are assessed here. In particular, a 2-hop and 3-hop network with one to four r-MTs per relaying tier is investigated. The first hop is therefore comprised of one to four SISO channels ($t_1 = 1$), whereas the second and third hops realize MISO channels ($t_{2,3} = 1,\ldots,4$).

First, the the precision of the suggested approximation of the Shannon capacity is examined in Fig. 2, which depicts the error in percent between the exact and approximated capacities for various communication configurations. Clearly, the error is very low and lies within 10% for practical regions of operation.

As an example, Fig. 3 plots the numerically obtained optimum end-to-end capacity allocation and the derived

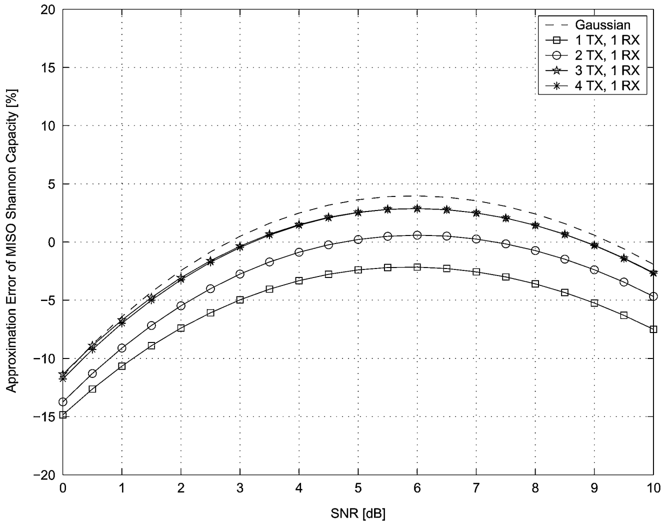

Fig. 2. Error in percent between the exact and approximated capacities for various MISO communication configurations.

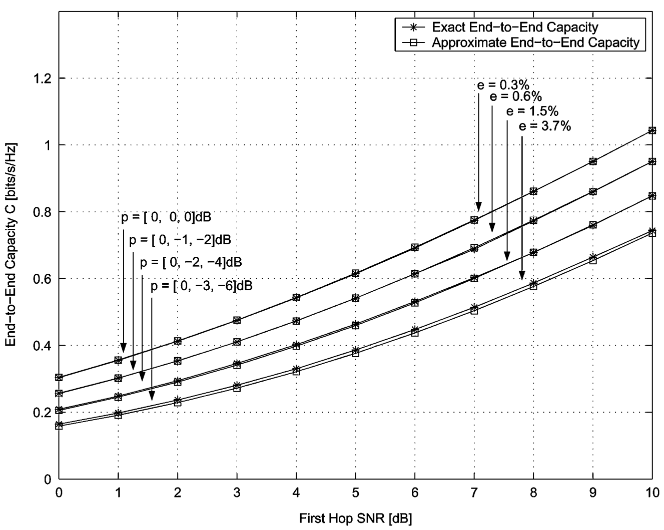

Fig. 3. Optimum versus near-optimum end-to-end capacity for the 3-hop case with three r-MTs per relaying tier.

approximated allocation for the 3-hop case with three r-MTs per relaying tier. The depicted SNR equates to $\gamma_1 S/N$ of the first hop. Performance is parameterised by p, where $p \equiv [(\gamma_1/\gamma_1),(\gamma_2/\gamma_1),(\gamma_3/\gamma_1)]$ for the 3-hop case.

The figure shows that the capacity is very well approximated by the capacity allocation scheme developed over $\mathrm{SNR} = (0, 10)$ dB and for the given link attenuations. The maximum error, e, between the exact and approximate expression is $e = (0.3, 0.6, 1.5, 3.7)\%$ for $p = ([0, 0, 0], [0, -1, -2], [0, -2, -4], [0, -3, -6])$ dB respectively. For an increasing attenuation in the second hop, the maximum end-to-end capacity decreases. The maximum error between the exact and approximate end-to-end capacity of the remaining distributed MIMO multi-hop configurations is summarized in Table I. The theory developed here has demonstrated a high-precision capacity allocation strategy without the need for extensive numerical optimization.

TABLE I
MAXIMUM PERCENTAGE ERROR BETWEEN THE EXACT AND APPROXIMATE END-TO-END CAPACITY ALLOCATION OF VARIOUS DISTRIBUTED MIMO MULTI-HOP CONFIGURATIONS PARAMETERIZED BY p

2-Hop	[0,0]dB	[0,-3]dB	[0,-6]dB	[0,-9]dB
1 r-MT per Hop	0.0%	0.8%	1.9%	3.1%
2 r-MT per Hop	0.1%	0.8%	1.9%	3.1%
3 r-MT per Hop	0.1%	0.9%	2.0%	3.2%
4 r-MT per Hop	0.2%	0.9%	2.0%	3.3%
3-Hop	**[0,0,0]dB**	**[0,-1,-2]dB**	**[0,-2,-4]dB**	**[0,-3,-6]dB**
1 r-MT per Hop	0.6%	0.5%	1.9%	3.9%
2 r-MT per Hop	0.7%	0.8%	1.7%	3.7%
3 r-MT per Hop	0.3%	0.3%	1.5%	3.7%
4 r-MT per Hop	0.3%	0.5%	1.7%	3.8%

V. CONCLUSION

The concept of distributed MIMO multi-hop communication systems has been introduced. It was assumed that a source MT delivers data to a target MT via a plurality of relaying hops, each of which consists of distributed but coordinated MTs. The relaying MTs were grouped into tiers, where each tier formed a virtual transmit antenna array to which space-time capacity enhancing techniques are applicable. An explicit capacity allocation strategy for such a topology utilising FDMA-based regenerative orthogonal relaying over ergodic narrowband fading channels under given constraints on bandwidth and transmission power has been presented. A simplification has been proposed that enables the nonlinear optimization problem to be reduced to a linear optimization problem. The error between the exact and approximated resource allocation method was shown to be less than 5% for $\mathrm{SNR} = (0, 10)$ dB with link qualities between relaying hops differing by up to 9 dB. The concept is equally applicable to distributed MIMO multi-hop communication systems where the r-MTs have more than one antenna element or are capable of communicating among each other.

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REFERENCES

[1] C. E. Perkins, *Ad-Hoc Networking*. Reading, MA: Addison-Wesley, 2001.
[2] A. J. Goldsmith, "The capacity of downlink fading channels with variable rate and power," *IEEE Trans. Veh. Technol.*, vol. 46, pp. 569–580, Aug. 1997.
[3] M. Dohler, E. Lefranc, and H. Aghvami, "Virtual antenna arrays for future wireless mobile communication systems," in *IEEE ICT 2002*, Beijing, China, 2002, CD-ROM.
[4] M. Dohler, J. Dominguez, and H. Aghvami, "Capacity of virtual antenna arrays," in *IEEE VTC—Fall 2002*, Vancouver, Canada, 2002, CD-ROM.
[5] M. Dohler, F. Said, A. Ghorashi, and H. Aghvami, "Improvements in or relating to electronic data communication systems," Patent Publ. No. WO 03/003672, June 2001.
[6] E. Telatar, "Capacity of multi-antenna Gaussian channels," *Eur. Trans. Telecommun.*, vol. 10, no. 6, pp. 585–595, Nov./Dec. 1999.
[7] I. S. Gradshteyn and I. M. Ryshik, *Table of Integrals, Series, and Products*, 6th ed. New York: Academia, 2000.
[8] T. Cover and J. A. Thomas, *Elements of Information Theory*: Wiley, 1991.