Average Packet Delay of CSMA/CA with Finite User Population

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***Abstract*— In this paper, a simple closed form solution for the packet delays of the basic Carrier Sense Multiple Access with Collision Avoidance system is derived. Simulation results confirm the applicability and correctness of the derivation.**

***Index Terms*— CSMA/CA, system delay.**

I. Introduction

CARRIER Sense multiple Access with Collision Avoidance (CSMA/CA) is a highly efficient random access scheme, which is widely used in wireless communications systems such as wireless local area networks (LANs) and in random access channels in cellular mobile systems. Packet delays in CSMA/CA systems occur either when a terminal senses the channel to be busy during a packet arrival, or due to the fact that each ready terminal does not start a transmission with probability $1-p$, and also, due to collisions occurring in the channel when more than one terminal attempts to transmit within the same "vulnerable" period a.

The CSMA protocol was first proposed by Kleinrock and Tobagi in [1]. An initial study of throughput and delay characteristics for a single receiver in CSMA based on an infinite population model was performed in the same paper. Since then, the performance of CSMA has been extensively analyzed in the literature. Another analytical approach was suggested in [2], for a finite population model. More recently, the throughput and delay performance of CSMA/CA protocols with capture effects has been analyzed in [3]. [4] Adopts and modifies the analytical approach of [3] without considering the capture effect phenomenon, which results in a different close-form equation for the time spent in successful transmissions. However, in their delay analysis, both [3] and [4] improperly use the "residual life" interval from renewal theory [5] since the "age" interval clearly depends on the probability of new arrivals g and the beginning of the transmission period. In addition, the ratio G/S for the mean number of retransmissions has to be based on an infinite population with poisson arrivals assumption. In [6], recursive processes are used to compute the moments of packet delays and inter-departure times, and closed-form expressions for their generating functions are derived for the finite population case. However, the approach followed in that work may not prove to be very practical for numerical computation since it requires symbolic inversion of matrices whose elements are themselves z-transforms.

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As indicated by our prior observations, many past approaches concentrate on an infinite population model, or are analytically complicated and intractable when the population size is large. On the other hand, there are publications in the literature that improperly use assumptions for an infinite number of terminals with Poisson arrivals in the delay analysis of the finite population case. Therefore, it is the aim of this paper to derive a simple closed form of the delay performance for the slotted p-persistent CSMA system with collision avoidance for a finite number of users and a single receiver in line-of-sight of all users. These results should guide further studies and improvements on CSMA/CA based MAC protocols like the the Distributed Coordination Function of the IEEE 802.11 WLAN standard. Particularly, the analysis in Section III facilitates an estimation of the average delay occurring in a packet transmission over WLAN systems.

The remaining of this paper is organized as follows. In Section II, the underlying system model and its assumptions are thoroughly explained. The expected normalized packet delays are calculated in Section III. Finally, numerical results are given and conclusions are drawn in Sections IV and V, respectively.

II. System Model

Our derivation of packet delays follows the approach in [2], for slotted p-persistent CSMA/CA channel throughput for a finite number of transmitters and a single receiver in line-of-sight of all transmitters. We hence use the same notation which defines the channel states.

All transmitted packets have the same length, which for simplicity is assumed to equal unity. Each terminal can sense any transmission that occurs within the Carrier Sensing area and delay its own transmission. All the transmissions inside the Transmission Range are successful, i.e. the Signal to Interference Ratio is greater than a threshold that allows error-free reception. The slot duration denoted by a is chosen to be equal to the signal propagation delay. All the users are synchronized to start their transmissions at the beginning of a slot. Each terminal has periods which are independent and geometrically distributed, in which there are no packets. In each slot an (empty) terminal generates a new packet with probability g ($0< g <1$), where we assume that g is comprised of both new and rescheduled packets. Since we consider the p-persistent protocol each ready terminal starts transmitting in the next slot with probability p ($0< p \leq 1$).

A terminal will be called *empty* if it has no packets in its buffer awaiting transmission and *ready* if there are. Each user is assumed to have at most one packet requiring transmission

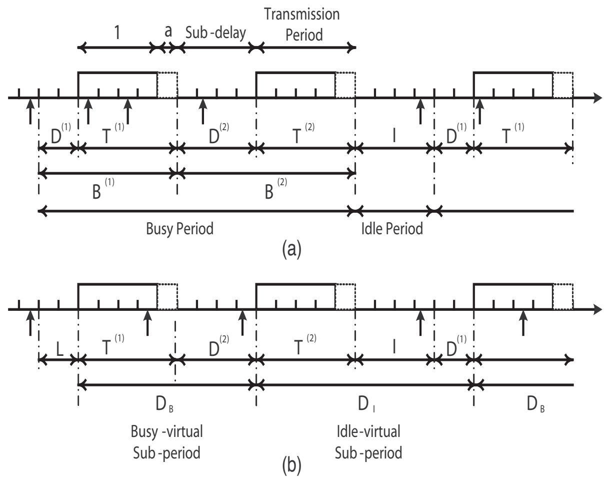

Fig. 1. Time sequence of events for basic p-persistent CSMA/CA.

at any one time (including any previously blocked packet). This means that all users make room for new packet arrivals from the beginning of $T^{(j-1)}$ by putting aside the already buffered packets; this is the same assumption as in [1] and [2]. Furthermore, for the delay analysis we assume that blocked packets are not actually discarded but are virtually buffered, and they appear again as a virtually new arrival with probability g.

The system state consists of a sequence of regeneration cycles composed of consecutive busy and idle periods. We define the *idle period* (I) as the time at which the channel is idle and all the terminals in this area are empty. We define *busy period* (B) as the time at which there is a transmission (successful or not) or where at least one of the M terminals is ready. A busy period ends if no packets have accumulated at the end of a transmission.

III. Delay Analysis

The channel busy period is divided into several sub-busy periods such that the jth subperiod, denoted by $B^{(j)}$, comprises a transmission delay, denoted by $D^{(j)}$, followed by a transmission time, denoted by $T^{(j)}$ Fig. 1(a). $D^{(1)}$ occurs when one or more packets arrive in the last slot of the idle period and $D^{(j)}$ $j \geq 2$ occurs when one or more packets arrive in the previous transmission time $T^{(j-1)}$. The transmission period is $T^{(j)} = 1 + a$, whether the transmission is successful or not. Therefore, $B^{(j)} = D^{(j)} + 1 + a$.

Let J denote the number of sub-busy periods in a busy period B. Since the busy periods continue as long as there is at least one arrival amongst the M terminals, during the last transmission time, the expectation of J, as derived in [2], is given by $\bar{J} = 1/(1-g)^{(1+1/a)M}$ and the average duration of the idle period, $\bar{I} = a/(1-(1-g)^M)$.

Let $Pr[N_0^{(j)} = n]$ be the probability that n packets arrive in M users during X_j slots, given that $n \geq 1$ we have

$$\Pr[N_0^{(j)} = n] = \frac{1}{1-(1-g)^{X_j M}} \binom{M}{n} \left[1-(1-g)^{X_j}\right]^n (1-g)^{X_j(M-n)} \tag{1}$$

where $X_j = 1$ slot for $j = 1$ and $X_j = 1 + 1/a$ slots for $j \geq 2$. This is the distribution of the number of packets awaiting transmission at the beginning of $B^{(j)}$. The distribution of $D^{(j)}$ given $N_0^{(j)} = n$, as derived in [2], is given by

$$\begin{aligned} &Pr[D^{(j)} \geq ka, N_k^{(j)} = n+m \mid N_0^{(j)} = n] = \\ &(1-p)^{kn}(1-g)^{k(M-n)} \\ &\left\{ \binom{M-n}{m} \left[\left(\frac{g}{p-g}\right) \frac{(1-g)^k - (1-p)^k}{(1-g)^k} \right]^m \right\} \end{aligned} \tag{2}$$

and the expected value of $D^{(j)}$ is

$$\begin{aligned} \overline{D^{(j)}} = &\frac{a}{1-(1-g)^{X_j M}} \sum_{k=1}^{\infty} \left\{ (1-p)^k \right. \\ &\left. - (1-g)^{X_j} p \left[\frac{(1-p)^k - (1-g)^k}{p-g} \right] \right\}^M \\ &- \frac{a(1-g)^{X_j M}}{1-(1-g)^{X_j M}} \sum_{k=1}^{\infty} \left[\frac{p(1-g)^k - g(1-p)^k}{p-g} \right]^M . \end{aligned} \tag{3}$$

The probability of successful transmission for a specific user can hence be derived as

$$\begin{aligned} &Pr[S^{(j)} \mid D^{(j)} \geq ka, N_k^{(j)} = n+m, N_0^{(j)} = n] = \\ &\frac{n+m}{M} p(1-p)^{n+m-1}. \end{aligned} \tag{4}$$

A failure occurs if there is a collision, or if another terminal occupies the channel first. Unconditioning (4) on $N_k^{(j)}$ and $D^{(j)}$, using (2), and on $N_0^{(j)}$, using (1), the probability of successful transmission given that $n \geq 1$ can be derived as

$$\begin{aligned} P_s^{(j)} = &\frac{p}{1-(1-g)^{X_j M}} \\ &\sum_{k=0}^{\infty} \left\{ (1-p)^k - (1-g)^{X_j} \left[\frac{p(1-p)^k - g(1-g)^k}{p-g} \right] \right\} \\ &\left\{ (1-p)^{k+1} - p(1-g)^{X_j} \left[\frac{(1-p)^{k+1} - (1-g)^{k+1}}{p-g} \right] \right\}^{M-1} \\ &- \frac{pg(1-g)^{X_j M}}{1-(1-g)^{X_j M}} \sum_{k=1}^{\infty} \left[\frac{(1-g)^k - (1-p)^k}{p-g} \right] \\ &\left[\frac{p(1-g)^{k+1} - g(1-p)^{k+1}}{p-g} \right]^{M-1} . \end{aligned} \tag{5}$$

This is the probability of a terminal generating and successfully transmitting a packet in the j-th sub-busy period.

The probability of failure for a specific terminal in the j-th subperiod, given that the terminal has generated a packet is given by $P_f^{(j)} = 1 - \frac{P_s^{(j)}}{1-P_e^{(j)}}$, where $P_e^{(j)}$ is the probability of a specific user not having any packet arrivals during the j-th sub-busy period, and it is given by

$$P_e^{(j)} = \frac{1-(1-g)^{X_j(M-1)}}{1-(1-g)^{X_j M}} (1-g)^{(X_j + \frac{\overline{D^{(j)}}}{a})}. \tag{6}$$

Therefore, the corresponding probability of failure in the first attempt to access the channel is given by

$$P_f = \frac{\bar{I} + \overline{D^{(1)}}}{\bar{I} + \bar{B}} \left[1 - \frac{P_s^{(1)}}{1-P_e^{(1)}} \right] + \frac{\bar{B} - \overline{D^{(1)}}}{\bar{I} + \bar{B}} \left[1 - \frac{P_s^{(2)}}{1-P_e^{(2)}} \right] \tag{7}$$

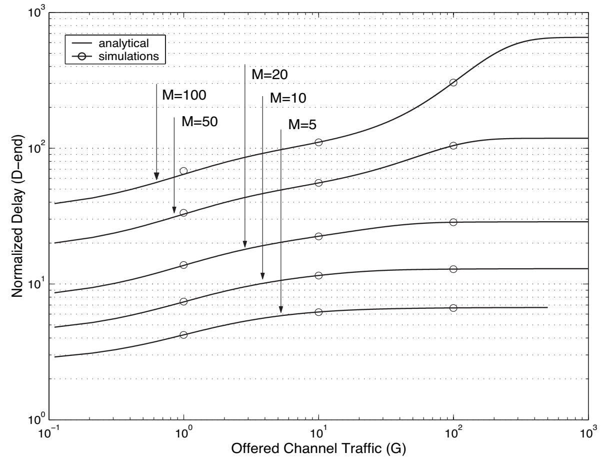

Fig. 2. Average normalized delay (D_{end}) versus offered load (G) for a varying number of terminals (M).

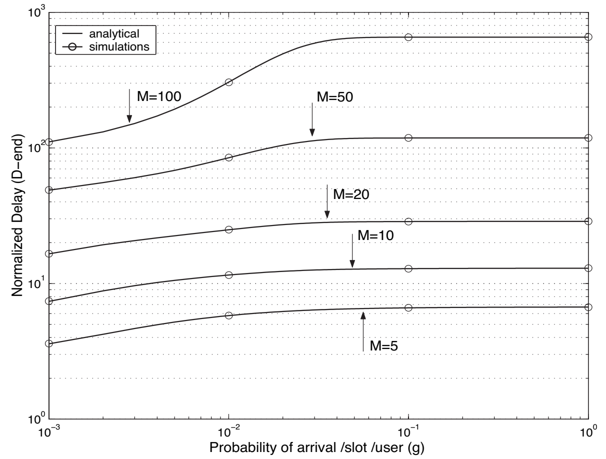

Fig. 3. Average normalized delay (D_{end}) versus probability of packet arrival per slot per user (g) for a varying number of terminals (M).

The average delay $\bar{L}$, from the time that a packet from a specific terminal senses the channel, to the time that a transmission (from the specific terminal or any other terminal) starts can be calculated as follows. First we introduce two kinds of virtual subperiods Fig. 1(b). These subperiods denote how a user that failed to transmit observes the channel, and they continue until the user successfully transmits. The busy-virtual subperiod of duration $(1+a+D^{(2)})$, and the idle-virtual subperiod of duration $(1+a+I+D^{(1)})$. The average time interval for a packet arrival in the $j-th$ virtual sub-busy period given that there is an arrival is

$$\overline{Lo^{(j)}} = \frac{a}{1-(1-g)^{(X_j+\frac{\overline{D^{(j)}}}{a})}} \sum_{k=1}^{\lfloor (X_j+\frac{\overline{D^{(j)}}}{a})\rfloor} k(1-g)^k g$$
$$= \frac{a(1-g)}{(1-(1-g)^{(X_j+\frac{\overline{D^{(j)}}}{a})})g} \left[1-\left(1+\lfloor X_j+\frac{\overline{D^{(j)}}}{a}\rfloor g\right)(1-g)^{\lfloor (X_j+\frac{\overline{D^{(j)}}}{a})\rfloor}\right] \quad (8)$$

where $\lfloor x \rfloor$ is the floor function. Therefore, the average delay from the time that a packet from a specific terminal senses the channel to the time that the next transmission takes place is given by $\overline{L^{(j)}} = X_j + \overline{D^{(j)}} - \overline{Lo^{(j)}}$, and the average time delay $\bar{L}$ by

$$\bar{L} = \frac{\bar{I}}{\bar{I}+\bar{B}}\overline{D^{(1)}} + \frac{\overline{D^{(1)}}}{\bar{I}+\bar{B}}\overline{L^{(1)}} + \frac{\bar{B}-\overline{D^{(1)}}}{\bar{I}+\bar{B}}\overline{L^{(2)}} \quad (9)$$

Finally the expected normalized overall delay due to re-transmissions is given by

$$\overline{D_r} = \frac{P_f}{\bar{J}P_s^{(1)}}\left(1+a+I+\overline{D^{(1)}}\right) + \frac{(\bar{J}-1)P_f}{\bar{J}P_s^{(2)}}\left(1+a+\overline{D^{(2)}}\right) \quad (10)$$

Concluding, the expected normalized end-to-end delay from the time a packet senses the channel to the end of its successful transmission is given by $\overline{D_{end}} = \bar{L} + \overline{D_r} + 1 + a$.

IV. Numerical Results

Numerical results for the average packet delay are given in this section, where the accuracy of the derived delay has been validated by means of simulations. The transmission probability is set to $p = 0.03$ and the slot duration is set to $a = 0.01$. Fig. 2 illustrates the dependence of the normalized delay (D_{end}) upon the total offered traffic ($G = gM/a$). We let $g = \min[1, aG/M]$. In Fig. 3, the average normalized packet delay versus the probability of packet arrival per slot per user (g) is presented for a varying number of terminals (M). It is noticeable that the packet delay tends to stabilize after a point (e.g. for $g > 0.5$) since a single packet buffer is used and the finite terminal population transmits with a relatively small probability p.

V. Conclusions

A closed form for the average packet delay of the basic p-persistent CSMA/CA system as a function of the offered load, the transmission probability and the number of terminals, was presented. The analytical results of delay performance have been verified by computer simulations.

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