A New Distributed Optimization Framework for Hybrid Ad-hoc Networks

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***Abstract*—The continuously increasing demand for resources in modern networks urges for more efficient resource allocation. Such an allocation of resources to network users can be formulated as an optimization problem. However, the existence of wireless links in modern networks and the competition for resources by multimedia applications turn the optimization problem into a non-convex one, which is in general difficult to solve. This paper presents a non-convex optimization formulation to describe the *Network Resource Allocation* problem in *hybrid* ad-hoc networks, i.e. networks with both wired and wireless links. To find the optimal solution to this problem, a novel general optimization framework, for non-convex optimization problems, is presented and the necessary and sufficient condition for the convergence of a distributed algorithm to the optimal solution is also proven. Moreover, based on this framework, a distributed joint power and rate adaptation algorithm is proposed to calculate the optimal solution, and finally, the convergence and optimality of the algorithm are verified by simulation.**

***Index Terms*—Hybrid Ad-Hoc Networks, Non-convex Optimization, Resource Allocation, Network Utility Maximization**

I. Introduction and Related Work

Modern networks must encompass and simultaneously support multiple users, services and applications with diverse demands and requirements that push networks' performance closer to their limit. Therefore optimum resource allocation between users and/or applications is of paramount importance in order to assure efficient utilization of the network. The *Resource Allocation* problem is one of the numerous research areas in which *Optimization Theory* has found extensive use, since it can lead to the development of distributed algorithms to assure optimal allocation of resources in a network.

Kelly et al. in their seminal paper [1] introduced the *Network Utility Maximization (NUM)* framework, where the *Resource Allocation* problem is expressed as an optimization problem. The authors propose an algorithm to determine the optimal way to share the link bandwidths among different traffic flows under the assumption that the *utility* functions are concave functions of the rate x_r. In 1999, Low et al. [2] proposed an alternative methodology as a solution to the exact same problem. They propose a distributed algorithm, based on *Duality Theory*, which allows the sources and network links to communicate and update their controls. This framework has found numerous applications in network resource allocation in wired ([3] and [4]) and wireless networks ([5], [6], [7],[8]).

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The main focus of existing pieces of work are on modeling applications that generate *elastic* traffic. These applications include FTP and HTTP, which were the majority of internet traffic until recently. TCP is an example of a protocol designed to perform optimally for this traffic in wired networks. However, modern internet traffic is dominated by real-time applications, such as video and audio streaming, that are considered *inelastic*, and make TCP operate suboptimally. Moreover, TCP operates suboptimally in networks with wireless links. **This absence of alternative transport protocols to allow network optimization for inelastic applications in heterogeneous networks is the main motivation behind this work.**

The main challenge for inelastic traffic is that it cannot be modeled using concave utility functions and therefore these applications cannot be optimized using *Convex Optimization*. *Convexity* of an optimization problem has always been considered the *"watershed"* for the differentiation between easy and hard problems. This is because, contrary to *convex optimization*, where the *duality gap* between the *primal* and the *dual* optimization problem, that is the gap between their optimal solutions, is always zero [9], in *non-convex* optimization problems it can be positive and then more sophisticated techniques must be employed to solve them [10].

Recent work removes the assumption of concave utilities in the context of *NUM* [11] [12]. Inelastic traffic is modeled either as a non-concave utility function or as a discontinuous utility function. An example of the first case could be a sigmoidal function and a step function could be considered an example of the discontinuous case. The authors in [13] [14] examine a non-convex NUM formulation in wired networks and prove the necessary and sufficient conditions so that the duality gap is zero and the standard distributed algorithm based on the *gradient method* [9] can solve this problem formulation optimally. However, these works are restricted only to a specific non-concave formulation for wired networks.

This paper makes the following contributions:

- Proposes a non-convex formulation of the *Resource Allo-*

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cation problem in *hybrid* ad-hoc networks, i.e, networks with both wireless and wired links.
- In order to solve this problem, it develops a non-convex optimization framework that removes the critical assumptions for convexity of the constraints and concavity of the objective function. The significance of this optimization framework is its generality and, therefore, its suitability to a wide range of applications.
- The aforementioned optimization framework is used to develop a distributed joint rate allocation and power control algorithm, which enables network nodes to optimize their performance, even for the case of inelastic traffic.
- Proves the necessary and sufficient condition so that the above mentioned optimization formulation can be solved distributedly by the *subgradient method.*

The rest of the paper is organized as follows. Section II presents the general optimization framework and proves a necessary and sufficient condition to assure optimality of the solution. In Section III, the framework is applied to the *resource allocation problem* in *hybrid ad-hoc* networks and the distributed algorithm is shown to converge to the optimal solution. Then, its performance is evaluated by simulations and, finally, our work is summarized in Section IV.

II. Framework Description

The optimization problem formulating the resource allocation problem in hybrid networks (see Section III), is evidently non-convex and therefore existent convex optimization frameworks cannot be applied. Hence, we begin this section by developing a general novel non-convex optimization framework and a necessary and sufficient condition for the convergence of any gradient based distributed algorithm.

First, consider a maximization problem over the vector of variables $\boldsymbol{x} = [x_1, x_2, \dots, x_n]$ of the form:

$$\begin{aligned} \max_{\boldsymbol{x}} \quad & f(\boldsymbol{x}) \\ \text{s. t.} \quad & h_i(\boldsymbol{x}) \geq 0, \quad \forall\ i \\ & \boldsymbol{x} \geq \mathbf{0}. \end{aligned} \tag{1}$$

Note that there are no assumptions regarding the concavity of the objective function and no assumptions about the convexity of the constraint functions $h_i(\boldsymbol{x})$. In order to solve the problem using *Duality Theory*, the Lagrangian function $L(\boldsymbol{x}, \boldsymbol{\lambda})$ must be first defined as $L(\boldsymbol{x}, \boldsymbol{\lambda}) = f(\boldsymbol{x}) + \sum_{i=0}^{M} \lambda_i h_i(\boldsymbol{x})$, where M is the number of constraints of the optimization problem and λ_i is the dual variable associated with the i^{th} constraint. Then, the *dual function* is defined as

$$d(\boldsymbol{\lambda}) = \sup_{x} L(\boldsymbol{x}, \boldsymbol{\lambda}) = \sup_{\boldsymbol{x}} \left\{ f(\boldsymbol{x}) + \sum_{i=0}^{M} \lambda_i h_i(\boldsymbol{x}) \right\}. \tag{2}$$

The Lagrange Dual optimization problem is:

$$\min d(\boldsymbol{\lambda}) = L(\boldsymbol{x}^*(\boldsymbol{\lambda}), \boldsymbol{\lambda}) \quad \text{s.t. } \boldsymbol{\lambda} \geq 0, \tag{3}$$

where $\boldsymbol{\lambda}$ is the optimization variable vector and $\boldsymbol{x}^*(\boldsymbol{\lambda})$ is a price-based function that maximizes the Lagrangian for a given price vector $\boldsymbol{\lambda}$, i.e.

$$\boldsymbol{x}^*(\boldsymbol{\lambda}) = \arg\max L(\boldsymbol{x}, \boldsymbol{\lambda}). \tag{4}$$

The dual function $d(\boldsymbol{\lambda})$ is always convex as a pointwise supremum of a family of affine functions of $\boldsymbol{\lambda}$ and hence problem (3) is always convex even if the primal problem (1) is not concave [9]. In other words, it is possible to solve the dual optimization problem optimally using the update equation

$$\lambda_i(t+1) = \lambda_i(t) - \alpha(t) g_i(\boldsymbol{x}^*(\boldsymbol{\lambda})) \tag{5}$$

where $\alpha(t)$ is the step size at time t and $g_i(\boldsymbol{x}^*(\boldsymbol{\lambda}))$ is the i^{th} component of the gradient of the dual objective function $d(\boldsymbol{\lambda})$ with respect to λ_i. Note that the uniqueness of the optimal vector $\boldsymbol{\lambda}$ is not guaranteed in all cases but the *Strict Mangasarian-Fromovitz Constraint Qualification* [15] provides a necessary and sufficient condition.

In concave maximization problems, equations (4) and (5) always converge to the optimal solution but this is not guaranteed for non-concave problem (1). Nonetheless, it is possible to identify the cases that (4) and (5) converge to the optimal solution using Theorem 1.

Theorem 1 (Necessary and Sufficient Condition): If the price based function $\boldsymbol{x}^*(\boldsymbol{\lambda})$ is continuous around at least one of the optimal lagrange multiplier vectors $\boldsymbol{\lambda}^*$ then the iterative algorithm consisting of equations (4) and (5) converges to the globally optimal solution.

Proof: To prove its sufficiency, it is necessary to show first that continuity of $\boldsymbol{x}^*(\boldsymbol{\lambda})$ around the optimal dual variables λ_i^* implies that *complementary slackness* is satisfied for problem (1). The *complementary slackness* condition states that $\lambda_i^* h_i(\boldsymbol{x}^*(\boldsymbol{\lambda}^*)) = 0$, $\forall i$ at the optimal solution $x^*(\boldsymbol{\lambda}^*)$.

First, the case where $\lambda_i^* > 0$ for an arbitrary chosen i is examined. A very small positive constant $\epsilon > 0$ and a new vector $\boldsymbol{\lambda}^-$ are defined where

$$\lambda_j^- = \begin{cases} \lambda_j^* - \epsilon & \text{, if } j = i \\ \lambda_j^* & \text{, if } j \neq i \end{cases} \tag{6}$$

Then, by definition of the subgradient [16],

$$\begin{aligned} & d(\boldsymbol{\lambda}^*) \geq d(\boldsymbol{\lambda}^-) + (\boldsymbol{\lambda}^* - \boldsymbol{\lambda}^-)^T \boldsymbol{g}(\boldsymbol{\lambda}^-) \Leftrightarrow \\ & d(\boldsymbol{\lambda}^*) \geq d(\boldsymbol{\lambda}^-) + \epsilon g_i(\boldsymbol{\lambda}^-) \Leftrightarrow \\ & d(\boldsymbol{\lambda}^*) - d(\boldsymbol{\lambda}^-) \geq \epsilon h_i(\boldsymbol{x}^*(\boldsymbol{\lambda}^-)). \end{aligned} \tag{7}$$

where $g_i(\boldsymbol{\lambda})$ is the i^{th} component of the subgradient of the dual objective function. But since the dual problem is a minimization problem and $\boldsymbol{\lambda}^*$ is its optimal solution, it follows that $d(\boldsymbol{\lambda}^*) \leq d(\boldsymbol{\lambda}^-)$ and hence by (7)

$$h_i(\boldsymbol{x}^*(\boldsymbol{\lambda}^-)) \leq 0. \tag{8}$$

Working at the same way, a second vector $\boldsymbol{\lambda}^+$ is defined as

$$\lambda_j^+ = \begin{cases} \lambda_j^* + \epsilon & \text{, if } j = i \\ \lambda_j^* & \text{, if } j \neq i \end{cases} \tag{9}$$

Again, by definition of the subgradient, it follows that

$$\begin{aligned} & d(\boldsymbol{\lambda}^*) \geq d(\boldsymbol{\lambda}^+) + (\boldsymbol{\lambda}^* - \boldsymbol{\lambda}^+)^T \boldsymbol{g}(\boldsymbol{\lambda}^+) \Leftrightarrow \\ & d(\boldsymbol{\lambda}^*) \geq d(\boldsymbol{\lambda}^+) - \epsilon g_i(\boldsymbol{\lambda}^+) \Leftrightarrow \\ & d(\boldsymbol{\lambda}^*) - d(\boldsymbol{\lambda}^+) \geq -\epsilon h_i(\boldsymbol{x}^*(\boldsymbol{\lambda}^+)). \end{aligned} \tag{10}$$

But for the same reason as before, $d(\boldsymbol{\lambda}^*) \leq d(\boldsymbol{\lambda}^+)$ and hence by (10), we conclude that

$$h_i(\boldsymbol{x}^*(\boldsymbol{\lambda}^+)) \geq 0. \tag{11}$$

From (8) and (11) we get to the conclusion that as long as $\boldsymbol{x}^*(\boldsymbol{\lambda})$ is continuous around $\boldsymbol{\lambda}^*$, then

$$h_i(\boldsymbol{x}^*(\boldsymbol{\lambda}^-)) = h_i(\boldsymbol{x}^*(\boldsymbol{\lambda}^+)) = h_i(\boldsymbol{x}^*(\boldsymbol{\lambda}^*)) = 0 \tag{12}$$

and hence *complementary slackness* is satisfied and the solution $\boldsymbol{x}^*(\boldsymbol{\lambda}^*)$ is primal feasible.

Then, the case where $\lambda_i^* = 0$ is examined. In this case, it is obvious that *complementary slackness* is satisfied. Primal feasibility of the solution can be shown using the positive constant ϵ and the price vector $\boldsymbol{\lambda}^+$ are defined as before. Equation (11) is reached again and under the continuity condition it follows that $h_i(\boldsymbol{x}^*(\boldsymbol{\lambda}^+)) \geq 0$. Hence, the *complementary slackness* condition is satisfied under the condition that the price-based function $\boldsymbol{x}^*(\boldsymbol{\lambda})$ is continuous at the optimal price vector $\boldsymbol{\lambda}^*$.

By definition of the dual problem,

$$d^* = \sup_x L(\boldsymbol{x}, \boldsymbol{\lambda}^*) = \sup_{\boldsymbol{x}} \left\{ f(\boldsymbol{x}) + \sum_{i=0}^{m} \lambda_i^* h_i(\boldsymbol{x}) \right\} \Leftrightarrow$$

$$d^* = f(\boldsymbol{x}^*(\boldsymbol{\lambda}^*)) + \sum_{i=0}^{m} \lambda_i^* h_i(\boldsymbol{x}^*(\boldsymbol{\lambda}^*)). \tag{13}$$

Since *complementary slackness* holds, (13) reduces to $D^* = f(\boldsymbol{x}^*(\boldsymbol{\lambda}^*))$, which by definition of the primal problem is $f(\boldsymbol{x}^*(\boldsymbol{\lambda}^*)) \leq f^*(\boldsymbol{x})$ and hence $d^* \leq p^*$. But by *weak duality* it is known that $d^* \geq p^*$ and therefore it follows that $d^* = p^*$, where p^* and d^* are the optimal values of the primal and the dual problem respectively.

So, it has been proven that continuity of the price based function (4) around at least one of the optimal price vectors implies that the *duality gap* is zero and that by solving the dual optimization problem the optimal solution $\boldsymbol{x}^*$ is also obtained. To prove that it is also a necessary condition, it is enough to consider that if the priced based function $\boldsymbol{x}^*(\boldsymbol{\lambda})$ is discontinuous around all optimal price vectors $\boldsymbol{\lambda}^*$, (12) does not hold and thus the duality gap will be strictly positive. ■

Theorem 1, compared to [14], provides a far more general optimization framework. [14] refers to a specific non-convex formulation in wired networks, whereas the necessary and sufficient condition in Theorem 1 here refers to the much more general problem (1) and therefore is widely applicable.

The wide applicability of the framework imposes some further research challenges as well. The calculation of the optimal solution relies on the shape of $x^*(\boldsymbol{\lambda})$, given by the non-concave problem (4). There are some cases where a closed form solution of this problem can be calculated but in general this is not possible and hence the need to develop a procedure to determine whether Theorem 1 holds or not is evident. Nonetheless, this is a significant result that shows that a family of non-convex problems can be solved distributedly using a gradient based method.

III. Network Utility Maximization in Hybrid Ad-Hoc Networks

To demonstrate the suitability of the framework, this section presents its application to the *Resource Allocation* problem in Hybrid Ad-Hoc Networks, i.e, in networks with both wired and wireless links. Such heterogeneous multi-hop networks provide increased flexibility but impose additional constraints in the resource allocation that can not be resolved by traditional *transport protocols* such as TCP.

A. Problem Formulation

Consider a multihop network where M nodes act as sources sending streams of traffic to a set of destination nodes using a set of K links. A single node can operate either as a source node or as a destination node or even as an intermediate node that just forwards traffic to its neighbors. Each link in the network can be either wired or wireless. The rate vector $\boldsymbol{r} = [r_1, r_2, \ldots, r_M]^T$ includes the transmission rates of all source nodes in the network and the power vector $\boldsymbol{p} = [p_1, p_2, \ldots, p_L]^T$ includes the transmission powers of all the wireless links. In addition, G is the path loss matrix of size $L \times L$ which depends on the physical characteristics of the wireless links. Its element G_{ij} is the path loss gain from the transmitter of link i to the receiver of link j. In addition, n_j is the power of *White Additive Gaussian Noise* at link j. Then, depending on the type of the application running at each source node i, node i receives a *utility* when sending traffic through the network which is depicted by a *utility function* $U_i(r_i)$. Typical example of $U_i\,(r_i)$ for applications such as HTTP or FTP would be a logarithmic function of rate, while some non-concave sigmoidal function should be used to model multimedia (real-time) applications. Moreover, wireless link j has a cost for transmitting at a specific power depicted by a cost function $V_j(p_j)$ that represents the cost of using the limited power resources of the wireless medium.

The optimization of the network's performance can be formulated as a maximization problem of the form:

$$\max_{\boldsymbol{r},\boldsymbol{p}} \quad \sum_{i=1}^{M} U_i(r_i) - \sum_{j=1}^{L} V_j(p_j) \tag{14}$$

$$\text{s. t.} \quad \sum_{i \in Z(k)} r_i \leq C_k, \qquad \forall \text{ links } k$$

$$\frac{p_j G_{jj}}{\sum_{l \neq j} p_l G_{lj} + n_j} \geq \gamma_j \quad \forall \text{ wireless links } j$$

where $Z(k)$ represents the set of traffic flows passing through link k, which can be either wired or wireless. The rates r_i and powers p_j are positive variables.

The formulation of problem (14) is an extension of the classic *NUM* framework described in [1]. The main advantages of the formulation described in (14) include the removal of the assumption for concave utility functions and the addition of a power control constraint to incorporate the main characteristic of the wireless medium, that is the interference among links. The second constraint of the formulation represents the requirement that the Signal-to-Interference plus Noise Ratio (SINR) of each link is at least equal to a minimum accepted value γ. This γ_j is also used to calculate the capacity of wireless link j using $C_j = B{\cdot}log_2\,(1 + \gamma_j)$, where B is the channel bandwidth. It is therefore evident that the problem consists of two distinct optimization subproblems, optimizing the rate allocation and the power control respectively, connected by the objective function.

B. Distributed Solution

In order to solve the problem using a gradient based distributed algorithm, it is necessary to form the Lagrangian function of (14), which is:

$$L(\boldsymbol{r},\boldsymbol{p},\boldsymbol{\lambda},\boldsymbol{\mu}) = \sum_{i=1}^{M}\left\{U_i(r_i) - r_i\left(\sum_{l\in S(i)}\lambda_l\right)\right\} + \sum_{k=1}^{K}\lambda_k C_k + \sum_{j=1}^{L}\left\{\mu_j\left(G_{jj}p_j - \gamma_j\sum_{l\neq j}G_{jl}p_l - \gamma_j n_j\right) - V_j(p_j)\right\} \quad (15)$$

where $S(i)$ is the set of links (wireless or wires) that source i is using to send its traffic, μ_j and λ_k are the dual variables and K is the total number of links in the network (wired and wireless). Then, assuming that there can be some information exchange between the wireless links, the distributed algorithm that can solve the dual optimization problem would consist of equations:

$$\lambda_k(t+1) = \lambda_k(t) - \alpha_\lambda(t)\frac{\partial L(\boldsymbol{r},\boldsymbol{p},\boldsymbol{\lambda},\boldsymbol{\mu})}{\partial \lambda_k} \quad (16)$$

$$\mu_j(t+1) = \mu_j(t) - \alpha_\mu(t)\frac{\partial L(\boldsymbol{r},\boldsymbol{p},\boldsymbol{\lambda},\boldsymbol{\mu})}{\partial \mu_j} \quad (17)$$

$$r_i^*(\boldsymbol{\lambda}) = \arg\max\left\{U_i(r_i) - r_i\left(\sum_{k\in S(i)}\lambda_k\right)\right\} \quad (18)$$

$$p_j^*(\boldsymbol{\mu}) = \arg\max\left\{\mu_j\left(G_{jj}p_j - \gamma_j\sum_{l\neq j}G_{jl}p_l - \gamma_j n_j\right) - V_j(p_j)\right\} \quad (19)$$

The formulation in (14) is evidently non-concave even though the constraints are both convex, as linear combinations of the optimization variables, due to the lack of assumption for concave utility function. Therefore, the distributed algorithm consisting of equations (16)-(19) can converge to the actual optimal solution only if Theorem 1 holds.

In order to verify that the condition in Theorem 1 holds, it is necessary to have information regarding the continuity of the functions $r_i^*(\boldsymbol{\lambda})$ and $p_j^*(\boldsymbol{\mu})$. Even though the development of a detailed procedure to determine the continuity of $r_i^*(\boldsymbol{\lambda})$ and $p_j^*(\boldsymbol{\mu})$ in the general case is an open research issue, it is possible to make some assumptions to assure the existence of these properties for this specific problem formulation. First, the utility function of each source will be assumed to have a sigmoidal shape. Sigmoidal utilities are ideal to model real time applications, such as video streaming, VoIP etc, which are responsible for the majority of network traffic in current networks. Moreover, for sigmoidal utility functions, $r_i^*(\boldsymbol{\lambda})$ is discontinuous at only one point and a heuristic has also been suggested for overcoming this discontinuity problem in case that point is an optimal $\boldsymbol{\lambda}$ [11]. Then, we assume that the cost function $V_j(p_j)$ is a convex function of power so that (19) is a convex problem and therefore continuous around the optimal $\boldsymbol{\mu}$.

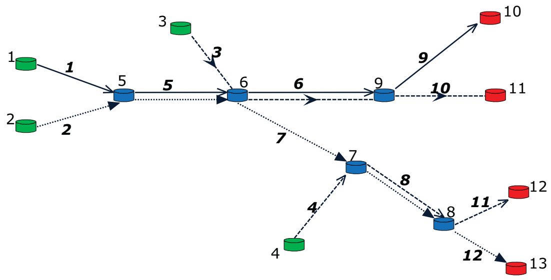

Fig. 1. Network Topology

Under these assumptions, the distributed algorithm in equations (16)-(19) will converge to the optimal solution if that exists even though the formulation is non-concave. Regarding the existence of a feasible power vector, [17] provides a necessary and sufficient condition for the existence of a feasible solution of the power control problem. The optimal rates and powers can be calculated using the gradient based iterative equations:

$$r_i(t+1) = r_i(t) - \alpha_r(t)\frac{\partial L(\boldsymbol{r},\boldsymbol{p},\boldsymbol{\lambda},\boldsymbol{\mu})}{\partial r_i} \quad (20)$$

$$p_j(t+1) = p_j(t) - \alpha_p(t)\frac{\partial L(\boldsymbol{r},\boldsymbol{p},\boldsymbol{\lambda},\boldsymbol{\mu})}{\partial p_j} \quad (21)$$

while restricting the range of the possible rates at the concave region of the sigmoidal utility function.

C. Simulation Results

This section presents numerical results that verify the performance and convergence of the gradient based algorithm. The distributed algorithm has been simulated in various topologies and under various interference conditions in order to evaluate its robustness and convergence. For illustrative purposes, Figure 1 shows an example of a simple hybrid network with 4 source nodes, 5 intermediate and 4 destination nodes. The links connecting the intermediate nodes are wired while the connections to the end nodes are wireless. In other words, links 1-4 and 9-12 are wireless while links 5-8 are wired. The utility functions of the sources were chosen as $U_i(r_i) = \frac{1}{1+e^{-2x+5}}$, the power cost functions as $V_j(p_j) = p_j$ and the gradient step sizes were $\alpha = 0.05$ for all variables. Moreover, the path-loss coefficients G_{ij} were calculated using the large-scale attenuation model, where each coefficient is a decreasing function of the distance between the transmitter and the receiver, while making sure that the necessary and sufficient condition in [17] for the feasibility of the power control problem is satisfied. The bandwidth B of the wireless links used was 2 MHz. Finally, the capacity of the wireless links was calculated as 6.9 MB/s, based on the SINR target $\gamma_j = 10dB$ and the channel bandwidth B. The capacity of the wired links was assumed equal to 6.9 MB/s.

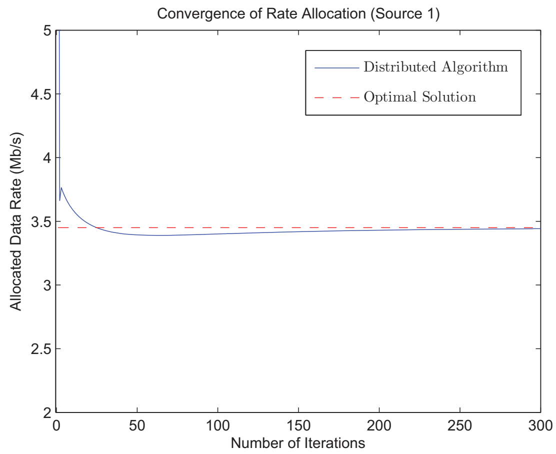

Fig. 2. Rate Convergence

Figures 2 and 3 show the convergence of the rate and power allocation for the specific scenario. It is clearly shown that even though the maximization problem solved is not concave, the iterative gradient based algorithm converges to the optimal solution since the necessary and sufficient condition in Theorem 1 holds. The convergence speed of the variables depends on the value of the gradient step size and there exists a trade-off between the convergence speed and the convergence distance from the actual optimum [10]. Specifically, the greater the gradient step size, the faster the convergence but also the larger the convergence distance from the actual optimal solution.

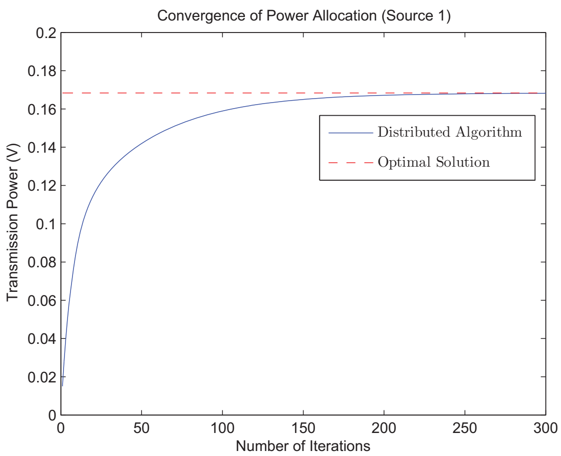

Fig. 3. Power Convergence

IV. Concluding Remarks

A non-convex optimization formulation to describe the *resource allocation* problem in *hybrid* ad-hoc networks, i.e. networks with both wired and wireless links, was proposed. To solve this problem, we developed a generic non-convex optimization framework and proposed a joint rate and power distributed gradient based algorithm together with the necessary and sufficient condition for which the algorithm converges to the optimal solution. Finally, we verified the convergence of the algorithm and we assessed its performance by simulations.

The proposed non-convex optimization framework can be used as a foundation for developing novel transport layer resource allocation protocols that would operate optimally for all range of applications. One of the key open issues to make full use of this necessary and sufficient condition is the development of a procedure for determining whether this theorem holds for a general optimization problem, which is part of our ongoing and future work. Nonetheless, the condition in Theorem 1 shows that a family of non-convex optimization problems can be solved distributedly using a gradient based method.

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