# Self-stabilising adaptive three-phase transforms via widely linear modelling

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The Clarke and Park transforms have been standard analysis techniques in three-phase power system analysis for over a century. However, these transforms were originally designed for static electric grids and therefore yield non-ideal oscillatory outputs in dynamic smart grids. Using recent advances in widely linear adaptive filtering algorithms, the authors propose a self-stabilising three-phase transformation which is capable of generating synchronous reference frames regardless of the type of voltage imbalance or frequency deviation.

*Introduction:* The performance of power electronic devices relies upon their ability to synchronise to the grid. This is enabled by synchronous reference frame (SRF) controllers which use the grid voltage and currents as reference operating points [1].

A crucial part of high-performance SRF control systems are three-phase voltage transformations, typically the Clarke and Park transforms. These transformations are necessary in order to solve equations involving time-varying power quantities by casting the original three-phase voltages to a common frame of reference. However, the frame of reference chosen by the Clarke, Park and similar transformations was designed based on static, balanced three-phase grids (equal phase voltages, equal angles between the phase voltages and no deviation of system frequency). Any imbalance between the three-phase voltages or frequency deviations from the nominal frequency of 50 Hz/60 Hz would change the optimal frame-of-reference and therefore, applying the classical Clarke/Park transforms for unbalanced grids yields wrong inputs to the controllers.

To this end, we propose an adaptive Clarke–Park transform which is capable of automatically generating an optimal reference frame for SRF controllers for both balanced and unbalanced systems with varying frequency deviations. The key benefit of the proposed adaptive Clark–Park transform is that it removes the need for present control schemes to be modified for future smart grids, by yielding reference voltages and currents which are structurally identical to the balanced case.

*Problem formulation:* Consider a sampled three-phase voltage measurement vector, $\boldsymbol{s}_k$, at a time-instant $k$, given by

$$\boldsymbol{s}_k = \begin{bmatrix} v_{a,k} \\ v_{b,k} \\ v_{c,k} \end{bmatrix} = \begin{bmatrix} V_a \cos(\omega k + \phi_a) \\ V_b \cos\left(\omega k + \phi_b - \frac{2\pi}{3}\right) \\ V_c \cos\left(\omega k + \phi_c + \frac{2\pi}{3}\right) \end{bmatrix} \tag{1}$$

where $V_a, V_b$ and $V_c$ are the amplitudes of the phase voltages, $\omega = 2\pi f/f_s$ is the fundamental angular frequency, with $f$ being the fundamental power system frequency and $f_s$ is the sampling frequency. The corresponding phase values are denoted by $\phi_a$, $\phi_b$ and $\phi_c$. The Clarke or $\alpha\beta$ transform projects the three-phase signal $\boldsymbol{s}_k$ in (1) onto a 2D subspace, to yield

$$\begin{bmatrix} v_{\alpha,k} \\ v_{\beta,k} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_{a,k} \\ v_{b,k} \\ v_{c,k} \end{bmatrix} \tag{2}$$

where $v_{\alpha,k}$ and $v_{\beta,k}$ are referred to as the $\alpha$ and $\beta$ sequences and can be conveniently represented as a complex variable, in the form

$$s_k \overset{\text{def}}{=} v_{\alpha,k} + \mathrm{j}v_{\beta,k}. \tag{3}$$

It can be shown that the complex-valued Clarke voltage in (3) can be expressed in terms of the positive, $\bar{V}_+$, and negative, $\bar{V}_-$, sequence voltages as

$$s_k = \frac{1}{\sqrt{2}}\left(\bar{V}_+ \mathrm{e}^{\mathrm{j}\omega k} + \bar{V}_-^* \mathrm{e}^{-\mathrm{j}\omega k}\right) \tag{4}$$

where the scalar phasors $\bar{V}_+$ and $\bar{V}_-^*$ are given by

$$\begin{aligned} \bar{V}_+ &= \frac{1}{\sqrt{3}}\left[V_a \mathrm{e}^{\mathrm{j}\phi_a} + V_b \mathrm{e}^{\mathrm{j}\phi_b} + V_c \mathrm{e}^{\mathrm{j}\phi_c}\right] \\ \bar{V}_-^* &= \frac{1}{\sqrt{3}}\left[V_a \mathrm{e}^{-\mathrm{j}\phi_a} + V_b \mathrm{e}^{-\mathrm{j}(\phi_b + (2\pi/3))} + V_c \mathrm{e}^{-\mathrm{j}(\phi_c - (2\pi/3))}\right]. \end{aligned} \tag{5}$$

On the other hand, the Park transform (also known as the $dq$ transform) projects the three-phase signal $\boldsymbol{s}_k$ onto an orthogonal, time-varying frame, which by virtue of rotating at the fundamental frequency $\omega_\circ$, yields stationary outputs. The Park voltages $v_{d,k}, v_{q,k}$ are obtained from the $\alpha\beta$ voltages in (2) using a time-varying transformation [2]

$$\begin{bmatrix} v_{d,k} \\ v_{q,k} \end{bmatrix} = \begin{bmatrix} \cos(\omega_\circ k) & \sin(\omega_\circ k) \\ \sin(\omega_\circ k) & \cos(\omega_\circ k) \end{bmatrix} \begin{bmatrix} v_{\alpha,k} \\ v_{\beta,k} \end{bmatrix} \tag{6}$$

where the orthogonal direct and quadrature components, $v_{d,k}$ and $v_{q,k}$, can be combined into a complex variable $v_k = v_{d,k} + \mathrm{j}v_{q,k}$.

*Adaptive Clarke/Park transform:* In real-world power systems, the three-phase voltages are rarely perfectly balanced and the system frequency is never exactly the fundamental frequency [3]. Therefore, the Clarke and Park transforms do not yield the ideal intended outputs. To this end, we require adaptive transformations, capable of tracking: (i) the voltage imbalances within an 'adaptive' Clarke transform and (ii) the system frequency for an 'adaptive' Park transform.

We now develop the adaptive Clarke and Park transforms by first introducing the widely linear autoregressive (WLAR) model of the $\alpha\beta$ voltage in (4). The WLAR model for the Clarke transform is [4]

$$s_k = h^* s_{k-1} + g^* s_{k-1}^* \tag{7}$$

where the WLAR coefficients $h$ and $g$ contain the information of the system frequency, $\omega$, and the level of imbalance in the system defined through the voltage unbalance factor (VUF) given by $\kappa \overset{\text{def}}{=} \bar{V}_-/\bar{V}_+$ [3].

Comparing the output of the WLAR model in (7) with the actual $\alpha\beta$ voltage in (4) gives the relationships [4]

$$\mathrm{e}^{\mathrm{j}\omega} = h^* + g^*\kappa \quad \text{and} \quad \mathrm{e}^{-\mathrm{j}\omega} = h^* + \frac{g^*}{\kappa^*}. \tag{8}$$

Solving the simultaneous equations in (8) gives the expressions for the system frequency and VUF in the form

$$\mathrm{e}^{\mathrm{j}\omega} = \mathrm{Re}\{h\} + \mathrm{j}\sqrt{\mathrm{Im}^2\{h\} - |g|^2} \tag{9}$$

$$\kappa = \frac{\bar{V}_-}{\bar{V}_+} = \frac{\mathrm{j}}{g^*}\left(\mathrm{Im}\{h\} + \sqrt{\mathrm{Im}^2\{h\} - |g|^2}\right). \tag{10}$$

*Balancing transform:* It was shown in [5] that the VUF, $\kappa$, defined in (10) can be used to eliminate the negative sequence phasor, $\bar{V}_-$, from the $\alpha\beta$ voltage $s_k$, through

$$\begin{aligned} m_k &\overset{\text{def}}{=} \sqrt{2}(s_k - \kappa^* s_k^*) \\ &= \bar{V}_+ \mathrm{e}^{\mathrm{j}\omega k} + \bar{V}_-^* \mathrm{e}^{-\mathrm{j}\omega k} - \frac{\bar{V}_-^*}{\bar{V}_+^*}\left(\bar{V}_+^* \mathrm{e}^{-\mathrm{j}\omega k} + \bar{V}_- \mathrm{e}^{\mathrm{j}\omega k}\right) \end{aligned} \tag{11}$$

$$= \bar{V}_+(1 - |\kappa|^2)\mathrm{e}^{\mathrm{j}\omega k}. \tag{12}$$

*Remark 1:* The voltage $m_k$ in (12) can be regarded as the output of an adaptive Clarke transform as it closely resembles the $\alpha\beta$ voltage for a balanced system.

The method in [5] requires the evaluation of the second-order statistics of the $\alpha\beta$ voltage $s_k$. In this Letter, we show that the WLAR coefficients $h$ and $g$ in (8) are sufficient to obtain the adaptive Clarke voltage in (11), while also catering for an adaptive Park transform. Specifically, using the estimated time-varying values of $\mathrm{e}^{\mathrm{j}\omega_k}$ in (9) and $\kappa_k$ in (10), the proposed adaptive Clarke and Park transforms are defined as

$$m_k = \sqrt{2}(s_k - \kappa_k^* s_k^*) \tag{13a}$$

$$\tilde{m}_k = \mathrm{e}^{-\mathrm{j}\omega_k k} m_k \tag{13b}$$

where $m_k$ is the adaptive $\alpha\beta$ (Clark) voltage while $\tilde{m}_k$ is the adaptive $dq$ (Park) voltage.

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*Algorithm 1:* Adaptive Clarke and Park transforms

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**Input**: Three-phase voltages, $\mathbf{s}_k$
**At each time instant** $k > 0$
1: Obtain Clarke transform: $s_k = \sqrt{2}\mathbf{c}^{\mathrm{H}}\mathbf{s}_k$
2: Update ACLMS weights

$$\begin{aligned} \varepsilon_k &= s_k - (h_{k-1}^* s_{k-1} + g_{k-1}^* s_{k-1}^*) \\ h_k &= h_{k-1} + \mu s_{k-1}\varepsilon_k^* \\ g_k &= g_{k-1} + \mu s_{k-1}^* \varepsilon_k^* \end{aligned}$$

3: Use (11) and (12) to obtain $\kappa_k$ ad $e^{j\omega_k}$.
4: Adaptive Clarke transform: $m_k = s_k - \kappa_k^* s_k^*$
5: Adaptive Park transform: $\tilde{m}_k = e^{-j\omega_k k} m_k$

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Adaptive Clarke and Park transforms can therefore be implemented using (13a) and (13b), with a suitable adaptive algorithm (e.g. least mean square, Kalman filter) employed to track the VUF, $\kappa_k$, and system frequency $\omega_k$. For simplicity, we present the adaptive Clarke/Park transform in Algorithm 1 which utilises the augmented complex least mean square (ACLMS) to estimate the WLAR coefficients $h$ and $g$.

*Remark 2:* The proposed adaptive Clarke and Park transforms are now able to yield non-oscillatory outputs, regardless of the system frequency or level of imbalance, as shown in Table 1. This crucially enables standard techniques designed for nominal conditions to be applied to a general set of unbalanced voltages, resulting in a bias-free operation.

**Table 1:** Output of the various three-phase transformations

| Transforms | System condition | |
|---|---|---|
| | Balanced | Unbalanced |
| Clarke [6] | $\bar{V}_+ e^{j\omega k}$ | $\bar{V}_+ e^{j\omega k} + \bar{V}_- e^{-j\omega k}$ |
| Balancing [7] | $\bar{V}_+ e^{j\omega k}$ | $(1-\lvert\kappa\rvert^2)\bar{V}_+ e^{j\omega k}$ |
| Park [3] | $\bar{V}_+$ | $\bar{V}_+ e^{j(\omega-\omega_\circ)k} + \bar{V}_-^* e^{-j(\omega+\omega_\circ)k}$ |
| This work | $\bar{V}_+$ | $(1-\lvert\kappa\rvert^2)\bar{V}_+$ |

*Simulations:* The performance of the ACLMS-based adaptive Clarke/Park transform in Algorithm 1 was tested on a three-phase signal under nominal (balanced voltages and frequency, $f = 50$ Hz) and off-nominal conditions (unbalanced voltages with $f \neq 50$ Hz), over a period of 4 s. The three-phase sinusoidal voltages were generated with phasors (amplitude and phase)

$$\bar{V}_a = R, \quad \bar{V}_b = \frac{1}{2}(-R - j\sqrt{3}), \quad \bar{V}_c = \frac{1}{2}(-R + j\sqrt{3})$$

where $R = 1$ reflected a balanced operating condition while $R = 0.5$ corresponded to a type D voltage sag [6]. The proposed algorithm applies to any type of voltage imbalance but for illustration we have chosen to show its operation under a type D sag.

The sampling frequency was set to $f_s = 500$ Hz while the step-size for the ACLMS was chosen arbitrarily as $\mu = 0.05$ since the claims in these simulations only serve to demonstrate that the proposed adaptive transform is able to yield the correct non-oscillatory outputs regardless of the operating condition of the three-phase voltages.

In this Letter, the three-phase signal was configured to operate under a balanced condition for the first 2 s and was subjected to both a type D sag and a change of the system frequency from the nominal 50–53 Hz. Fig. 1 shows the direct and quadrature Park voltages, $v_{d,k}$ and $v_{q,k}$, from both the original Park transform defined in (6) and the adaptive Park transform in Algorithm 1. Observe that the original Park transform yielded an oscillating output when the system frequency was at an off-nominal value with a type D voltage sag, while the adaptive Park transform was able to converge to a stationary (non-oscillatory) phasor after the frequency change point.

The circularity diagram (a scatter plot of $\alpha$ versus $\beta$ voltages) in Fig. 2 reveals that the output of the adaptive Clarke transform in Algorithm 1 (Step 4) was able to change the profile of the unbalanced voltage into a profile corresponding to a balanced system. See Table 1 which lists the output of the Park transform and the proposed adaptive Clark/Park transform for both balanced and unbalanced conditions.

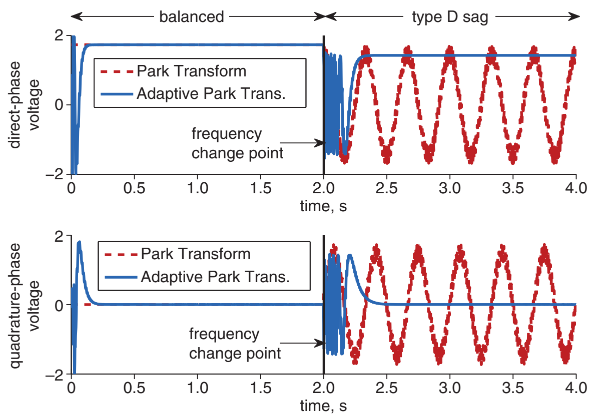


**Fig. 1** *Performance of fixed Park transform in (6) and adaptive Park transform in Algorithm 1 under a voltage imbalance and frequency deviation*

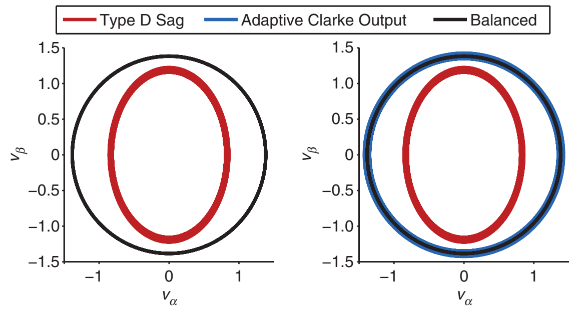


**Fig. 2** *Circular profile for αβ voltage indicates balanced three-phase system whereas non-circular trajectory indicates a voltage imbalance*

*Conclusion:* An adaptive three-phase transformation for dynamic smart grids has been proposed. This algorithm has been developed based on the widely linear autoregressive modelling of the $\alpha\beta$ voltage. This has allowed a wide class of widely linear adaptive filters to be applied to yield a self-stabilising three-phase transform which we refer to as the adaptive Clarke/Park transform. Simulations over synthetic data show the effectiveness of the scheme over traditional static transforms, and its promise in the future dynamically changing smart grid.


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One or more of the Figures in this Letter are available in colour online.

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