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New Sparse Sampling Methods: Time-based sampling and sampling along trajectories.

Pier Luigi Dragotti

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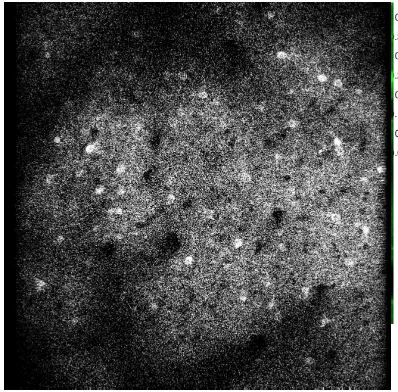
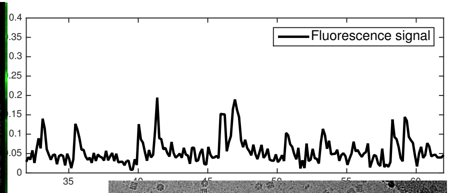
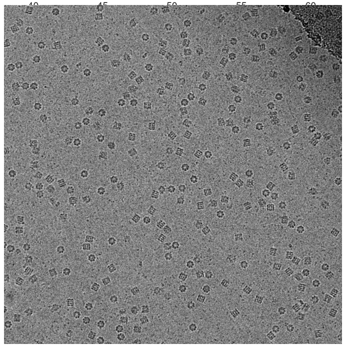
Motivation

- The revolution in sensing, with the emergence of many new sensing and imaging techniques, offers the possibility of gaining unprecedented access to the physical world

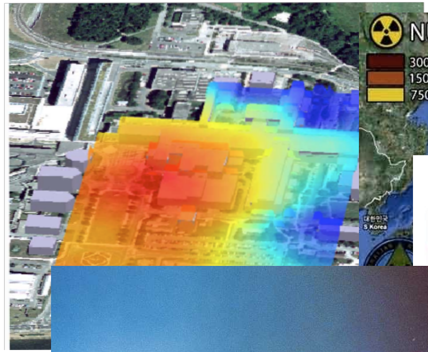




Two-photon microscopy



Electron Microscopy[*]



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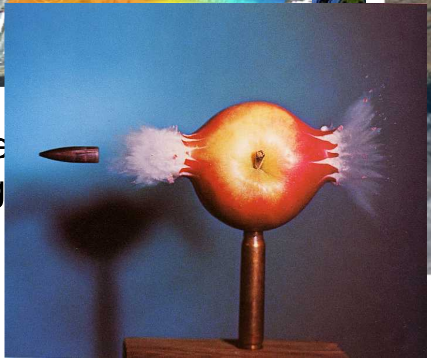

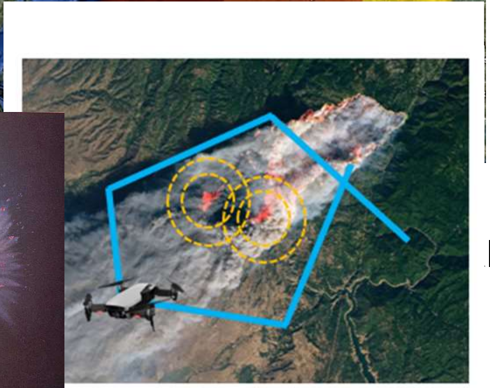

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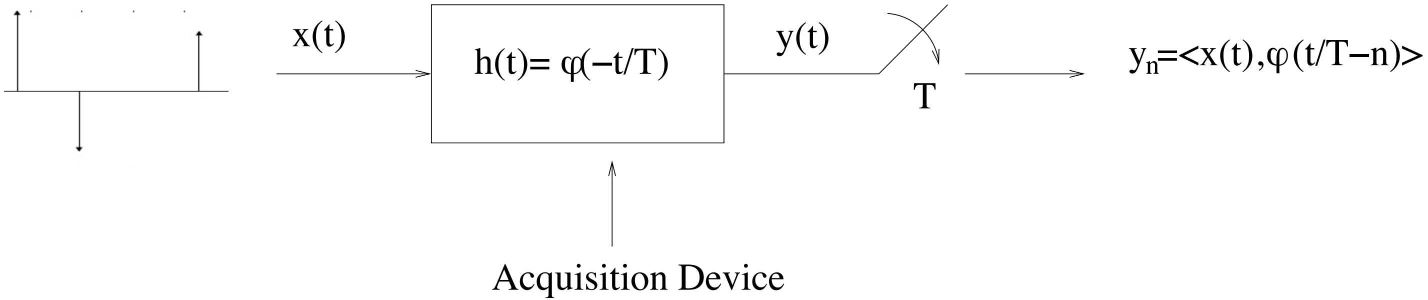
High-speed cameras

[*]From Wikimedia Commons, By Vossman - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=4560672

Outline

- The sampling problem
- A bit of history:
  - The linear case: Shannon sampling theorem
  - The non-linear case: sparse sampling and sampling signals with finite rate of innovation
- Bio-Inspired energy-efficient sampling and sampling based on timing
  - Integrate and fire system
- Sampling along unknown trajectories
  - Estimating diffusion fields using mobile sensors
- Conclusions and outlook

Sampling: The Set-up



Note that

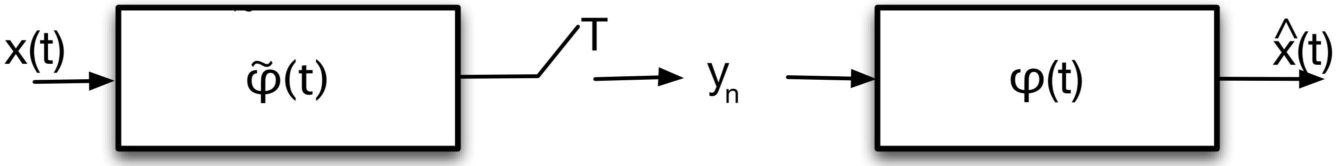
- $y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$
- $$\begin{aligned} y_n &= y(nT) = \int_{-\infty}^{\infty} x(\tau)h(nT-\tau)d\tau = \int_{-\infty}^{\infty} x(\tau)\varphi(\tau/T-n)d\tau \\ &= \langle x(t), \varphi(t/T-n)\rangle \end{aligned}$$
- $\varphi(t)$ is the time reversed version of the acquisition device and is called **sampling kernel**.

A bit of History: the shift-invariant model

- Typically, signals are represented as follows (e.g., bandlimited functions):

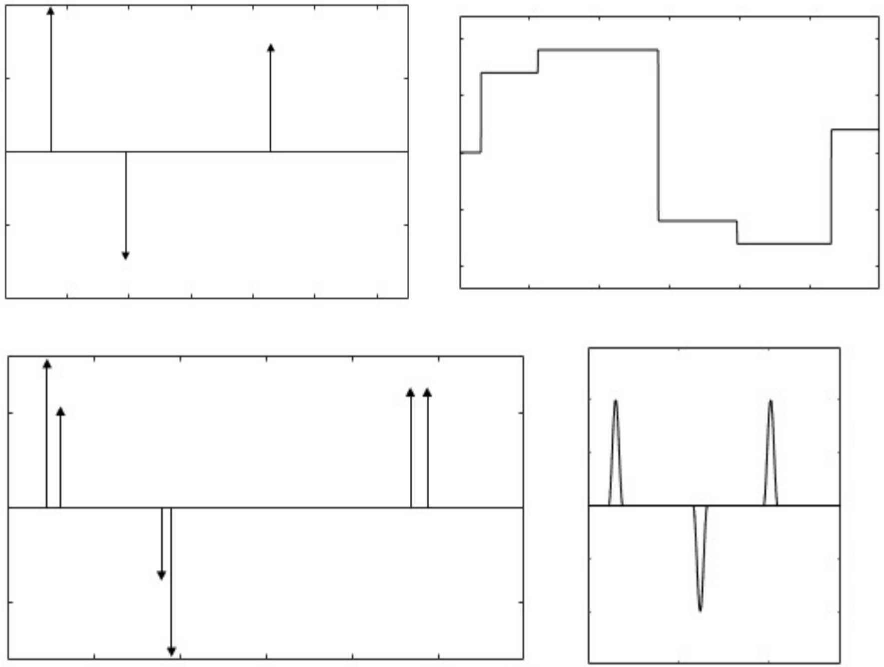
$$x(t) = \sum_n y_n \varphi\left(\frac{t}{T} - n\right)$$

- which leads to this sensing system and to a linear reconstruction process



A bit of History: Sparsity and Sampling

- No real-life signals are bandlimited, but they might be sparse in a domain or in a parametric space



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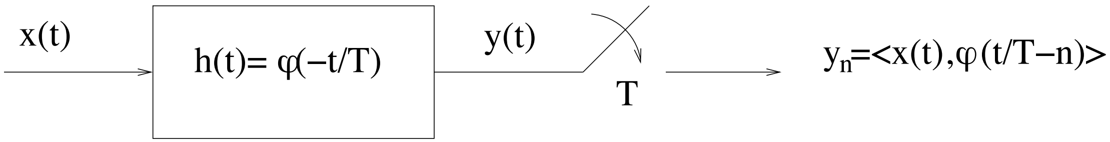
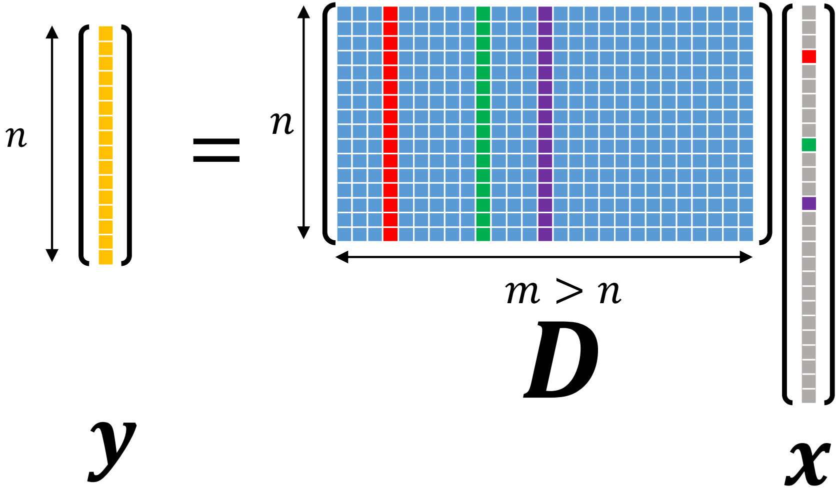
Compressed Sensing Formulation



- Discretize the input (this leads to a sparse vector $\boldsymbol{x}$)
- Discretize the sampling kernel (this leads to a fat matrix $\boldsymbol{D}$)
- Reconstruct the signal from a small number of measurements $\boldsymbol{y}$ using convex optimization methods (l_1 minimization)
- Strong recovery guarantees [Donoho:06, Candes et al.:06]



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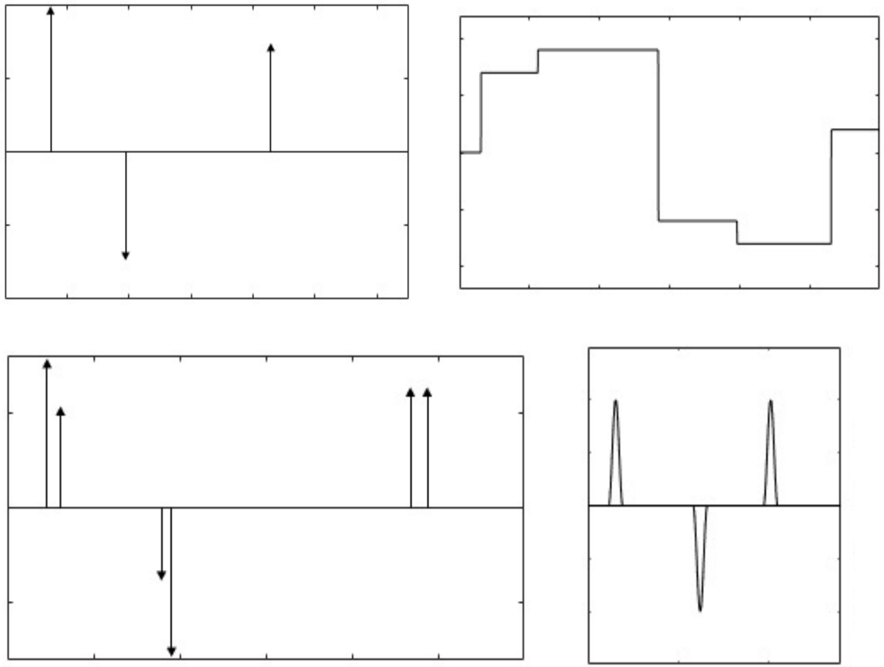
A bit of History: Sparsity and Sampling

- No real-life signals are bandlimited, but they might be sparse in a domain or in a parametric space
- For example, sparse parametric signals (i.e., **signals with finite rate of innovation (FRI) [VetterliMB:01]**).

$$x(t) = \sum_{k} x_k \varphi(t - t_k)$$

- Key issue is how to retrieve the free parameters of these signals from samples



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Sparse Sampling: Core Approach

- Assume that $x(t) = \sum_{k=1}^{K} x_k \delta(t - t_k)$
- The key idea is to connect sparse sampling to a method broadly used in e.g. array signal processing and known as Prony's method

$$y_n \to s_m = \sum_{k=1}^{K} b_k u_k^m ,$$

where $b_k = x_k e^{j\omega_0 t_k}$, $u_k = e^{j\lambda t_k}$

- Retrieving the pulse locations u_k and the amplitudes x_k from s_m is a classical problem first solved by Baron de Prony in 1795.

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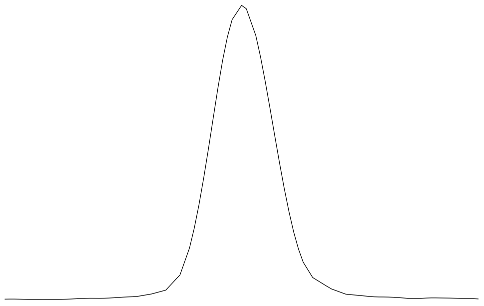
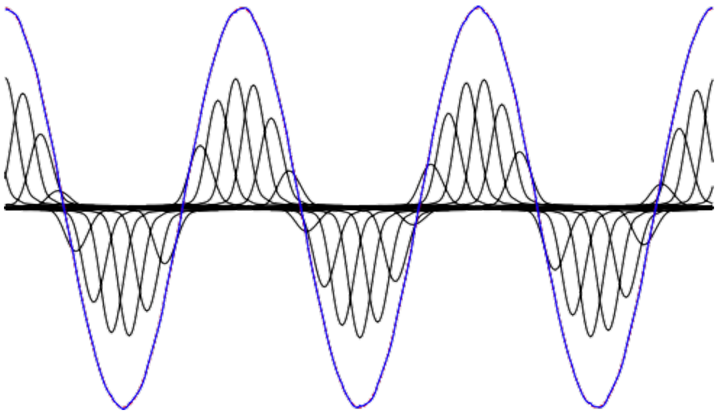
Sparse Sampling: Core Approach

$$y_n = \langle x(t), \varphi(t-n) \rangle$$

$$\sum_n c_{m,n}\varphi(t-n) \approx e^{j\omega_m t}$$



Shape of the sampling kernel



Reproduction of exponentials

Computation of the coefficients $c_{m,n}$

- We want to find coefficients $c_{m,n}$ such that $\sum_n c_{m,n}\varphi(t-n) \approx f_m(t)$ in the least-square sense.
- We need to compute the orthogonal projection of $f_m(t)$ onto $span\{\varphi(t-n)\}_n$
- This means $\langle f_m(t) - \sum_n c_{m,n}\varphi(t-n), \varphi(t-k)\rangle = 0$ (orthogonality principle)
- Leveraging the fact that we are considering uniform shifts of $\varphi(t)$ and that in our case $f_m(t) = e^{j\omega_m t}$, we end-up with an exact expression[1]:

$$c_{m,n} = \frac{\hat{\varphi}(\omega_m)e^{j\omega_m n}}{\widehat{a_\varphi}(e^{j\omega_m})}$$

where $\widehat{a_\varphi}(e^{j\omega_m})$ is the z-transform of $\langle \varphi(t-n), \varphi(t)\rangle$ at $z = e^{j\omega_m}$.

[1]J. Urigüen, T. Blu, and P. Dragotti, "FRI sampling with arbitrary kernels," IEEE Transactions on Signal Processing, vol. 61, no. 21, pp. 5310–5323, 2013.

Sparse Sampling: Core Approach

- Compute a linear combination of the samples: $s_m = \sum_n c_{m,n} y_n$ for some choice of coefficients $c_{m,n}$
- Because of linearity of inner product, we have that

$$
\begin{aligned}
s_m &= \sum_n c_{m,n} y_n \\
&= \sum_m c_{m,n} \langle x(t), \varphi(t/T - n) \rangle \qquad m = 1, 2, \ldots, L. \\
&= \langle x(t), \sum_n c_{m,n} \varphi(t/T - n) \rangle \qquad m = 1, 2, \ldots, L.
\end{aligned}
$$

- Assume that $\sum_n c_{m,n} \varphi(t/T - n) \simeq e^{j\omega_m t/T}$ for some frequencies ω_m $m = 1, 2, \ldots, L$

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Sparse Sampling: Core Approach

Then

$$
\begin{aligned}
s_m &= \sum_n c_{m,n} y_n \\
&= \langle x(t), \sum_n c_{m,n} \varphi(t/T - n) \rangle \\
&= \int_{-\infty}^{\infty} x(t) e^{j\omega_m t} dt, \qquad m = 0, 1, ..., L.
\end{aligned}
$$

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Sparse Sampling: Core Approach

- Assume $x(t)$ is a stream of K Diracs on the interval of size N: $x(t) = \sum_{k=0}^{K-1} x_k \delta(t - t_k),\ t_k \in [0, N)$.
- We restrict $j\omega_m = j\omega_0 + jm\lambda \quad m = 1, ..., L$ and $L \geq 2K$.
- We have N samples: $y_n = \langle x(t), \varphi(t-n) \rangle,\ n = 0, 1, ... N-1$:
- We obtain

$$
\begin{aligned}
s_m &= \sum_{n=0}^{N-1} c_{m,n} y_n \\
&= \int_{-\infty}^{\infty} x(t) e^{j\omega_m t} dt, \\
&= \sum_{k=0}^{K-1} x_k e^{j\omega_m t_k} \\
&= \sum_{k=0}^{K-1} \hat{x}_k e^{j\lambda m t_k} = \sum_{k=0}^{K-1} \hat{x}_k u_k^m, \qquad m = 1, ..., L.
\end{aligned}
$$

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Sparse Sampling: Core Approach

- To summarize:

$$\boxed{s_m} = \sum_{n=0}^{N-1} c_{m,n} \boxed{y_n} = \sum_{k=1}^{K} x_k \sum_{n=0}^{N-1} c_{m,n} \varphi[t_k - n]$$

$$\approx \sum_{k=1}^{K} x_k e^{j\omega_m t_k} = \sum_{k=1}^{K} x_k e^{j\omega_0 t_k} \left(e^{j\lambda t_k}\right)^m = \boxed{\sum_{k=1}^{K} b_k u_k^m},$$

where $b_k = x_k e^{j\omega_0 t_k}$, $u_k = e^{j\lambda t_k}$

- The amplitudes x_k and locations t_k can now be retrieved using Prony's method.

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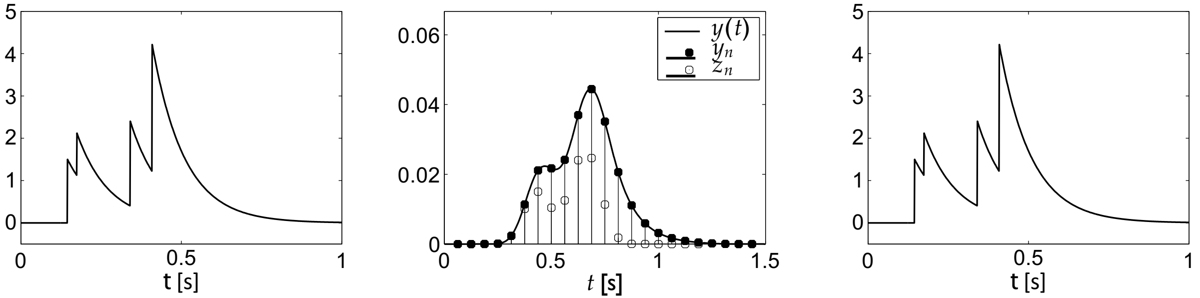
Sampling Stream of Decaying Exponentials



(a) Input signal, $x(t)$ **(b)** Filtered and sampled signal **(c)** Reconstructed signal

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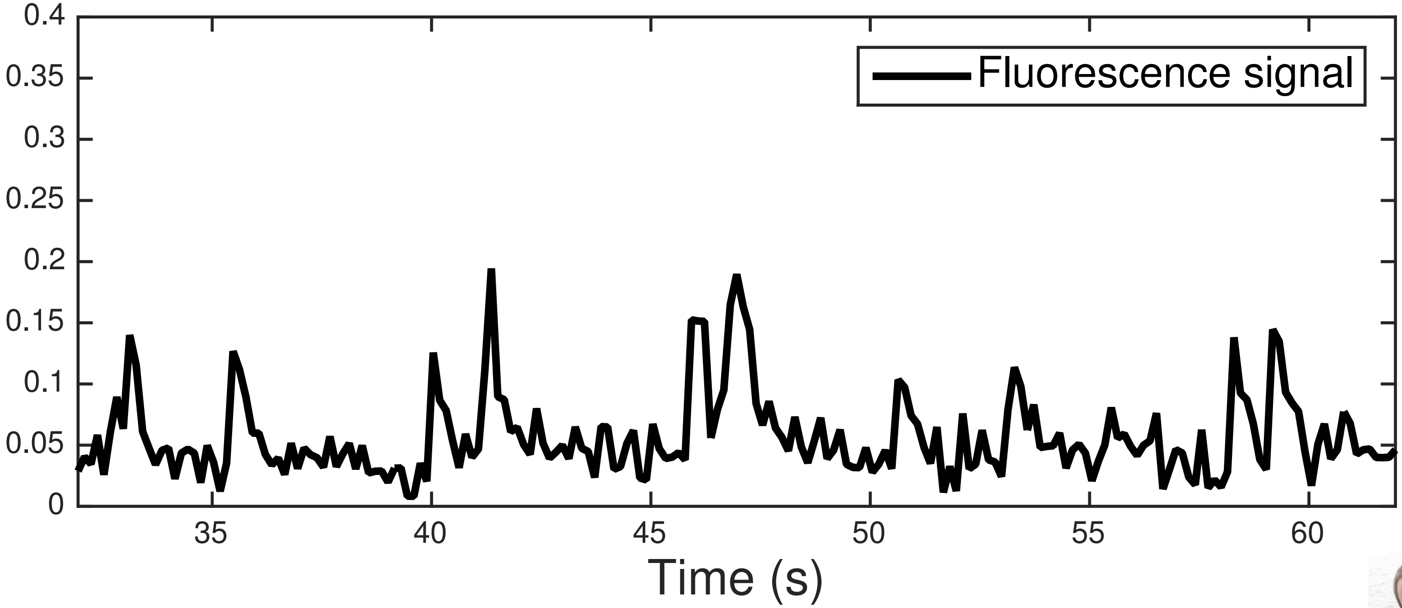
Sparse Sampling – Application in Neuroscience



J.Onativia, S. Schultz and P.L. Dragotti, A finite rate of innovation algorithm for fast and accurate spike detection from two-photon calcium imaging, Journal of Neural Engineering, 10 (4), August 2013.

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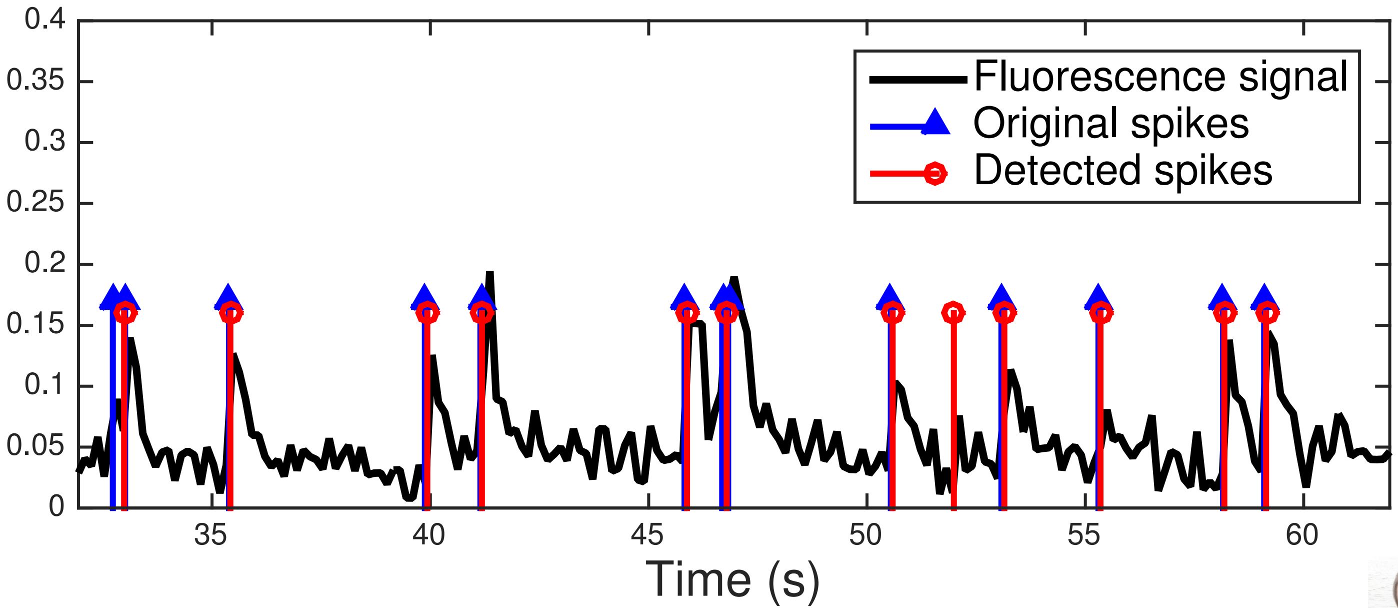

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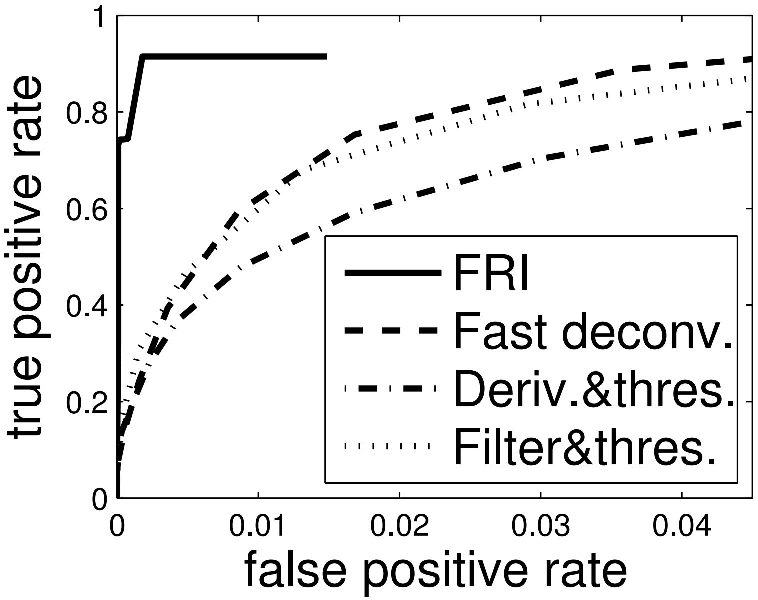

- The algorithm **outperforms** state-of-the art methods
- Can operate in **real-time** simultaneously on 80 streams
- Increase in **resolution** by factor 3

Sparse Sampling – Image Super-resolution

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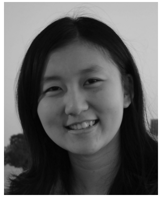
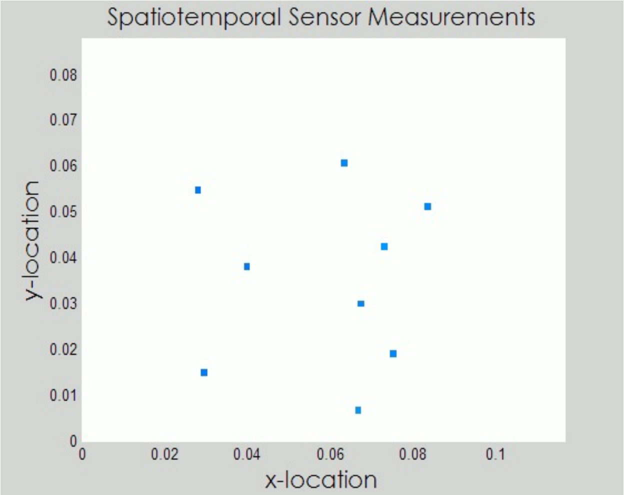
Low-res input 64 x64 pixels

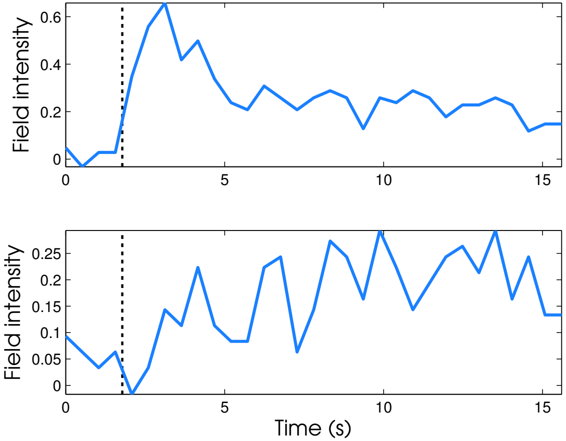
Final result 256x256 pixels



X. Wei and P. L. Dragotti, FRESH -FRI-based single image super-resolution algorithm, IEEE Trans on Image Processing, Vol.25(8), pp. 3723-3735, August 2016.

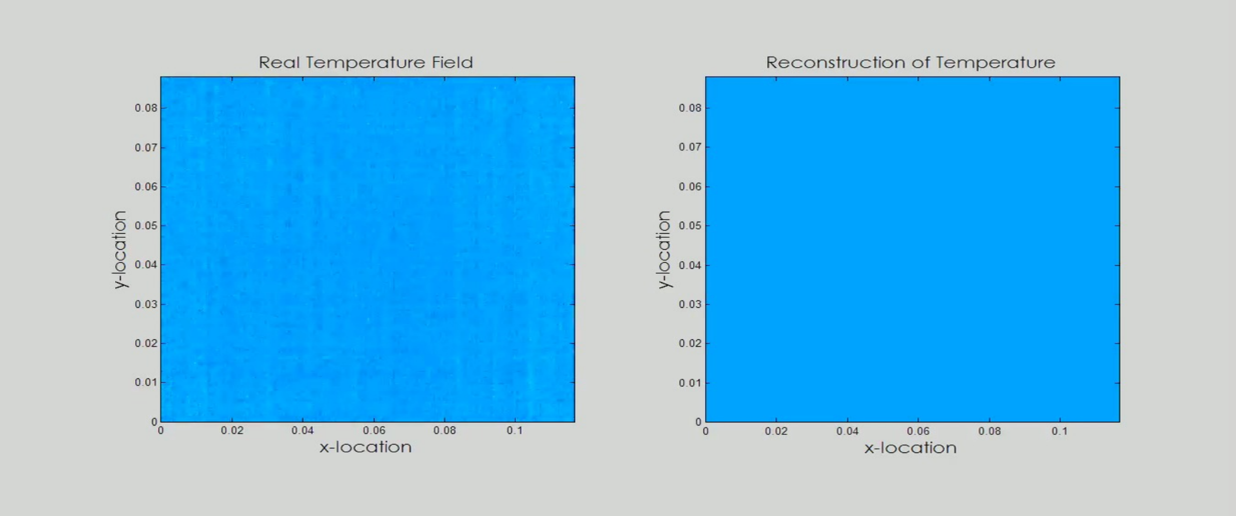
Estimating Temperature Fields with Sensors





J. Murray-Bruce and P.L. Dragotti, A Sampling Framework for Solving Physics-driven Inverse Source Problems, IEEE Trans. on Signal Processing, Vol. 65(24), pp. 6365-6380, December 2017

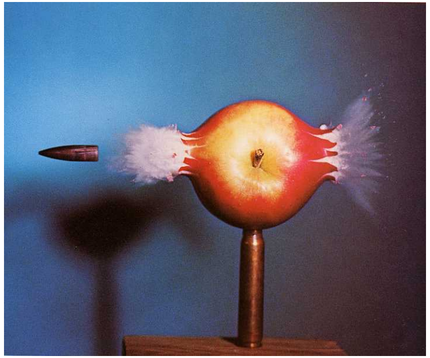
Estimating Temperature Fields with Sensors



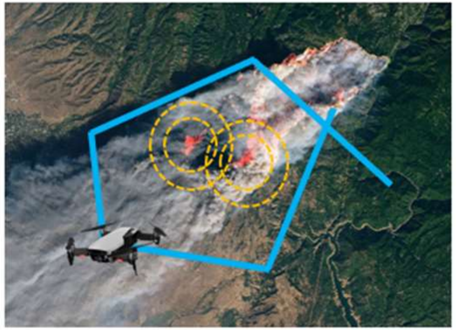
J. Murray-Bruce and P.L. Dragotti, A Sampling Framework for Solving Physics-driven Inverse Source Problems, IEEE Trans. on Signal Processing, Vol. 65(24), pp. 6365-6380, December 2017

Sparse Sampling – New Challenges

- Current sensing methods are energy inefficient especially when low-latency is needed (e.g., commercial ultra-fast cameras)



- Often sampling happens at unknown locations (e.g, unknown trajectories, unknown projections)



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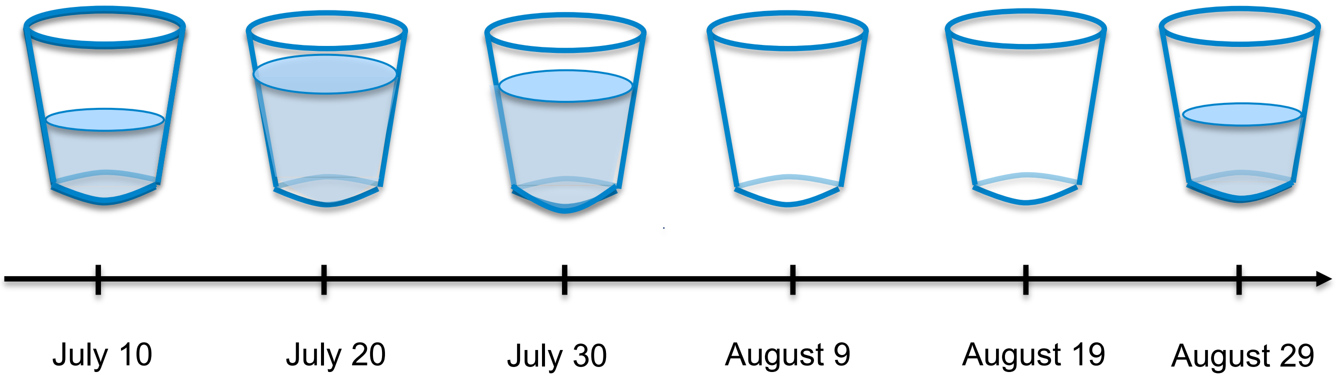
Bio-Inspired Energy Efficient Sensing

- Current sensing methods are energy inefficient especially when low-latency is needed.
- Example: Rainfall estimation



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Bio-Inspired Energy Efficient Sensing

Approach 2

- Only record the day when the bucket is full and then empty it

Bio-Inspired Energy Efficient Sensing

Approach 2 maps analogue information into a time sequence and is used by nature (e.g., integrate-and-fire neurons)

Time encoding appears in nature, as a mechanism used by neurons to represent sensory information as a sequence of action potentials, allowing them to process information **very efficiently**.

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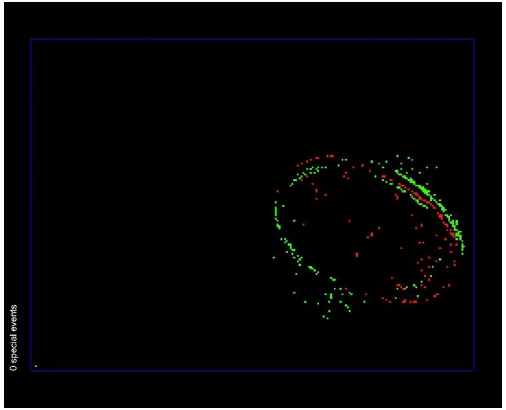
Sensing based on Timing Information

- Energy-efficient sensing inspired by nature raises a fundamental representation question:
  - How can we embed information related to complex signals into the timing information of spikes?
  - Besides its theoretical implications, addressing this question will lead to new neuromorphic sensing devices



Video taken from Inivation.com

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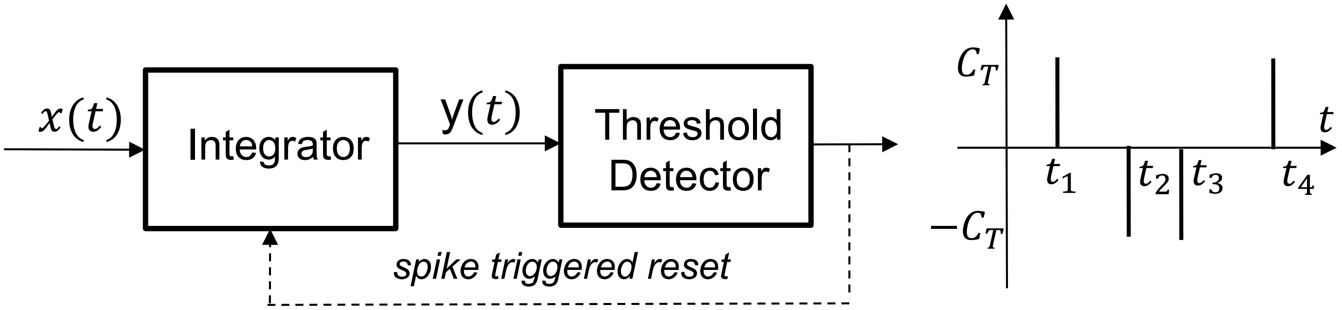
Time-Encoding Machines

Integrate-and-fire System



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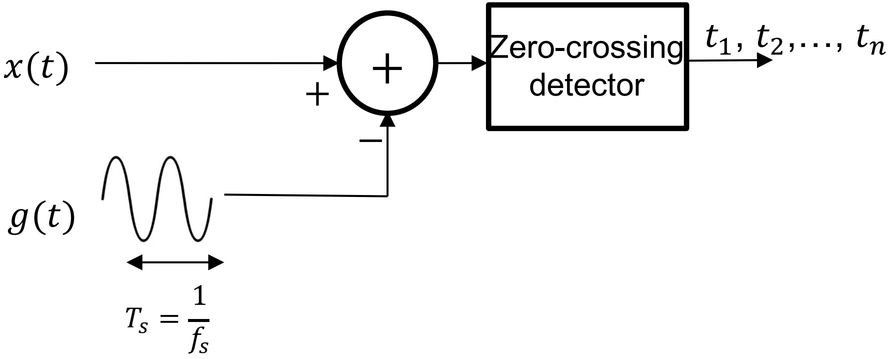
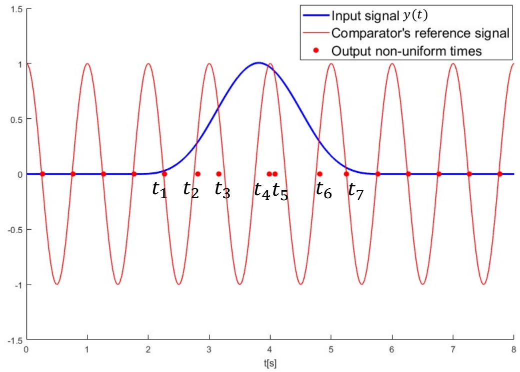
Time-Encoding Machines

Comparator System





- At the crossing times, $x(t_n) - g(t_n) = 0$ hence $x(t_n) = g(t_n)$.

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Reconstruction from time-encoded information

- Given the retrieved non-uniform samples $x(t_1), x(t_2), \dots, x(t_n)$ can we reconstruct $x(t)$?

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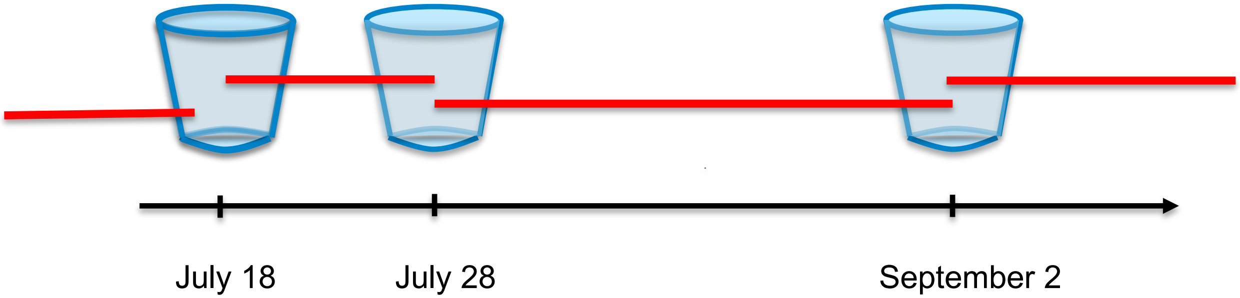
Reconstruction from time-encoded information

- **Key result**:[2] if the density of samples $\mathrm{D} \geq 1$ then perfect reconstruction can be using an iterative approach proposed by Aldroubi and Grochenig[1]



[2] A. Aldroubi and K, Grochenig, "Non-Uniform Sampling and Reconstruction in shift-invariant spaces" SIAM Review 2001

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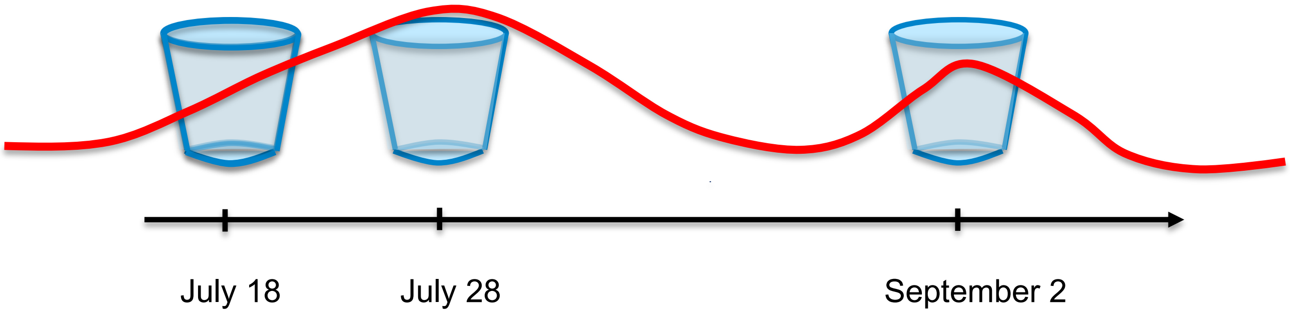

[2]A. Aldroubi and K, Grochenig, "Non-Uniform Sampling and Reconstruction in shift-invariant spaces" SIAM Review 2001

Reconstruction from time-encoded information

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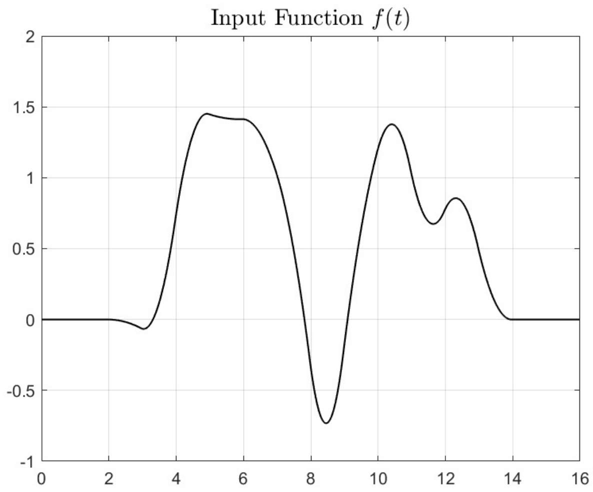
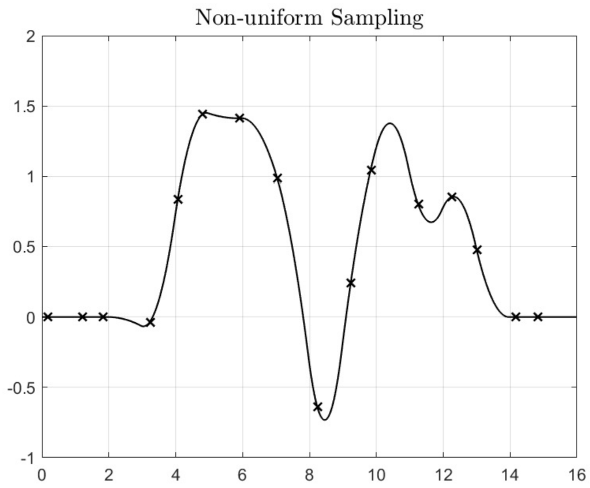
- The iterative approach proposed by Aldroubi and Grochenig



Reconstruction from time-encoded information

- The iterative approach proposed by Aldroubi and Grochenig



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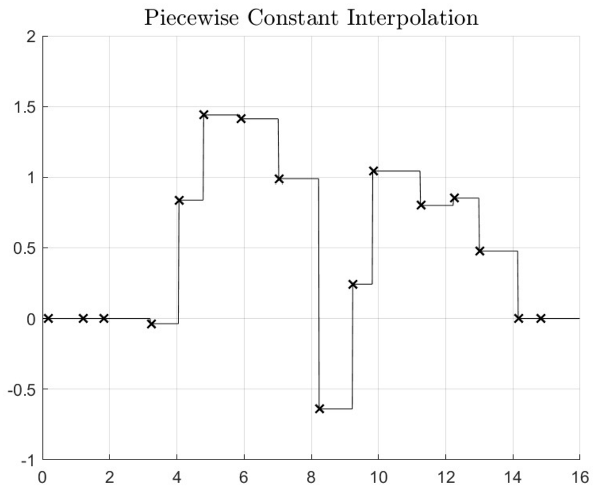

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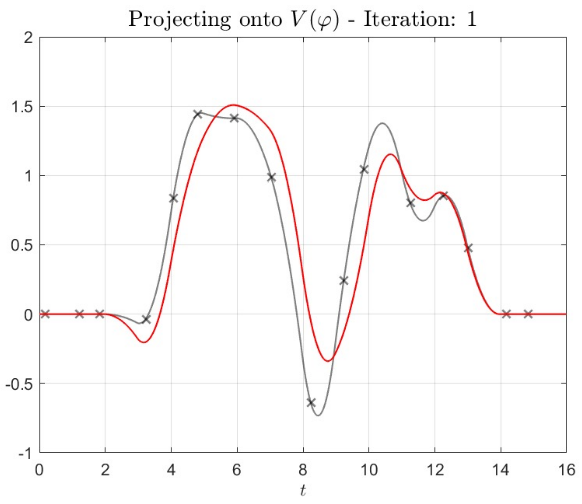

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Reconstruction from time-encoded information

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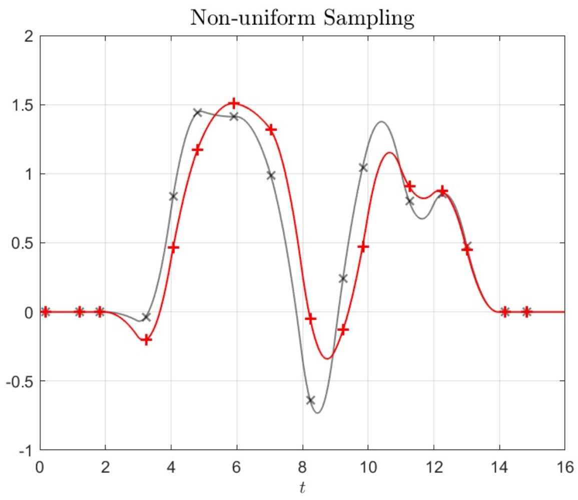

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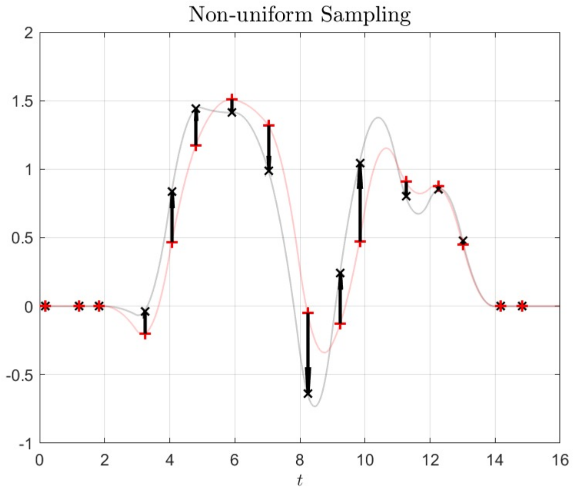

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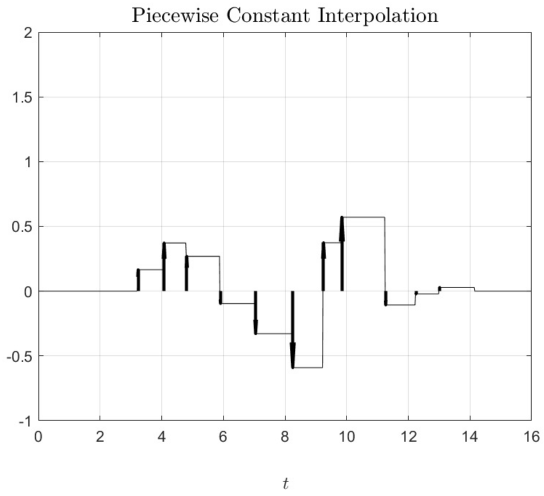

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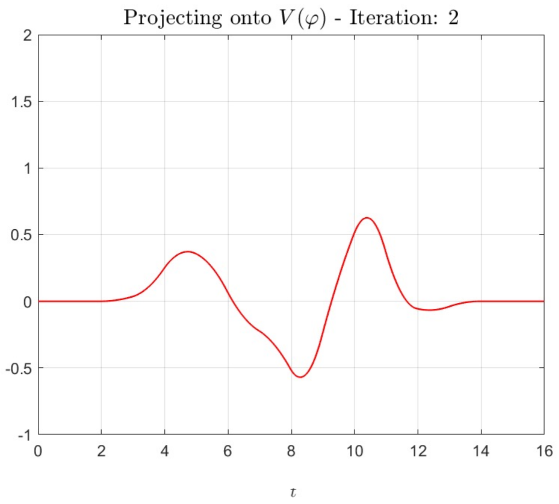

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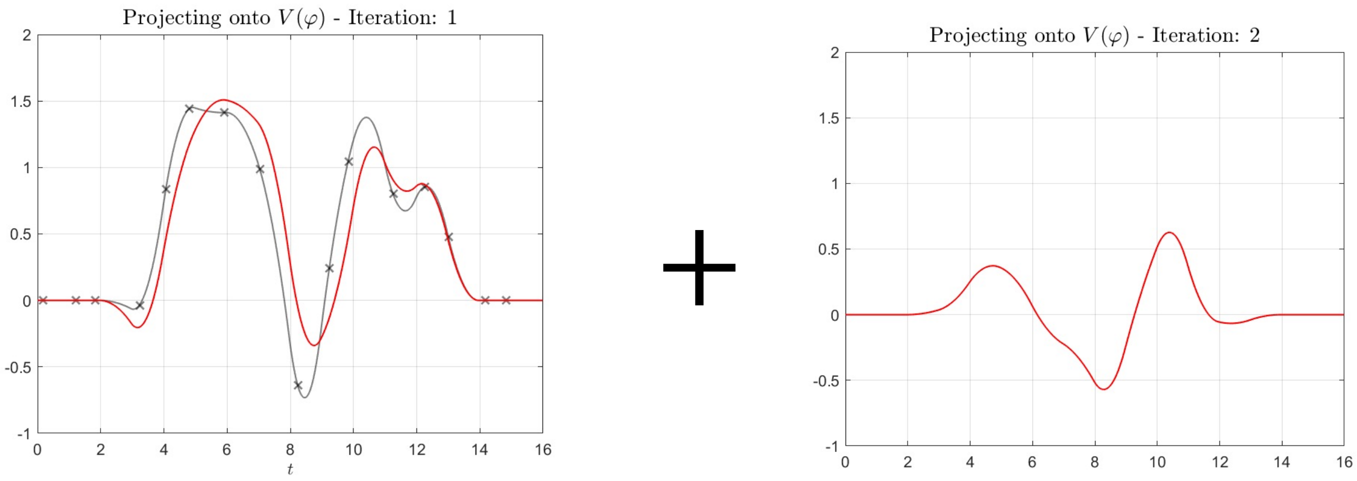

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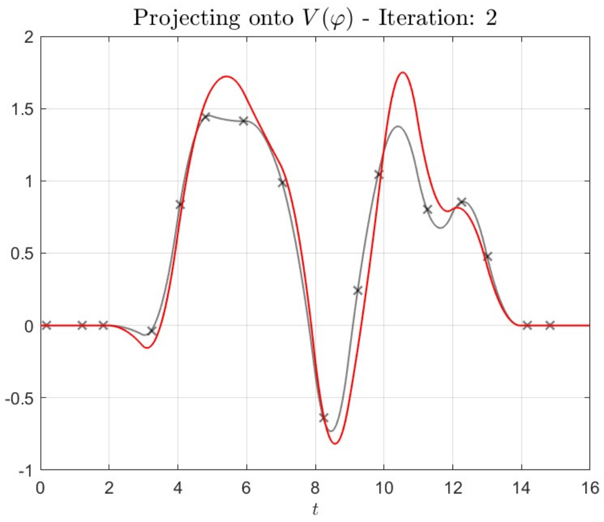

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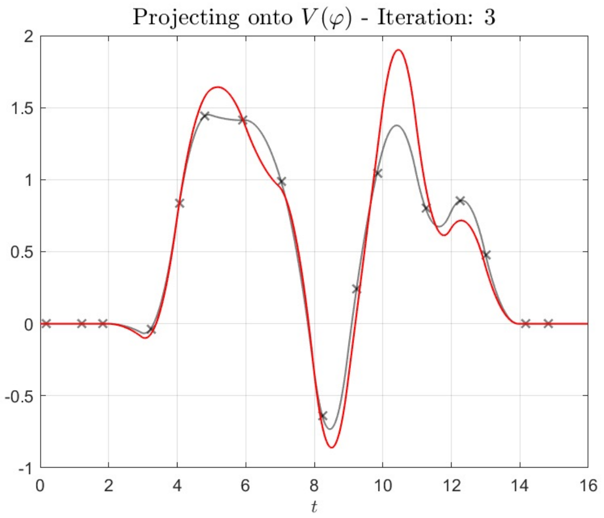

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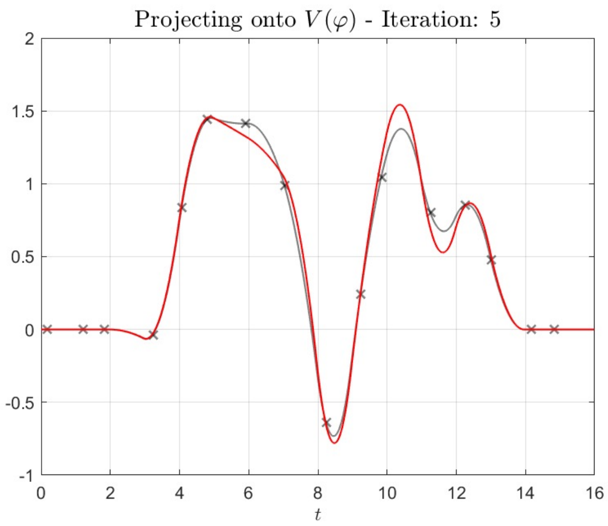
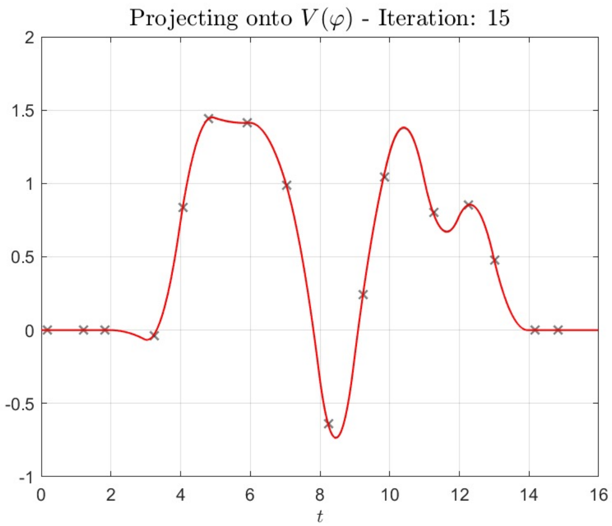


Reconstruction from time-encoded information

- The iterative approach proposed by Aldroubi and Grochenig



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Reconstruction from time-encoded information

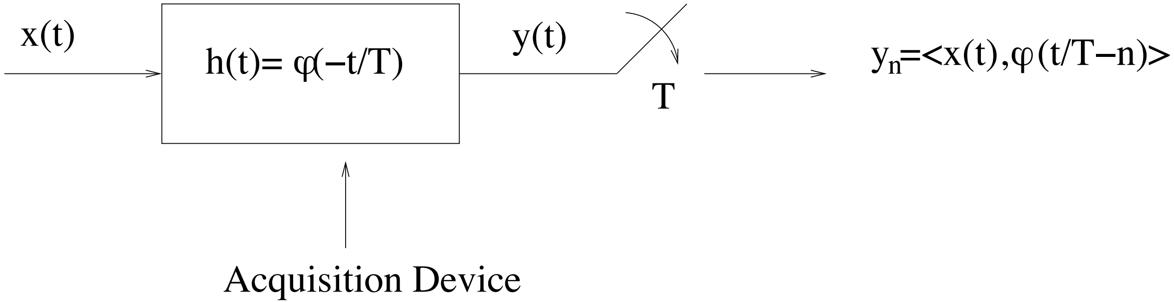
- **Key result**: if the density of samples $\mathrm{D} \geq 1$ then $K_{t_j}(t)$ form a basis

- **Key Issue 1**: In the case of uniform sampling the density is $D = 1$. This means that current TEMs are **less** energy efficient than uniform sampling!

- **Key Issue 2:** Cannot sample sparse (non-bandlimited) signals with the current methods.

Time-based Sampling of Sparse Signals

- We leverage two main ideas from sparse sampling:
  - The sampling kernels can reproduce exponentials
  - Reconstruction is achieved using Prony's method



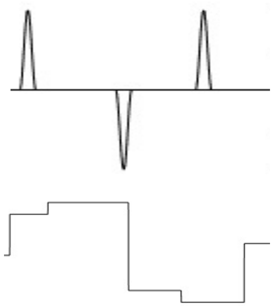
Time-based Sampling of Sparse Signals

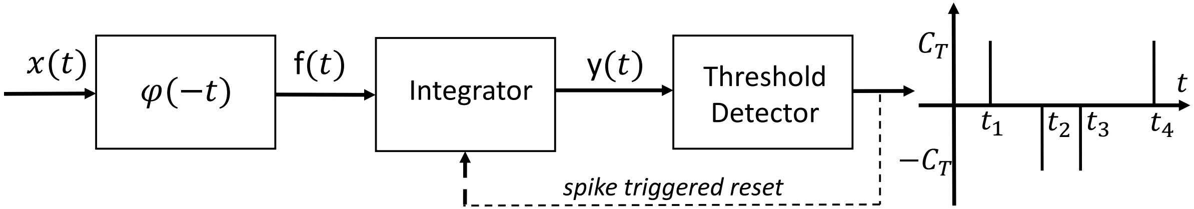
Signals:

- We consider sparse continuous-time signals like stream of pulses, piecewise constant or regular signals

Sensing Systems:

- We filter before using a TEM



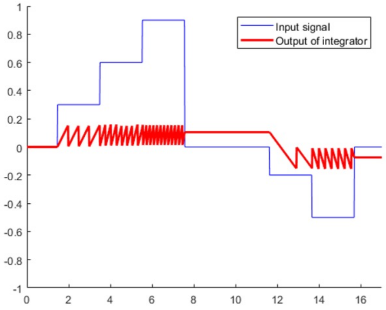


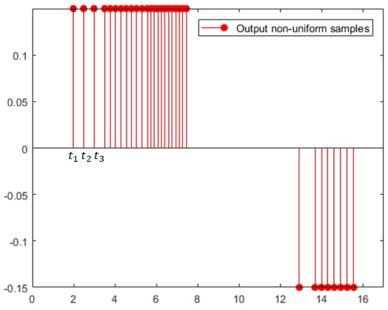
$$y(t_n) = \langle x(t), \varphi_n(t - t_n) \rangle$$

Our approach for time decoding of signals

- Reconstruction of $x(t)$ depends on the
  – sampling kernel $\varphi(t)$
  – the density of time instants $\{t_n\}$

- We achieve a sufficient density of output samples by imposing conditions on:
  – The trigger mark of the integrator (**integrate-and-fire TEM**).





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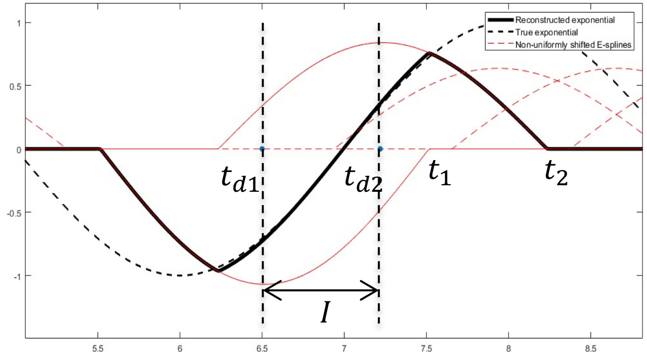
Reproduction of Exponentials

- **Key Insight**: Reproduction of exponentials can be achieved locally in I, using at least two non-uniform shifts of the kernel:

$$\sum_{n=1}^{N} c_{m,n}\varphi(t-t_n) = e^{-\alpha_m t}, N \geq 2$$

- The kernels should be continuous within that local interval I.



t_{d1} - discontinuity of $\varphi(t-t_1)$

t_{d2} - discontinuity of $\varphi(t-t_2)$

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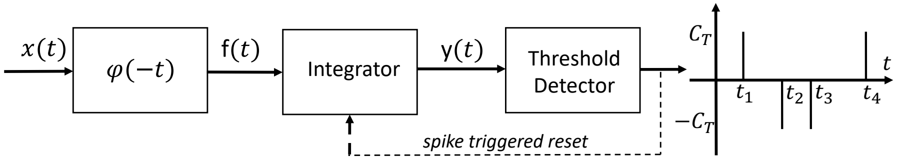
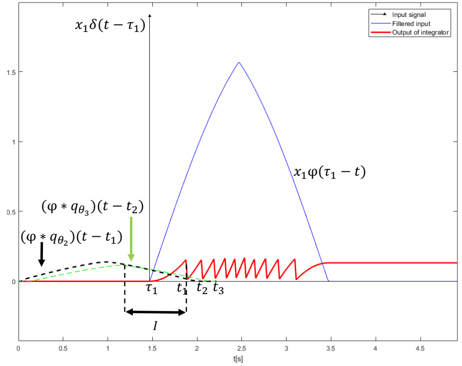
Integrate and Fire TEM





- The sampling kernel $\varphi(t)$ and its non-uniform shifts reproduce $e^{j\omega_0 t}$ and $e^{-j\omega_0 t}$ and $0 < \omega_0 \le \frac{\pi}{L}$ where L is the support of $\varphi(t)$.

- What is the minimum value of the trigger mark C_T that would allow the perfect reconstruction of stream of pulses or piecewise constant signals?

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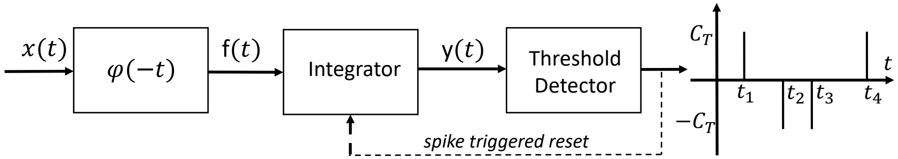
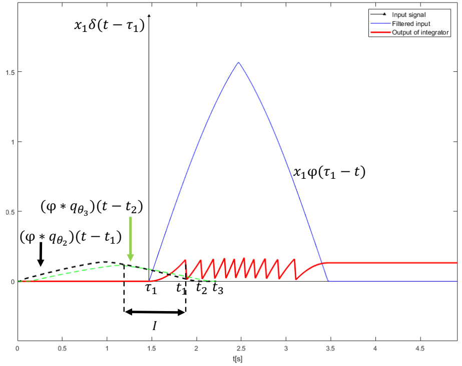
Integrate and Fire TEM





- Trigger mark must guarantee enough samples (three samples) in a short interval
- *Proposition:* when $C_T < \frac{A_{min}}{4\omega_0^2}\left(1-\cos\left(\frac{\omega_0 L}{2}\right)\right)$ then $t_1, t_2, t_3 \in \left[\tau_1, \tau_1 + \frac{L}{2}\right]$ and perfect reconstruction is possible

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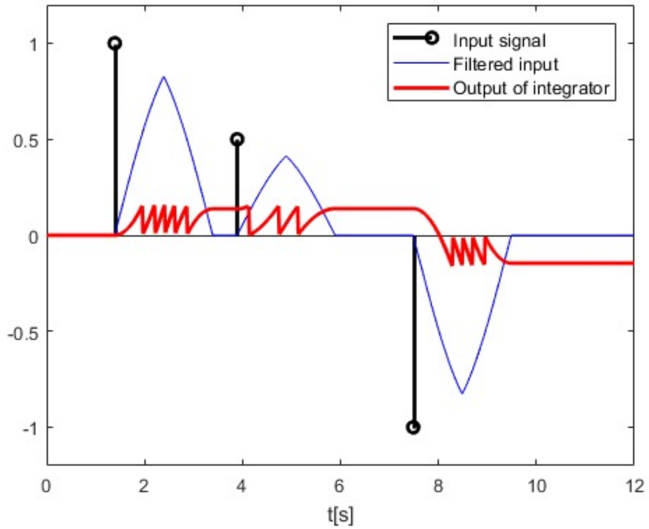
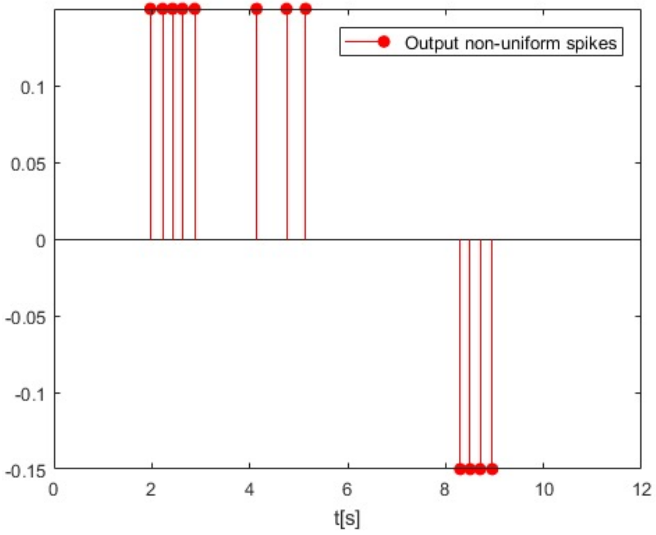
Integrate and Fire – Reconstruction of Pulses





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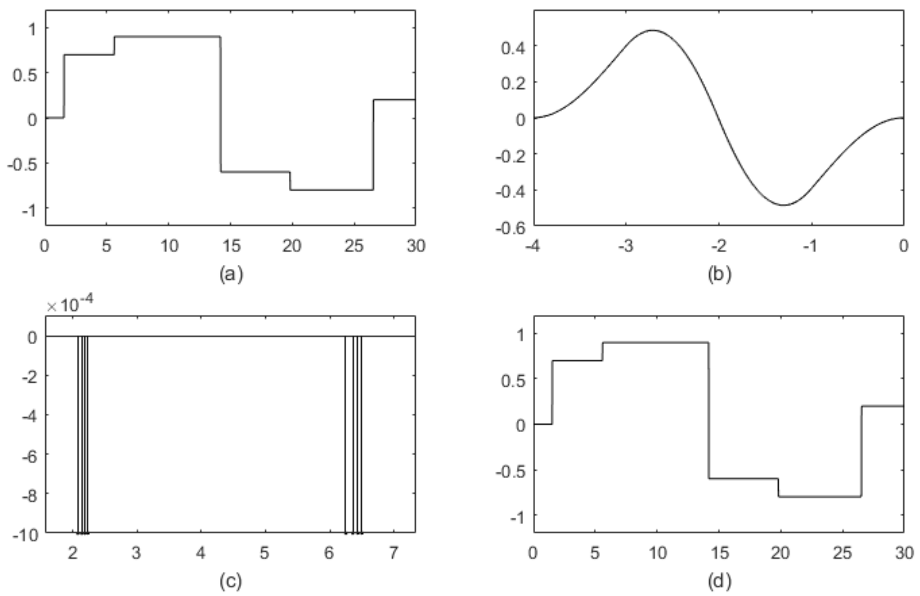
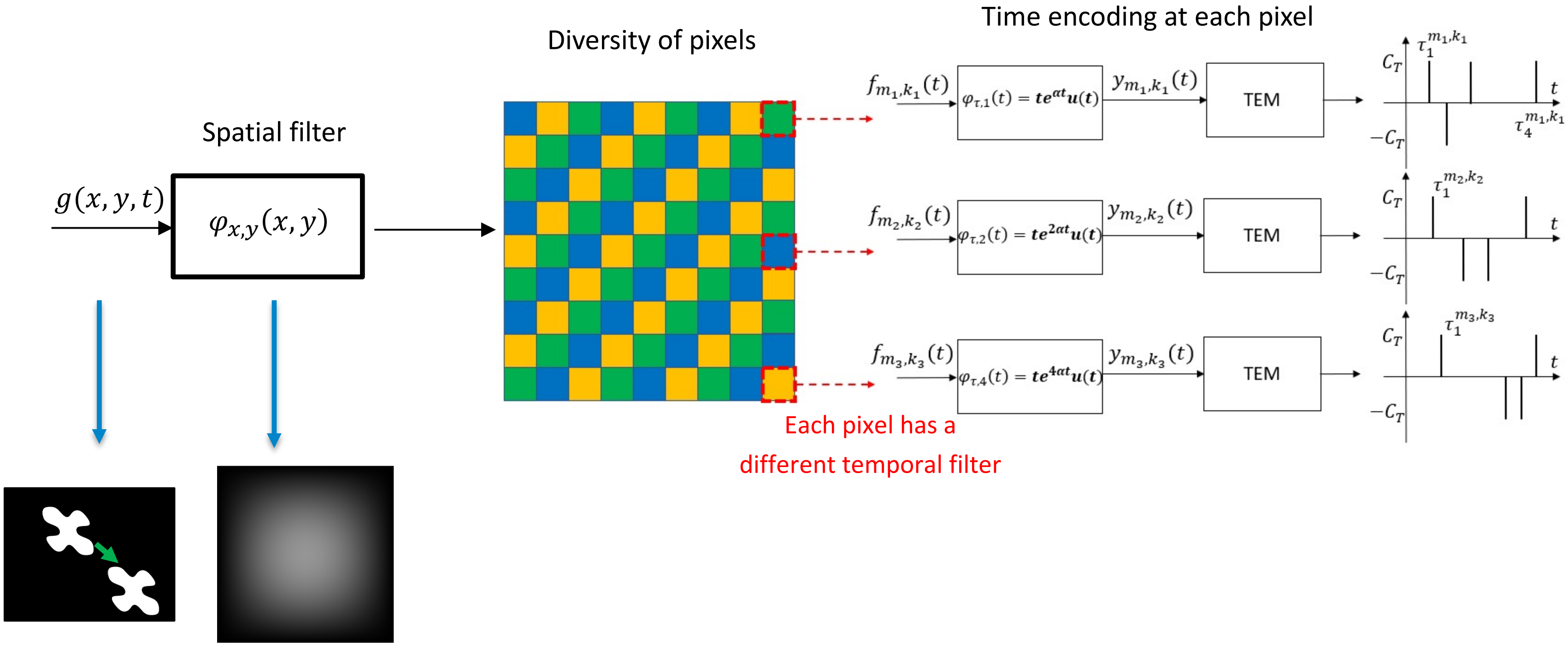
Energy Efficient Sampling -Results



If the distance S between discontinuities is on average $S > (L-1)T$ with T being the sampling period in uniform sparse sampling then the new time encoding framework[3] is **more efficient** than sparse sampling (lower sampling density

[3]R. Alexandru and P.L. Dragotti, Reconstructing Classes of Non-bandlimited Signals from Time Encoded Information, IEEE Trans. on Signal Processing, vol.68, 2020.

Integrate and Fire and Neuromorphic Cameras



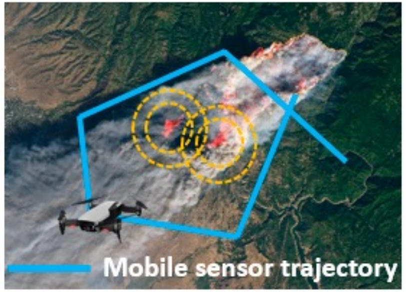
Integrate and Fire and Neuromorphic Cameras

Sampling Diffusion Fields along Trajectories


(a)


(b)


(c)



(d)

The problem we consider is localising diffusion sources and estimating the trajectory of the mobile sensor, from samples taken along unknown trajectories.

This is similar to the classic SLAM problem in computer vision, but is now driven by the physics of the field (Diffusion-SLAM)

Sampling Diffusion Fields along Trajectories

- The problem of sampling at unknown location is not new, e.g, [Browning TSP 2007]
- The problem can be combinatorial
- The solution is normally not unique
- Many applications: cryo-EM, SLAM
- In our case (Diffusion-SLAM)
    - The problem is sufficiently constrained to admits an algebraic solution
    - Solution is unique up to a rigid rotation

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Diffusion Fields

The diffusion field generate by a source k will propagate according to the Green's function, as follows:

$$f_k(\boldsymbol{x}, t) = \frac{1}{4\pi\mu(t - \tau_k)} a_k e^{-\frac{|\boldsymbol{x}-\boldsymbol{S_k}|^2}{4\mu(t-\tau_k)}} H(t - \tau_k)$$

where:

$\boldsymbol{x}$ → location of measurement $f_k(\boldsymbol{x}, t)$

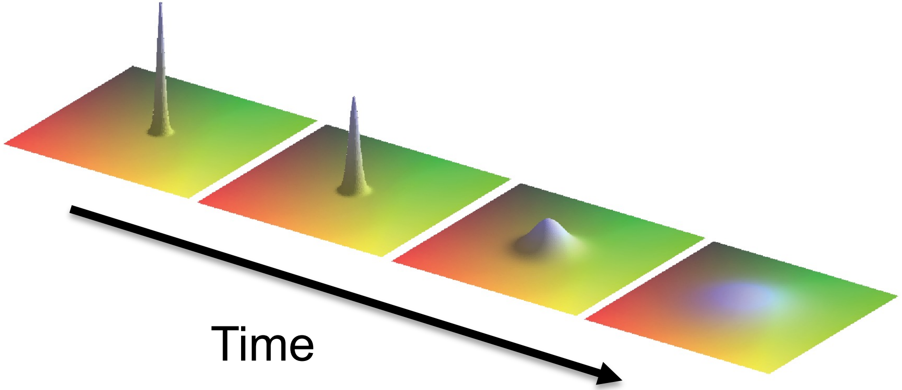
a_k → amplitude of the diffusion source

τ_k → activation time of the diffusion source

$\boldsymbol{S_k}$ → coordinates of source k in $\mathbb{R}_2$ space

$H(t)$ → unit step function

μ → diffusivity of the medium



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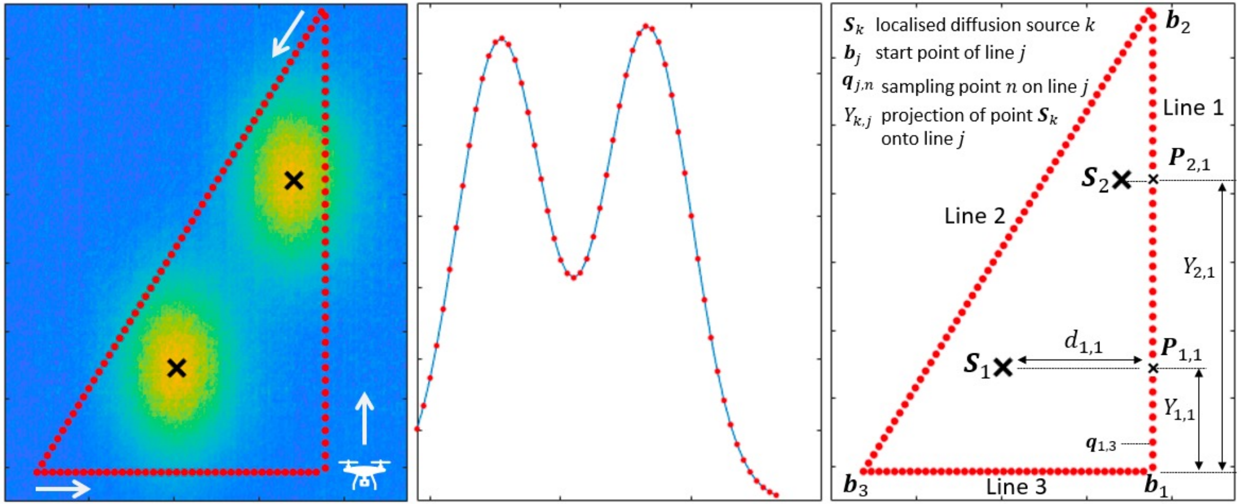
Problem Set-up

Given spatial measurements of the diffusion field the aim is to:

1. Estimate the locations and amplitudes of the sources
2. Reconstruct the trajectory of the mobile sensor



(a) (b) (c)

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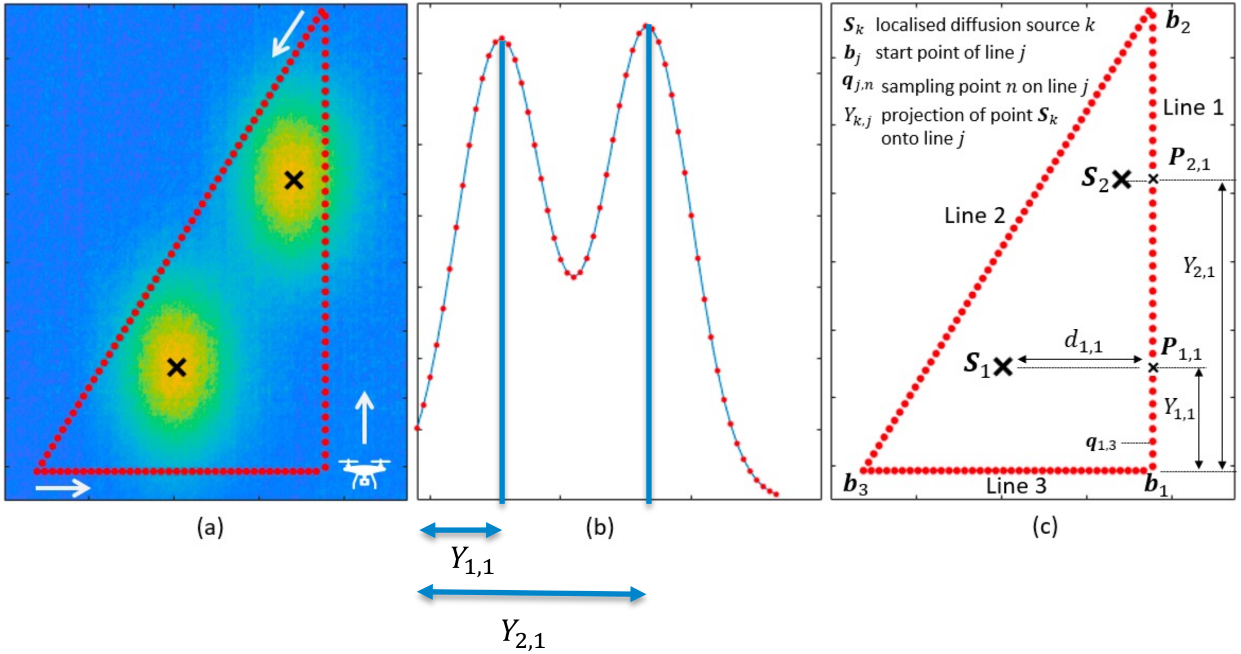
Problem Set-up

The problem is *sparse*

1. K sources to estimate
2. Trajectory is piecewise linear so only the vertices of each line needs to be estimated
3. The projection of the field on each line is an FRI signal (stream of pulses)
4. The location of each pulse can be estimated using sparse sampling theory



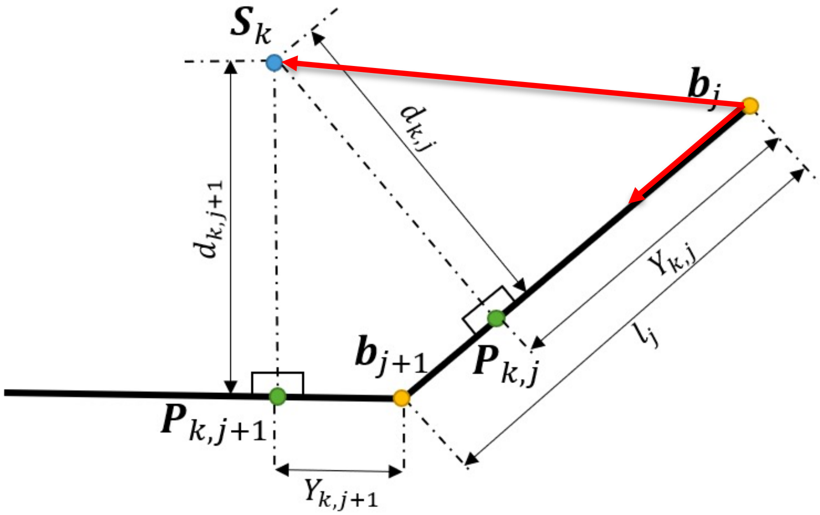
(a) (b) (c)

Estimation using Matrix Factorization

- The slope of the line is $\boldsymbol{c}_j = \frac{\boldsymbol{b}_{j+1} - \boldsymbol{b}_j}{l_j}$
- So $Y_{k,j} = \left(\boldsymbol{b}_j - \boldsymbol{S}_k\right)^T \boldsymbol{c}_j$
- We define $\Omega_{k,j} = Y_{k+1,j} - Y_{k,j} = (\boldsymbol{S}_{k+1} - \boldsymbol{S}_k)^T \boldsymbol{c}_j$

$$\boldsymbol{\Omega} = \begin{bmatrix} (\mathbf{S}_2 - \mathbf{S}_1)^T \mathbf{c}_1 & \dots & (\mathbf{S}_2 - \mathbf{S}_1)^T \mathbf{c}_L \\ (\mathbf{S}_3 - \mathbf{S}_2)^T \mathbf{c}_1 & \dots & (\mathbf{S}_3 - \mathbf{S}_2)^T \mathbf{c}_L \\ \vdots & \ddots & \vdots \\ (\mathbf{S}_\mathbf{k} - \mathbf{S}_{K-1})^T \mathbf{c}_1 & \dots & (\mathbf{S}_\mathbf{k} - \mathbf{S}_{K-1})^T \mathbf{c}_L \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} (\mathbf{S}_2 - \mathbf{S}_1)^T \\ (\mathbf{S}_3 - \mathbf{S}_2)^T \\ \vdots \\ (\mathbf{S}_\mathbf{k} - \mathbf{S}_{K-1})^T \end{bmatrix}}_{(K-1)\times 2} \underbrace{\begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_2 & \dots & \mathbf{c}_L \end{bmatrix}}_{2\times L} := \mathbf{SC}.$$

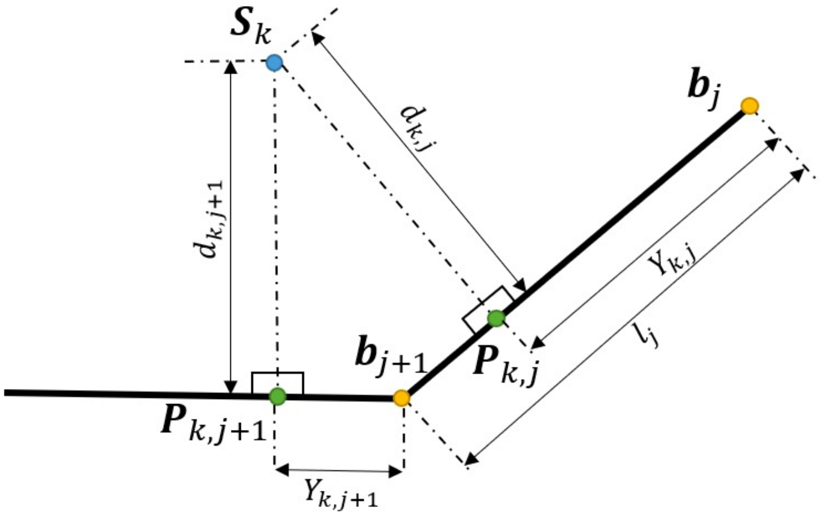


Estimation using Matrix Factorization

- We factorize $\mathbf{\Omega}$ using the SVD
- Given $\boldsymbol{C}$ we find the trajectory
- We then find the locations of the sources

$$\mathbf{\Omega} = \begin{bmatrix} (\mathbf{S}_2 - \mathbf{S}_1)^T \mathbf{c}_1 & \dots & (\mathbf{S}_2 - \mathbf{S}_1)^T \mathbf{c}_L \\ (\mathbf{S}_3 - \mathbf{S}_2)^T \mathbf{c}_1 & \dots & (\mathbf{S}_3 - \mathbf{S}_2)^T \mathbf{c}_L \\ \vdots & \ddots & \vdots \\ (\mathbf{S_k} - \mathbf{S}_{K-1})^T \mathbf{c}_1 & \dots & (\mathbf{S_k} - \mathbf{S}_{K-1})^T \mathbf{c}_L \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} (\mathbf{S}_2 - \mathbf{S}_1)^T \\ (\mathbf{S}_3 - \mathbf{S}_2)^T \\ \vdots \\ (\mathbf{S_k} - \mathbf{S}_{K-1})^T \end{bmatrix}}_{(K-1)\times 2} \underbrace{\begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_2 & \dots & \mathbf{c}_L \end{bmatrix}}_{2\times L} := \mathbf{SC}.$$



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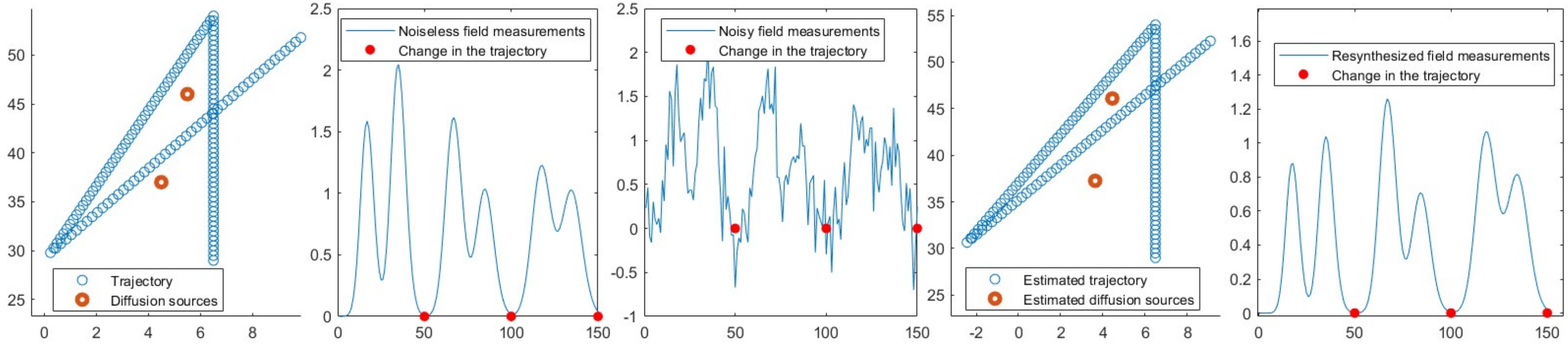
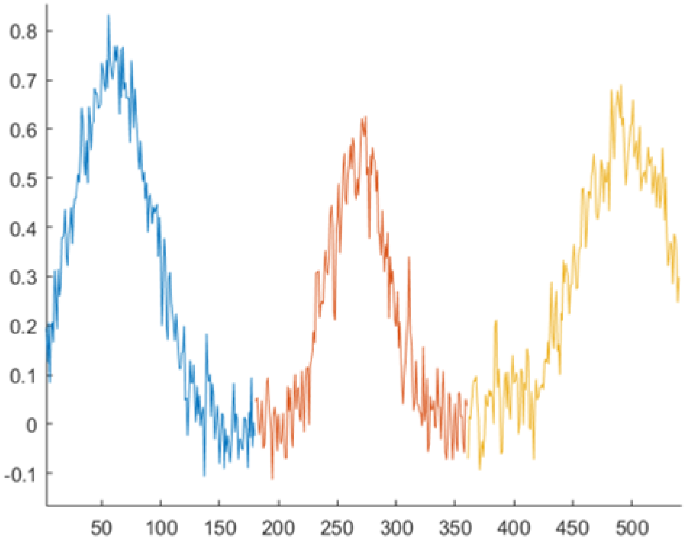
Results

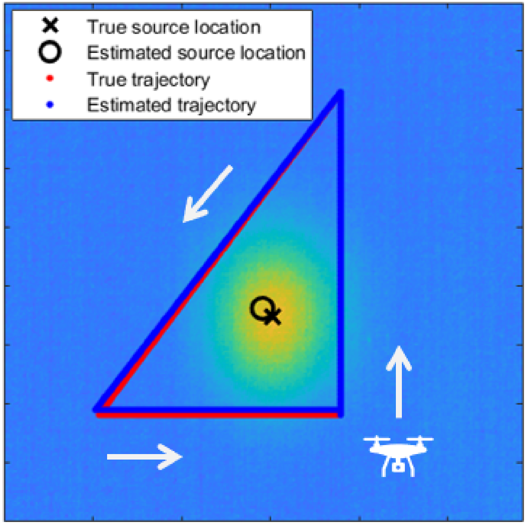


R. Alexandru, T. Blu and P.L. Dragotti, "D-SLAM: Diffusion Source Localization and Trajectory Mapping," IEEE International Conference on Acoustics, Speech and Signal Processing, Barcelona, Spain, 2019.

Results



(a)



(b)

R. Alexandru, T. Blu and P.L. Dragotti, "D-SLAM: Diffusion Source Localization and Trajectory Mapping," IEEE International Conference on Acoustics, Speech and Signal Processing, Barcelona, Spain, 2019.

Conclusions

Sampling is alive and well 😁

- In sampling, sparsity is king
- In many real-life problems:
  - We always need better resolution (e.g. neuroscience)
  - We often sample at unknown locations (SLAM, Cryo-EM)
  - Sampling might be irregular (e.g., along trajectories)
  - Time-based sampling is still un-explored but essential for fast energy efficient devices

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Thank you!

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Application in Neuroscience

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- John Murray-Bruce and Pier Luigi Dragotti, Estimating localized sources of diffusion fields using spatiotemporal sensor measurements, IEEE Trans. on Signal Processing, June 2015.