Widely Linear Adaptive Frequency Estimation for Unbalanced Three-Phase Power Systems with Multiple Noisy Measurements

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***Abstract*—We address the problem of adaptive frequency estimation of unbalanced three-phase power systems in practical situations when the voltage samples are contaminated with measurement noise. The complex-valued -transformed voltages are used in order to utilise all the available information in the three-phase reference channels, at the expense of modest addition in computational complexity. A widely linear predictive model is established over multiple noisy voltage measurements to cater for the system unbalance conditions, which are manifested in noncircular empirical distributions. To obtain frequency estimates in an adaptive manner, the total least-squares fitting and gradient descent optimisation techniques are adopted based on the augmented complex statistics. The so introduced augmented complex total least mean square (ACTLMS) algorithm is shown to enable, by design, more reliable frequency estimates over its augmented complex least mean square (ACLMS) counterpart. The ACTLMS is also shown to provide the user with a choice in the degrees of freedom to control the trade-off between tracking speed and estimation accuracy. Simulations on both synthetic and real-world noisy unbalanced power systems support the analysis.**

***Index Terms*—Unbalanced three-phase power systems, multiple noisy measurements, adaptive frequency estimation, widely linear model, augmented complex total least mean squares (ACTLMS)**

I. Introduction

The frequency of a power system is a crucial power quality parameter and is allowed to vary around its nominal value only within a prescribed tolerance level. Even a small deviation from the nominal value of an electric power system is an indicator of the generation-consumption mismatch in power grids. Therefore, fast and precise frequency tracking and estimation is of prime importance in power system analysis, by enabling to monitor the health state of the power grid and to assure reliable measurement of other system parameters such as voltages and currents [1]–[6]. The importance of frequency estmation becomes even more pronounced in the context of smart grid, where frequent switching from the main grid to microgrids, and electricity islands, together with dual natures of some loads, such as plug in hybrid electric vehicles (PHEVs), all trigger imbalance in the power generation-load chain, and hence frequency variations [7].

Since in three-phase power systems, none of the single phases can faithfully characterise the whole system and its properties, a robust frequency estimator should take into account the information of all three phases; this would enable a unified estimation of system frequency as a whole and provide enhanced robustness. To this end, Clarke's $\alpha\beta$ transformation has been introduced to construct a complex-valued signal with the information provided by all the three-phase voltages in a simultaneous way, and has equipped the classical single phase methods with more robustness in characterising system frequency [8]. Most complex-valued frequency estimation algorithms applied to the $\alpha\beta$-transformed voltage work well under normal balanced power system conditions [8]–[10]. However, balanced system conditions are likely to be violated in practice. A major cause of voltage unbalance is the uneven distribution of single-phase loads. This can happen, for example, in rural electric power systems with long distribution lines, as well as in large urban power systems where heavy single-phase demands, such as lighting loads, are imposed by large commercial facilities. Additional causes of power system voltage unbalance can be asymmetrical transformer winding impedances and asymmetrical transmission impedances. Such unbalanced system problems give rise to the so called negative sequence, a complex exponential rotating clockwise at the system frequency, within the $\alpha\beta$-transformed voltage [11], [12]. Due to the explicit or implicit omission of the negative sequence component, standard phase angle calculation techniques employed by the conventional complex-valued frequency estimators encounter unavoidable estimation bias and oscillations, resulting in incorrect frequency estimates and perhaps false alarm spread through the system, although the system frequency was indeed in its nominal range [13].

Our earlier work showed that this complex-valued $\alpha\beta$-transformed voltage under unbalanced system conditions admits a first-order widely linear autoregressive (AR) predictive model in the time domain [7], [14]. This observation enables us to exploit the noncircular statistical nature of the $\alpha\beta$-transformed voltage and to extend standard strictly linear model based frequency estimation algorithms into the more general widely linear form. This makes it possible to take into account the frequency information contained both in the positive sequence component and the negative sequence component and to achieve unbiased frequency estimates. Examples include the augmented (widely linear) complex least mean square (ACLMS), augmented minimum variance distortionless response (AMVDR) spectrum, and the augmented Kalman filtering based adaptive frequency estimators [14]–[17].

By considering the practical situations, where several kinds of error would contaminate the measurements, e.g., sampling, quantisation, and instrument errors, in this paper, we first extend the first order widely linear predictive model to cater for multiple noisy voltage measurements. Next, to achieve frequency estimates in an adaptive manner, the total least-squares fitting and gradient descent optimisation techniques are employed. By virtue of such a combination of enhanced memory and advanced learning strategies, the so introduced augmented complex total least mean square (ACTLMS) algorithm by design enables more reliable frequency estimates than its scalar-based version [18] and its noise-free counterpart, the augmented CLMS (ACLMS) algorithm [14]. Simulations on both synthetic and real-world noisy unbalanced power systems support the analysis.

II. Unbalanced Three-Phase Power System

The three-phase voltages of a power system in a noise-free environment can be represented in a discrete time form as

$$\begin{aligned} v_a(k) &= V_a \cos(k\omega \triangle \mathrm{T} + \phi) \\ v_b(k) &= V_b \cos(k\omega \triangle \mathrm{T} + \phi - \frac{2\pi}{3}) \\ v_c(k) &= V_c \cos(k\omega \triangle \mathrm{T} + \phi + \frac{2\pi}{3}) \end{aligned} \tag{1}$$

where V_a, V_b, V_c are the peak values of each fundamental voltage component at time instant k, $\Delta T = 1/f_s$ is the sampling interval,where f_s is the sampling frequency, ϕ is the initial phase, and $\omega = 2\pi f$ is angular frequency of the voltage signal, with f being the system frequency. Although the frequency of the system can be estimated directly from any one of the three-phases, utilising the information from all three phases gives more robust frequency estimates [8]. To achieve this with modest complexity, the dimensionality of the signal is first reduced from $\mathbb{R}^3$ to $\mathbb{C}$ via the following $\alpha\beta$ transformation [19]

$$\begin{bmatrix} v_\alpha(k) \\ v_\beta(k) \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_a(k) \\ v_b(k) \\ v_c(k) \end{bmatrix} \tag{2}$$

where the v_α and v_β parts are used to form the complex-valued voltage $v(k)$, i.e., $v(k) = v_\alpha(k) + jv_\beta(k)$. When the three-phase power system deviates from its nominal condition, such as in the presence of unequal levels of voltage magnitude variations along the three phases, or the phase difference between any two phases deviates from the nominal $2\pi/3$, the $\alpha\beta$ transformed voltage $v(k)$ becomes

$$v(k) = Ae^{jk\omega\Delta T} + Be^{-jk\omega\Delta T} \tag{3}$$

where A and B are complex-valued, for which the detailed expressions can be found in [7], [14].

III. Proposed Augmented Complex Total Least Mean Square (ACTLMS) Algorithm

In a noiseless setting, the $\alpha\beta$-transformed voltage $v(k)$ of the unbalanced power system in (3), obeys a first order widely-linear AR predictive model, given by [7], [14], [16]

$$v(k)h^\circ + v^*(k)g^\circ = v(k+1) \tag{4}$$

where h° and g° are the optimal filter weight coefficients, and $(\cdot)^*$ is the complex conjugate operator. The values of both h° and g° depend on a system imbalance ratio B^*/A, and a phasor $e^{j\omega\Delta T}$ which contains the instantaneous frequency information. The widely linear nature of $v(k)$ requires adaptive frequency estimation algorithms, e.g., the ACLMS and augmented Kalman filtering algorithms [14], [16], which use a widely linear estimation model in order to achieve statistically unbiased estimates, given by [20], [21]

$$v(k)h(k) + v^*(k)g(k) \approx v(k+1) \tag{5}$$

where $h(k)$ and $g(k)$ are respectively the standard and conjugate filter weights used to track h° and g° in (4) in an adaptive manner. By taking into the consideration that in realistic situations, the noiseless voltage observations are almost never available since several kinds of imperfections, e.g., sampling, quantisation, and instrument errors, would contaminate the measurements, noisy voltage measurements should be used within the underlying widely linear estimation model, to give [18]

$$\tilde{v}(k)h(k) + \tilde{v}^*(k)g(k) \approx \tilde{v}(k+1) \tag{6}$$

where $\tilde{v}(k) = v(k) + \triangle v(k)$ and the perturbation term $\triangle v(k)$ represents the measurement imperfections. By further assuming that the system parameters are time-invariant within a short sliding window consisting of N consecutive voltage measurements, the underlying widely linear relationship in (6) still stands, and generalises to [22]

$$\begin{bmatrix} \tilde{v}(k) \\ \tilde{v}(k-1) \\ \vdots \\ \tilde{v}(k-N+1) \end{bmatrix} h(k) + \begin{bmatrix} \tilde{v}^*(k) \\ \tilde{v}^*(k-1) \\ \vdots \\ \tilde{v}^*(k-N+1) \end{bmatrix} g(k) \approx \begin{bmatrix} \tilde{v}(k+1) \\ \tilde{v}(k) \\ \vdots \\ \tilde{v}(k-N+2) \end{bmatrix}$$

which can be written in a compact vectorial form as

$$\tilde{\mathbf{v}}(k)h(k) + \tilde{\mathbf{v}}^*(k)g(k) \approx \tilde{\mathbf{v}}(k+1) \tag{7}$$

where $\tilde{\mathbf{v}}(k) = [\tilde{v}(k), \tilde{v}(k-1), \ldots, \tilde{v}(k-N+1)]^T$. The main motivation for the use a sliding window technique here is in that it provides enhanced robustness to interferences as compared to its scalar version in (6). It also provides a user-defined degree of freedom to compromise between tracking speed and estimation accuracy within the associated adaptive filtering algorithms [23]. It is well known that by defining an augmented data matrix, $\tilde{\mathbf{Z}}(k)$, and an augmented weight vector, $\mathbf{w}(k)$, as

$$\tilde{\mathbf{Z}}(k) = [\tilde{\mathbf{v}}(k), \tilde{\mathbf{v}}^*(k), \tilde{\mathbf{v}}(k+1)]^H \tag{8}$$

$$\mathbf{w}(k) = [h(k), g(k), -1]^T \tag{9}$$

and applying the singular value decomposition (SVD) on $\tilde{\mathbf{Z}}^H(k)$ gives the optimal total least squares (TLS) solution for (7), given by [24]

$$\mathbf{w}(k) = -\frac{[u_1, u_2, u_3]^T}{u_3} \tag{10}$$

where $[u_1, u_2, u_3]^T$ is the right singular vector corresponding to the smallest singular value of the augmented data matrix $\tilde{\mathbf{Z}}^H(k)$ or the eigenvector corresponding to the smallest eigenvalue of the matrix

$$\tilde{\mathbf{R}}(k) = \tilde{\mathbf{Z}}(k)\tilde{\mathbf{Z}}^H(k) \tag{11}$$

Following the analysis in [25], a more computationally efficient alternative to the TLS solution can be obtained by minimising the following Rayleigh quotient over $\mathbf{w}(k)$, given by

$$\begin{aligned} J(\mathbf{w}(k)) &= \frac{\mathbf{w}^H(k)\tilde{\mathbf{R}}(k)\mathbf{w}(k)}{\|\mathbf{w}(k)\|^2} \\ &= \frac{\mathbf{w}^H(k)\tilde{\mathbf{Z}}(k)\tilde{\mathbf{Z}}^H(k)\mathbf{w}(k)}{\|\mathbf{w}(k)\|^2} \\ &= \frac{\|\mathbf{e}(k)\|^2}{\|\mathbf{w}(k)\|^2} \end{aligned} \tag{12}$$

where

$$\begin{aligned} \mathbf{e}(k) &= \tilde{\mathbf{Z}}^H(k)\mathbf{w}(k) \\ &= \tilde{\mathbf{v}}(k)h(k) + \tilde{\mathbf{v}}^*(k)g(k) - \tilde{\mathbf{v}}(k+1) \end{aligned} \tag{13}$$

is the output error vector according to (7). Instead of achieving the TLS solution at the cost of updating and performing either the SVD of the augmented data matrix $\tilde{\mathbf{Z}}(k)$ or eigendecomposition of the matrix $\tilde{\mathbf{R}}(k)$, a more computationally efficient alternative is to employ a gradient descent method, e.g., like complex least mean square (CLMS), to recursively minimise the Rayleigh quotient based cost function in (12), where the gradient of the cost function $J(\mathbf{w}(k))$ is calculated as

$$\begin{aligned} &\nabla J(\mathbf{w}(k)) \\ &= \frac{\partial J(\mathbf{w}(k))}{\partial \mathbf{w}^*(k)} \\ &= \frac{2\tilde{\mathbf{R}}(k)\mathbf{w}(k)\mathbf{w}^H(k)\mathbf{w}(k) - 2\mathbf{w}^H(k)\tilde{\mathbf{R}}(k)\mathbf{w}(k)\mathbf{w}(k)}{(\|\mathbf{w}(k)\|^2)^2} \\ &= \frac{2\|\mathbf{w}(k)\|^2\tilde{\mathbf{Z}}(k)\mathbf{e}(k) - 2\mathbf{w}(k)\mathbf{e}^H(k)\mathbf{e}(k)}{(\|\mathbf{w}(k)\|^2)^2} \\ &= \frac{2\big(\|\mathbf{w}(k)\|^2\tilde{\mathbf{Z}}(k) - \mathbf{w}(k)\mathbf{e}^H(k)\big)\mathbf{e}(k)}{(\|\mathbf{w}(k)\|^2)^2} \end{aligned} \tag{14}$$

Subsequently, a gradient descent weight update can be iteratively performed as

$$\begin{aligned} \mathbf{w}(k+1) &= \mathbf{w}(k) - \frac{\mu}{2}\nabla J(\mathbf{w}(k)) \\ &= \mathbf{w}(k) + \frac{\mu\big(\mathbf{w}(k)\mathbf{e}^H(k) - \|\mathbf{w}(k)\|^2\tilde{\mathbf{Z}}(k)\big)\mathbf{e}(k)}{(\|\mathbf{w}(k)\|^2)^2} \end{aligned} \tag{15}$$

where μ is the step-size. Given an appropriate choice of μ, the global minimum of $J(\mathbf{w}(k))$ can be arrived from any initial point [24], [26], [27]. We shall refer to this proposed algorithm, which is based on the widely linear estimation model over multiple noisy measurements, as the augmented complex total least mean square (ACTLMS). When the length of the sliding window $N = 1$, it degrades into the method proposed in [18]. Accordingly, the instantaneous system frequency is adaptively estimated as [14]

$$f(k) = \frac{1}{2\pi\Delta T}\sin^{-1}\big(\sqrt{\Im^2(w_1(k)) - |w_2(k)|^2}\big) \tag{16}$$

where $\Im(\cdot)$ and $|\cdot|$ respectively denote the imaginary part and the absolute value of a complex-valued number, and $w_1(k)$ and $w_2(k)$ are the first and second elements in $\mathbf{w}(k)$, obtained in an adaptive manner by using (15).

IV. Simulations

To verify the benefits of the proposed ACTLMS algorithm and the associated sliding window technique over its scalar version and its noise-free ACLMS counterpart for adaptive frequency estimation of unbalanced power systems, numerical simulations on unbalanced power systems contaminated by noise and harmonics, as well as real-world measurements, were conducted in the MATLAB programming environment. The system frequency f and the sampling frequency f_s were fixed at $f = 50$ Hz and f_s = 5000 Hz, respectively, and the step-size μ of both algorithms was set to be $\mu = 0.01$ in all the simulations.

We first performed the statistical bias and variance analysis of the considered frequency estimators for unbalanced three-phase power systems with $V_a = 1.1$, $V_b = 0.9$, and $V_c = 1.05$, contaminated by white Gaussian noise with various signal-to-noise ratios (SNRs). Fig. 1(a) and Fig. 1(b) respectively illustrate the estimation bias and variance of the considered frequency estimators against different levels of noise. The results were obtained by averaging 1,000 independent trials. By design, the scalar version of ACTLMS algorithm always exhibited better noise rejection than ACLMS due to its consideration of noise within the underlying widely linear estimation model, especially in heavy noise situations. The performance advantages of using a sliding window technique within ACTLMS algorithm was more pronounced with a larger window length N. The tradeoff between the frequency tracking speed and estimation accuracy provided by the length of the sliding window N can be observed from Fig. 2 where the simulated unbalanced power system experienced higher order harmonic distortion composing of a 10% third harmonic, a 5% fifth harmonic and a 2% seventh harmonic on all the three phases at 0.2 sec. Both the ACTLMS frequency estimators with $N = 1$ and $N = 10$ were initialised at 50.2 Hz. Although a larger number of observations resulted in a slightly slower convergence, they equipped the proposed ACTLMS method with enhanced robustness against higher order harmonics.

We next considered a real-world power system. The three-phase voltages were recorded at a 110/20/10 kV transformer

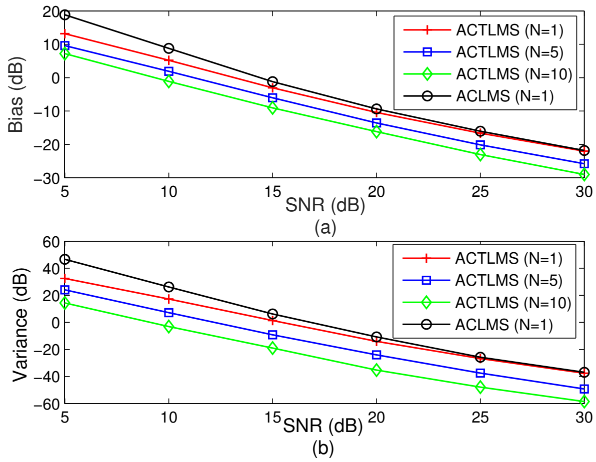

Fig. 1. Statistical evaluation of the considered frequency estimation methods for a noisy unbalanced power system against different SNRs. (a) Bias. (b) Variance.

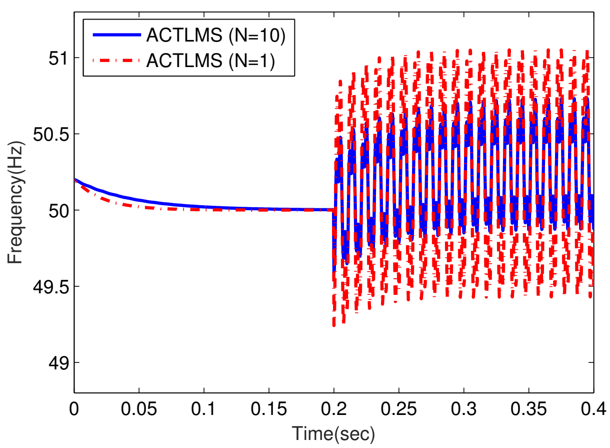

Fig. 2. Performance comparison of the proposed ACTLMS algorithm for different filter lengths, N.

station. The REL 531 numerical line distance protection terminal, produced by ABB Ltd., was installed in the station and was used to monitor changes in the three "phase-ground" voltages on the 20 kV busbars (neutral earthed by 40 Ω resistor). The measured three phase-ground voltages with a system frequency around 50 Hz were normalised with respect to the normal peak voltage values, as shown in Fig. 3(a). The three-phase power system was initially in a balanced condition, at around 0.13 sec, phase $v_b(k)$ experienced an earth fault, causing a 51.6% voltage drop, and a 48.5% and a 23.9% voltage swells in phases $v_a(k)$ and $v_c(k)$, respectively. The three phase-ground voltages were sampled at 1000 Hz by the terminal. The frequency tracking capabilities of the proposed ACTLMS and the ACLMS methods are illustrated in Fig. 3. After a short disturbance during the system transition from a balance to unbalance state, the ACTLMS provided more robust frequency estimates over ACLMS. This advantage, quantified by estimation variance, was more pronounced when multiple

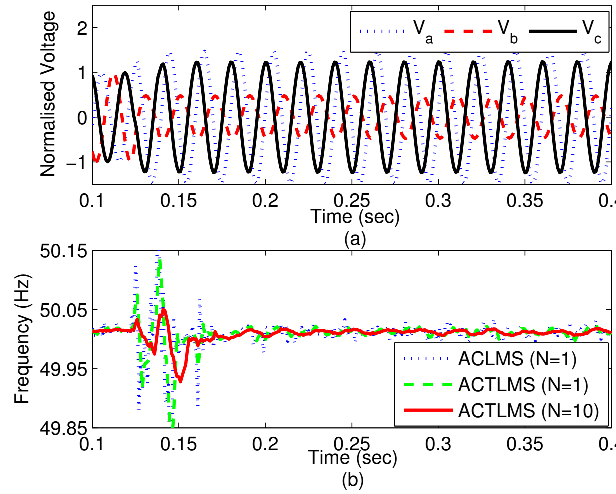

Fig. 3. Frequency estimation using the proposed ACTLMS based algorithms for a real-world unbalanced three-phase power system. (a) The waveform of the three-phase power system. (b) Comparison of frequency estimation results by the considered methods.

TABLE I
ESTIMATION VARIANCES OF THE CONSIDERED ALGORITHMS FOR THE REAL WORLD SYSTEM VOLTAGES.

ACLMS (N=1)	ACTLMS (N=1)	ACTLMS (N=10)
2.7e-003	1.6e-003	3.0e-004

noisy measurements were used underlying the widely linear estimation model of ACTLMS, which is further summarised by Table I.

V. Conclusion

We have introduced a widely linear predictive model for frequency estimation over multiple noisy voltage measurements in unbalanced three-phase power systems. To obtain frequency estimates in an adaptive manner, a gradient descent ACTLMS algorithm has been proposed based on the recursive minimisation of the Rayleigh quotient associated with this model. The sliding window framework enables the proposed ACTLMS frequency estimation method to be less sensitive to measurement noises and higher order harmonic distortion compared to its scalar version and a noise-free ACLMS method. It has also been shown that ACTLMS is inherently equipped with one more degree of freedom which allows to control the tradeoff between tracking speed and estimation accuracy. Simulations on both synthetic and real-world noisy unbalanced power systems support the analysis.

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